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Control of the Laminar Boundary Layer around a Profile by Suction

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Abstract

A laminar and two-dimensional incompressible boundary layer around a profile and its control using suction is studied numerically. The study is based on the Prandtl boundary layer model. Using the method of finite differences and the Crank-Nicolson scheme. The study concerns the flat plate case and the 0012 NACA profile. The velocity distribution, the boundary layer thickness and the friction coefficient distribution are determined and presented with and without control. The application of the control technique, has demonstrated its positive effect on the detachment point and the friction coefficient. The control procedure is applied for different porosity lengths, speeds and angles of suction

Keywords: boundary layer, laminar, two-dimensional, incompressible, profile control, suction, and friction coefficient

1 Introduction

The present work is to study the laminar boundary layer developed by two-dimensional incompressible flow of air around profiles airplane wing or vane compressor or turbine blades from the equations of the laminar boundary layer told Prandtl's equations and its control by suction. The equations governing this kind of problem, differential equations are nonlinear partial differential. Their solution can be only numerically. The boundary layer is the thin area in contact with the wall of the profile, is the seat of the phenomena causing viscous drag of friction[**1**] with the several recent studies estimate that about half of the total drag. Any reduction of the latter would result in an increase in performance or a reduction in energy consumption knowing that the fossil fuels are scarce by the day on the one hand; on the other hand, the excessive consumption of these reserves is constantly following increasing international demand [**2,3,4**]. One way to achieve this goal is to monitor the boundary layer pushing its point of detachment towards the trailing edge by applying suction [**5, 6**] in order to expand its area laminar. The study concerns the flat plate and NACA 0012 profile.

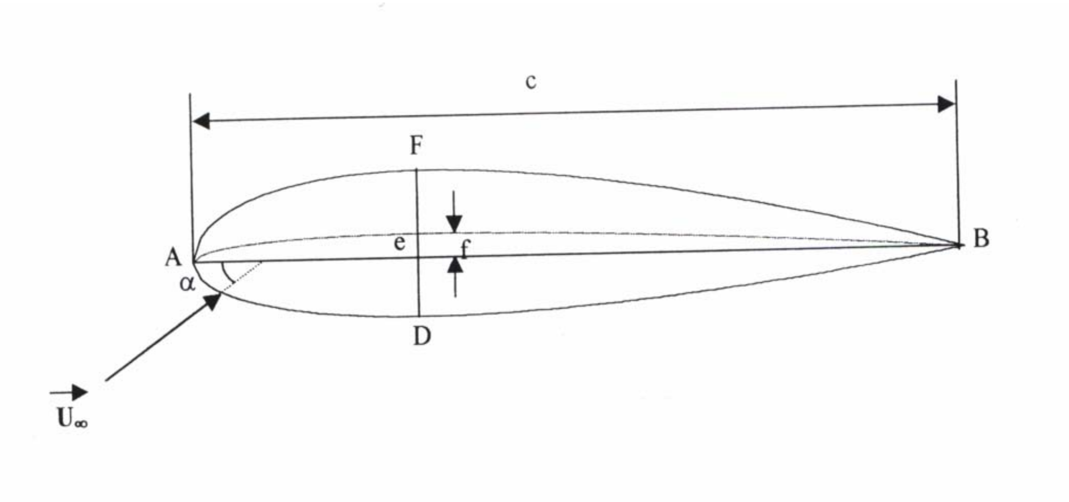

Fig. 1: Geometrical characteristics of a two-dimensional profile.

Nomenclature

x, y	:	Cartesian co-ordinate
s	:	Curvilinear co-ordinate.
η	:	Dimensionless co-ordinate
ξ	:	Dimensionless co-ordinate

∞	:	Outside the boundary layer
υ	:	kinematic viscosity
τ	:	Tangential constraint.
α	:	Angle of attack.
θ	:	Angles of suction
β	:	Dimensionless parameter $\beta(s) = (s.du_e)/(u_e(s).ds)$
p	:	Static pressure.
Cp	:	Coefficient of pressure $Cp = (p - p_\infty) \Big/ \left(\frac{1}{2} (\rho U_\infty^2) \right)$.
u	:	Speed Component following x
v	:	Speed Component following y
V	:	Dimensionless speed component following y
δ	:	Boundary layer thickness
f	:	Stream function
f'	:	Dimensionless speed component following x
Re	:	Reynolds number
$U\infty$	:	Velocity Outside the boundary layer
M∞	:	Mach number
vo /U∞	:	Velocity of suction
Xp	:	Porosity lengths

