Adv. Theor. Appl. Mech., Vol. 1, 2008, no. 3, 173 - 186

Semi Analytical Study of a Flat Surface Uniformly Accessible in a Permanent Laminar Flow and a Newtonian Fluid

Nabil KABOUCHE

Rectorat de l'Université El Hadj Lakhdar Batna Avenue chahid boukhlouf
Mohamed El hadi 05000 Batna
nkabouche@yahoo.fr

Azzedine NAHOUI

Rectorat de l'Université El Hadj Lakhdar Batna Avenue chahid boukhlouf
Mohamed El hadi 05000 Batna
enfantramzi@yahoo.fr

Abstract

The authors calculate the profile of a rectangular pipe by imposing the density $J_{(x)}$ of the limit flow of diffusion on a reactive surface placed at the centre of the pipe to fulfill the law $J_{(x)} = J_0 x^p$ (where J_0 is an independent factor of the co-ordinates and p a numerical constant). x is the co-ordinate in the flow direction. They consider the flow in the pipe of an Ostwald fluid and link the allowed values for p to the behavior index n of the fluid. The particular case of a surface uniformly accessible (p = 0) of a Newtonian fluid (n = 1) is studied by integrating the equation for mass transfer by approaching the transversal component of the speed V^* by polynomials in the interval $[0, y_w]$. As the Sherwood number should be independent of the x co-ordinate (accessibility hypothesis), they come to show, using the error function properties, that the average Sherwood number on the central material plan is written as $Sher = a \cdot Pem^{1/3}$ where a is a constant which is independent of the Reynolds number. The calculations treat the mass transfer case and they are transposable to the heat transfer by substituting the Sherwood number by that of Nusselt, and the mass fraction of the conveyed substance by temperature.

Keywords: Fluid mechanics, mass transfer, uniformly accessible surface

Introduction:

The term *uniformly accessible surface* from the point of view of diffusion seems to have been proposed by D.A. Frank-Kamenetskii **[1]** in the late thirties to declare that the diffusion flow density presents the same value at every point of a reactive surface. By using the "quasi-stationary" approximation he shows, for example, that a reactive surface of a body having any geometrical shape, placed in a large enclosure can be uniformly accessed provided it is smooth, without pores and that the reagent is diluted in a large amount of inert gas. The surfaces having this remarkable property are rare. The rotating disk, famous in electrochemistry **[2]** and some axisymmetric paraboloids **[4, 5, 6]** do possess this property. From the point of view of heat transfer, these surfaces are those which offer the least resistance to conduction and thus allow energy savings.

Starting from the approximations of the boundary layer, the authors study experimentally and numerically the influences of the natural convention or those of the rotary convection and /or those of the appearance factor on the accessibility of the surface immersed in a Newtonian or Ostwaldien fluid in external flows. It appears, for example, from the theoretical and the experimental studies of R.Bachrun et al that the presence of a natural convection superimposed to the flow generated by the rotation of a revolution surface tends to destroy the surface uniform accessibility contrarily to the superimposition of a forced flow parallel to the surface axis of rotation.

In this study we consider a flattened pipe of an indefinite length in the *x-axis* direction, of a rectangular section and of which the dimension along the *y-axis* is small compared to that along the z-axis, but larger than the thickness of the hydrodynamic boundary layers (Figure 1). Let us assume that an insulated Ostwald fluid, with constant physical properties, flows in the pipe as a steady streamline flow. And let us put in the centre of the pipe a reactive surface *S* with a very small thickness in order not to disturb the flow. We ask the question whether it is possible to make the surface uniformly accessible by giving the two walls of the pipe a particular profile. More generally, it is to know if, by changing the profile of the pipe, given the fluid behavior law, it is possible to impose on the flux density limit dissemination a law of variation with the co-ordinate x, of the form $J_{(x)} = J_0 x^p$ (Where J_0 is a factor independent from the co-ordinates and p a numerical constant).

We are studying the case of a uniformly accessible surface (p = 0) when it is the case of a Newtonian fluid (n = 1). We suggest a correlation in the form $Sher = a. Pem^{1/3}$ where *Sher* is the Sherwood average number on the central material plan and a a constant related to the geometry of the channel i.e. its length and its height of entrance. The calculations concern the case of material transfer but are also transposable to the transfer of heat by replacing the Sherwood number by that of Nusselt and the mass fraction of the transported substance by temperature.

NOMENCLATURE

h : half-height of the entrance section

h* : Dimensionless half-height of the entrance, =h / L
L : Channel length
D : Reagent diffusion coefficient
P : Fluid index behavior
Sc : Schmidt number , S_c= ν/D
Scher : Sherwood average number on the central material plane
Pem : Peclet mass number, = Sc.Re
Re : Reynolds number ,= (U_e . h)/ ν
*x** : Dimensionless co-ordinate following ***x*** **, =** ***x/L***
x : Cartesian co-ordinate defined in Figure 1
y : Cartesian co-ordinate defined in Figure 1
*y** : Dimensionless co-ordinate following y, = *y/L*
*V**: Dimensionless speed component following y, = V/U_e
V: Speed Component following y
U : Speed Component following *x*
*U** : Dimensionless speed component following *x*, = U/U_e
Ue : Velocity of the fluid at the channel entrance
C : Reagent concentration
*C** : Dimensionless reagent concentration, $C^* = \dfrac{C - C_e}{C_{pa} - C_e}$
C_e : Reagent concentration at the channel entrance
C_{pa} : Reagent concentration on the flat wall (Ox, Oz)

Greek Letters

δ : Boundary layer thickness.
ε : Coordinate when *V** = *V** max
ν : kinematic viscosity.

Lower indexes

e : Entrance
pa : Central material plane
w : When the co-ordinate is on the hyperbolic wall
max : maximum value
∞: Outside the boundary layer

Exponents

* : Dimensionless Values
m : Numerical exponent

Mathematical problem formulation and solving

The density of the limit flux of diffusion on the reactive surface is given by the expression:

$$J_{(x)} = D\left(\frac{\partial C}{\partial y}\right)_{y=0} \qquad (1)$$

Where D is the reagent diffusion coefficient and C is the concentration of the reagent, solution of the following differential system:

$$U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (2)$$

$$y=\infty, C=C_\infty=Cte \quad (3)$$
$$y=0, C=0 \quad (4)$$

U and V are the components following the x and y co-ordinates of the fluid relative velocity from the reactive surface. Let us clarify that the origin o of the repository (o, xyz) is placed in the middle of the edge of attack of the surface ; x is counted in the direction of the flow.

For the Schmidt numbers $Sc=\nu/D$, bigger than of the unit as it is usually the case in liquids, when U is close to the surface it can be written:

$$U \approx S_0 f_{(x)} y \quad (5)$$

Where S_0 is a coefficient that is independent from the co-ordinates, $f(x)$ a function of x and ν is a kinematic viscosity of the fluid.

A very general solution of the system (1) – (5) for the two-dimensional flow that concerns us was calculated by Suwono **[5.6]** is the expression of

$$J_{(x)}=\frac{3^{-1/3}}{\Gamma(4/3)}C_\infty D\left(\frac{S_0}{2}\right)^{1/2}\left(\frac{4}{3\phi}\right)^{1/3}\xi_{(x)}^{-1/3}f_{(x)}^{1/2} \quad (6)$$

With

$$\phi=2\sqrt{S_0} \quad (7)$$

$$\xi_{(x)}=\int_0^x f_{(x)}^{1/2}dx \quad (8)$$

We see that $J_{(x)}$ depends of x solely through the function $f(x)$. By solving the equation, comes

$$f_{(x)}=\frac{2}{3}\frac{x^{3p+1}}{p+1}J_0^3\left\{\frac{3^{-1/3}}{\Gamma(4/3)}C_\infty D\left(\frac{S_0}{2}\right)^{1/2}\left(\frac{4}{3\phi}\right)^{1/3}\right\}^{-3} \quad (9)$$

This is the condition that the function must meet in order for the limit flux density on the surface *S* is to be in the form $J_{(x)} = J_0 x^p$

We know **[7]** that there are affine solutions for the equation of motion of a fluid Ostwald in the boundary layer when hydrodynamics out of this layer, speed is of the form

$$Ue = U_1 X^m \tag{10}$$

Where U_1 is a coefficient that is independent from the co-ordinates and *m* a numerical exponent. If we take the first term of development of *y* of the component *U* of the fluid velocity in the boundary layer proposed by (7) and if we identify this term with the expression (5), we find, considering (9) and by noticing that *m* is bound to the fluid index of behaviour, the expression:

$$m = p(n+1) + \frac{n+2}{3} \tag{11}$$

We shall obtain now the profile of the walls of the pipe by realizing the surfaces of current of a perfect fluid which would pass by in pipe so as to have in the centre the speed given by (11). We find in this way:

$$r^{m+1} \sin[\theta(m+1)] = A = Cte \tag{12}$$

.Where r and θ are the polar co-ordinates of a current point *m* of the wall, Figure1.

Conclusion:

The equation (12) defines curves of hyperbolic shape that have the asymptotes:

$$\theta_1 = 0 \tag{13}$$

And

$$\theta_2 = \frac{\pi}{m+1} \tag{14}$$

Only values

$$\frac{\pi}{2} \le \theta_2 \le \pi \tag{15}$$

present physical interest **[7],** where you can deduct inequality

$$1 \geq m \geq 0 \tag{16}$$

In addition, the approximations of the boundary layer in a laminar Ostwaldien flow are valid for:

0 <n <2 **[8]** (17)

From (12) and taking into account (16) and (17), one finds, for example, that values allowed for p range from:

$+\frac{1}{3}$ et $-\frac{2}{3}$ pour $n = 0$; et $-\frac{1}{9}$ et $-\frac{5}{9}$ pour $n = 0.5$

0 et $-\frac{1}{2}$ pour $n = 1$; $-\frac{1}{15}$ et $-\frac{7}{15}$ pour $n = 1.5$

$-\frac{1}{9}$ et $-\frac{4}{9}$ pour $n = 2$

The value $p = 0$ corresponds to a surface S uniformly accessible and can be obtained both with a Newton fluid ($n = 1$) as with a pseudo-plastic fluid (n <1). The profile of the pipe corresponding to different values of n is then given by the equation:

$$p^{(n+5)/3} \sin\left(\theta \frac{n+5}{3}\right) = A = Cte \tag{18}$$

For the Newtonian fluid the pipe profile is a hyperbole equilateral of the following equation:

$$r^2 \sin 2\theta = A = Cte \tag{19}$$

For a rectilinear pipe $(\theta_2 = \pi \text{ et } m = 0)$ and a Newtonian fluid (n=1), we find $p = -\frac{1}{2}$. This result is in accordance with the data of the literature **[2, 9]** because the flow over the reactive surface is then a “Blasius flow”.

Study of a flat surface uniformly accessible (*p* = 0) in the case of a Newtonian fluid (*n* = 1):

The case of a flat surface immersed in a laminar and permanent flow of a Newtonian fluid is particularly interesting because of the applications it is likely to know, especially in the practice of deposits. For example, the manufacturer of photocells must obtain uniform layers for a better return; to overcome the phenomena of aberration, the manufacturer of mirrors (especially high resolution used in astronomy) is required to obtain uniform deposits.

Stating the problem:

Let us consider as schematized on figure 2, a convergent possessing two walls the profiles of which are given by the equation $r^2 \sin 2\theta = A = Cte$ and a Cartesian mark (o, x, y, z). The plan (o, x, z), supposed material, is a symmetry plan in consideration of both hyperbolic walls and we suppose that the width following the direction oz is very big in front of the length l and the height $2h$ of the entrance section of the convergent. C_e the concentration of both hyperbolic sides and C_{pa} the concentration of the reagent on the plan (ox, oz), also assumed constant. Let us force a Newtonian fluid to flow under laminar and permanent regime in the pipe following ox. We agree that the physical properties of the fluid are constant and that the viscous dissipation, the work of pressure forces and the radiation are negligible.When the surface is uniformly accessible from the point of view of mass transfer and considering the simplifying assumptions, the mass transfer equation is written under the following dimensionless form:

Equation of mass transfer in the dimensionless form:

$$V^* \frac{\partial C^*}{\partial y^*} = \frac{1}{Pem} \frac{\partial^2 C^*}{\partial y^{*2}} \qquad (20)$$

with the boundary conditions:

$$y^* = 0, \quad C^* = 1 \qquad (21)$$

$$y^* = y_w^*, \; C^* = 0 \qquad (22)$$

A double integration of the equation of mass transfer gives us:

$$C^*(y^*) = C1 + C2 \int_0^y F(t)\, dt \qquad (23)$$

with

$$F(t) = \exp\left[Pem \int_0^t V(z)\, dz \right]$$

Where $C1$ and $C2$ are constants.

Taking into account the requirement to limit $C^* = 0$, it comes:

$$C(y^*) = 1 + C2\int_0^{y^*} F(t)\, d(t) \qquad (24)$$

The second condition to limit $C^* = 1$, leads to:

$$C2\int_0^{yw} F(t)\, d(t) = -1 \qquad (25)$$

To determine the constant $C2$, we will cut the interval $[0\,; y_w]$ into three intervals:

$[0\,; y_w] = [0\,; \delta] \cup [\delta; \varepsilon] \cup [\varepsilon; y_w]$

δ : Boundary layer thickness.
ε : Co-ordinate when $V^* = V^*$ max .
$y\,{*}_{\mathrm{w}}$: When the co-ordinate is on the hyperbolic wall.

The previous relationship can be written as:

$C2\ (I_1 + I_2 + I_3) = -1$ (26)

With

$$I_1 = \int_0^{\delta} F(t) d(t) \quad ;\ I_2 = \int_{\delta}^{\varepsilon} F(t) d(t) \quad ;\ I_3 = \int_{\delta}^{y_w} F(t) d(t)$$

The curves representing the normal component V^* of the speed depending on y^* obtained by **[10]**, Figure 3, shows that in the interval $[0; y_w]$, the normal component V^* can be approximated by:

In the interval $[0\,; \delta]$,

$V^*(y^*) \approx \alpha_1 y^{*2} \quad avec \quad \alpha_1 \succ 0$

In the interval $[\delta; \varepsilon]$,

$V(y) \approx -\alpha_2 y + \alpha_3 \quad avec \quad \alpha_2 \succ 0$

In the interval $[\varepsilon; y_w]$,

$$V(y) \approx -\alpha_4 y + \alpha_5 \quad avec \quad \alpha_4 \succ 0$$

Calculating I_1

Suppose that in the boundary layer,

$$V^*(y^*) \approx \alpha_1 y^{*2} \quad avec \quad \alpha_1 \succ 0$$

It comes:

$$F(t) = \exp\left[\frac{-Pem\alpha_1}{3} t^3\right] = \exp\left[-(a_1 t)^3\right]$$

With:

$$a_1 = \left(\frac{Pem\alpha_1}{3}\right)^{1/3}$$

As a following:

$$I_1 = \frac{1}{3} a_1^{-1} \gamma \left[1/3; (\delta\, a_1)^3\right] \tag{27}$$

With

$$\gamma\left[1/3\ ; (\delta a_1)^3\right] = \int_0^R U^{-2/3} \exp(U)\, dU$$

$$U = (a_1 t)^3 \ . \ R = \left[(\delta\, a_1)^3\right]$$

Calculating I_2

Suppose that $V^*(y^*) \approx -\alpha_4 y^* + \alpha_5 \quad with \quad \alpha_4 \succ 0$

The integration of $F(t)$ in the interval $[\delta; \varepsilon]$, given the properties of integration of the error function exp ($-x^2$), provides:

$$I_2 = \frac{\sqrt{\pi}}{2} a_2 \exp\left[\left(\frac{-a_2\alpha_3}{\alpha_2}\right)^2 \left\{erf\ a_2\left(\varepsilon - \frac{\alpha_3}{\alpha_2}\right) + erf\ a_2\left(\delta - \frac{\alpha_3}{\alpha_2}\right)\right\}\right] \tag{28}$$

where

$$erf(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} \exp(-t^2)dt$$

$erf(\eta)$: Is the error function

And

$$a_2 = \left(\frac{Pem\alpha_2}{2} \right)^{1/2}$$

Calculating I_3

Let us put that:

$$V(y) \approx -\alpha_4 y^* + \alpha_5 \quad avec \quad \alpha_4 \succ 0$$

We get then:

$$I_3 = \frac{\sqrt{\pi}}{2} a_3 \exp\left[\left(\frac{a_3\alpha_5}{\alpha_4} \right) \left\{ erf\, a_3 \left(y_w^* - \frac{\alpha_5}{\alpha_4} \right) + erf\left(-a_2\right) \left(\varepsilon - \frac{\alpha_5}{\alpha_4} \right) \right\} \right] \qquad (29)$$

With

$$a_3 = \left(\frac{Pem\alpha_4}{2} \right)$$

Knowing I_1, I_2 and I_3 and taking into account the relationship $C2$ (I_1 + I_2 +I_3) = -1

It comes:

$$C_2 = \frac{-1}{1/3\, a_1^{-1}\, \gamma\left(1/3, (\delta\, a_1)^3\right) + \frac{\sqrt{\pi}}{2} a_2^{-1} \exp\left(a_2 y_w^*\right)^3 erf\left[a_2\left(y_w^* - \delta\right)\right]}$$

We deduct:

$$Sher = -\left(\frac{\partial C^*}{\partial y^*} \right)_{y^*=0}$$

It comes:

$$Sher = \frac{-1}{1/3\, a_1^{-1}\, \gamma\left[1/3, (\delta\, a_1)^3\right] + \frac{\sqrt{\pi}}{2} a_2^{-1} \exp\left[\left(a_2 y_w^*\right)^3 erf\left[a_2\left(y_w^* - \delta\right)\right]\right]}$$

When the surface is uniformly accessible, the Sherwood number is independent from x^*. As a result, *Sher* must be independent from y_w^*, which is a function of abscissa x^*

To be so, the second term in the denominator should be negligible. So:

$$Sher = \frac{-1}{\gamma\left[1/3, (\delta\, a_1)^3\right]} a_1$$

Assuming that $(\delta\ a_1)^3$ is very big, we have:

$$Sher \approx \ \ Cte\ \mathrm{a}\ _1^1 \ \ \approx Cte\,\alpha_1 Pem^{1/3}$$

The curves representing the variation of the velocity component V^* in function of y^* (Figure 3), obtained by (10) shows that α_1 was totally independent from the Reynolds number and depends only on h^*.

So:

$$Sher \approx Cte\, Pem^{1/3}$$

According to the results of the numerical analysis of **[10]**.

Conclusion: Now we can assert that a flat surface, situated in the centre of a pipe of which the walls are hyperbolic and crossed by a Newtonian fluid of constant physical properties in permanent laminar flow is uniformly accessible from the point of view of mass transfer and that the Sherwood average number on the central material plane is written $Sher = a.\ Pem^{1/3}$. The constant a is related to the geometry of the channel i.e. its length and its entrance height. The calculations concern the case of mass transfer but are also transposable to the transfer of heat by replacing the Sherwood number by that of Nusselt and the mass fraction of the transported substance by temperature.

Bibliography

[1] A. Ali Cherif, Ch. R. R. Raminosoa, M. Rakotomalala, A. Daif et M. Daguent, Contrôle hydrodynamique des couches limites thermiques en convection mixte autour d'ellipsoïdes aplatis axisymétriques, Int.J. Heat and Mass Transfer, Vol n° 3, 657-663 (1997).

[2] B.Rachmat and Daguenet ,Laminar boundary layer low near a rotating axisymmetric electrode of arbitrary shape in a power-law fluid, J. Electroanal. Chem., 124, 53 (1981).

[3] B.Rachmat, A. Suwono and Daguenet, Control o the growth o the diffusional boundary layer on a rotating axisymmetric electrod o arbitrary shape in an Ostwaldian fluid, Electrochim. Acta, 25, 1561-1570 (1980).

[4] D.A. Frank-Kamenetskiy, Difusion and Heat Exchange in Chemical Kineetics Princeton University Press (1955).

[5] H. Schlichting, Boundary Layer Theory, Academic Press, New-york1968)

[6] H. SCHLICHTING, Boundary Layer Theory, McGraw Hill, New York, 1968

[7] N. Kabouche, Cheikh Mbow, M. Daguenet Etude Numérique d'une surface plane uniformément accessible dans un écoulement permanent et laminaire d'un fluide newtonien Int. J. of Thermal Sci. 455-469 (2002)

[8] S.Y. LEE et W.F AMES A.I.Ch.E.J.12 ,1966,p.700.

[9] V.G. Levich, Physicochemical Hydrodynamics, trad.angl., Prentice-Hall, New-work(1962).

[10] W.R. SCHOWALER, A.I.Ch.E.J.6 ,1960,p.24.

Received: March 3, 2008

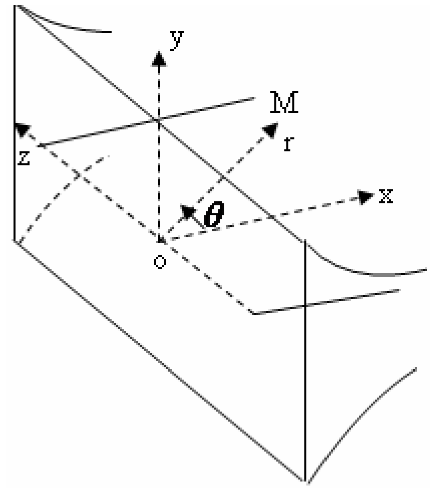
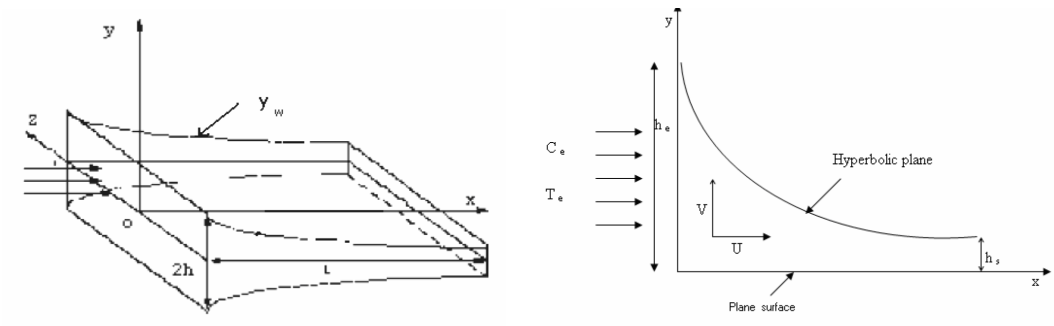


Figure 1: representation of a point *M* and co-ordinated *r* and *θ*



(a) **(b)**

Figure 2: Schematic representations: (a) of the pipe and of the co-ordinates; (b) of the longitudinal section of the pipe.

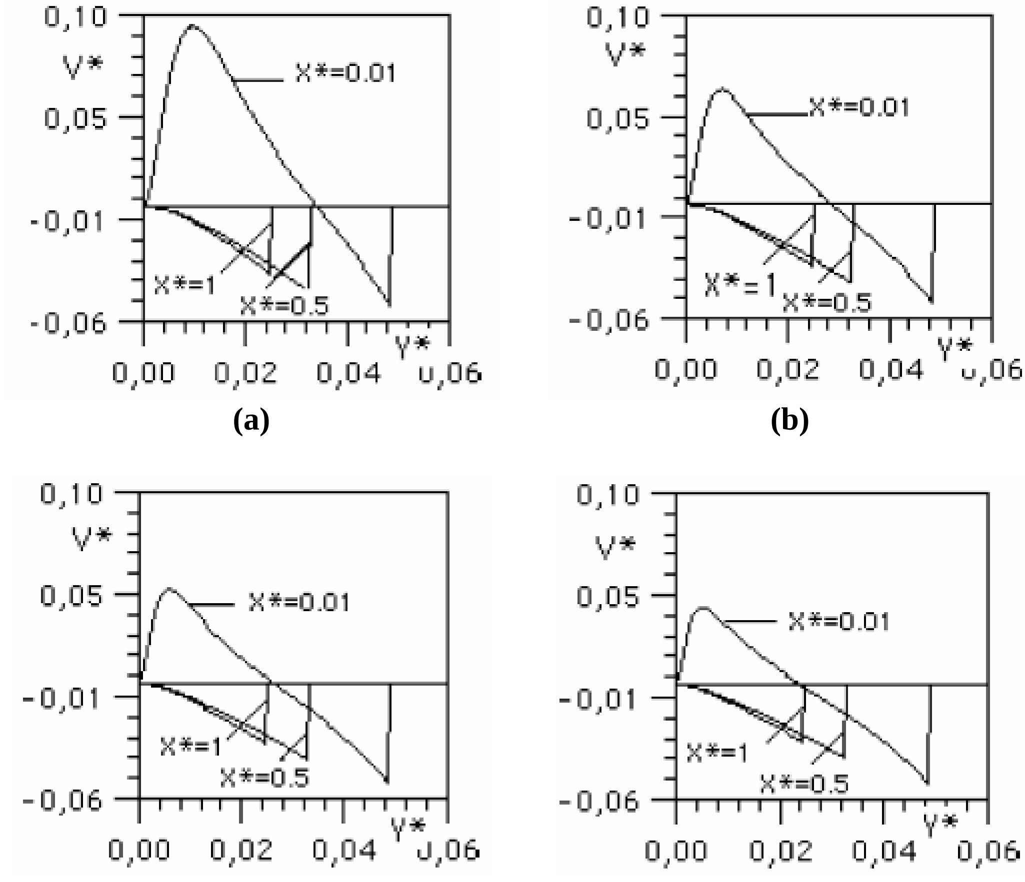


Figure3 : Variations of the dimensionless transversal velocity *V** versus, the dimensionless normal co-ordinate *y** for *h**=1/20, different values of *x** and Reynolds *Re* : (a) *Re*=100 ; (b) *Re*=300 ; (c) *Re*=500 ; (d) *Re*=800