2. Mathematical modelling

2.1 Modelling of the boundary layer

The problem of a laminar boundary layer dynamic and two-dimensional incompressible is governed by **[1,7,8,9]**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g_x \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g_y \tag{3}$$

2.1.1 Assumptions of the laminar boundary layer

The thickness of the boundary layer is small enough rope to the profile (δ << c) The component of the longitudinal velocity is greater than the transverse component ($v \ll u$)
The transversal pressure gradient is neglected ($(\partial p/\partial y) = 0$)
The forces of weight are neglected.
The gradient of the longitudinal velocity in the transverse direction is much higher than the gradient speed transverse $(\partial v/\partial y) \ll (\partial u/\partial y)$
By introducing these assumptions, we arrive at a scale model of Prandtl follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} \quad (4)$$

At the entrance

$$u(x = 0.y) = u(y) \quad (5)$$

$$v(x = 0.y) = v(y) \quad (6)$$

$$u(x,0) = 0 \quad (7)$$

$$v(x,0) = 0 \quad (8)$$

$$u(x \to \infty) = Ue(x) \quad (9)$$

The length of the laminar boundary layer with its endpoint point where detachment or the boundary layer changes in nature and becomes turbulent, is defined according to the classical theory by

$$\frac{\partial u}{\partial y} = 0 \quad (10)$$

2.2 Boundary layer on the flat plate

The model is reduced to **[1, 7, 8, 9]**:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \tag{11}$$

Among the results of this study, we quote:
The thickness of the boundary layer is given by

$$\delta(x)=\frac{4.9\cdot x}{\sqrt{R_{ex}}} \tag{12}$$

Or the Reynolds number is given by

$$R_{ex}=\frac{U_\infty\cdot x}{\upsilon} \tag{13}$$

The local friction coefficient is given by

$$C_f(x)=\frac{0.664}{\sqrt{R_{ex}}} \tag{14}$$

2.3 On the boundary layer profiles NACA

2.3.1 Equations of the boundary layer

2.3.1.1 Change landmark

We propose to study the boundary layer in curvilinear coordinates (s, y); with the coordinated who represents the surface contour profile and perpendicular to the coordinated s. First, we must make the mathematical model no dimensional by making the following changes **[7]**

$$\xi=\frac{s}{L} \tag{15}$$

$$\eta=y\sqrt{\frac{U_e(s)}{\upsilon\cdot s}} \tag{16}$$

$$f'(\xi,\eta)=\frac{\partial f}{\partial\eta}=\frac{u}{Ue} \tag{17}$$

$$V(\xi,\eta)=\frac{v}{U\ (s)}\sqrt{\frac{s\cdot U_e(s)}{\upsilon}} \tag{18}$$

The mathematical model takes the following form [2]

$$\xi\frac{\partial f'}{\partial\xi}+\beta f'+\frac{\eta}{2}(\beta-1)\frac{\partial f'}{\partial\eta}+\frac{\partial V}{\partial\eta}=0 \qquad (19)$$

$$f'\xi\frac{\partial f'}{\partial\xi}+\bar{V}\frac{\partial f'}{\partial\eta}=\left[1-f'^2\right]\beta+\frac{\partial^2 f'}{\partial\eta^2} \qquad (20)$$

The parameter β depends on the speed distribution potential beyond the boundary layer, with:

$$\bar{V}=V+\frac{1}{2}\eta f'(\beta-1) \qquad (21)$$

$$f'(\xi,0)=0 \qquad (22)$$

$$V(\xi,0)=0 \qquad (23)$$

$$f'(\xi,\eta\to\infty)=1 \qquad (24)$$

At the entrance (ξ = 0) is arbitrarily chosen, away from the edge, so the mathematical model is reduced to the equation Falkner-Skan **[7]**

$$\frac{\partial^3 f}{\partial\eta^3}+\beta_i\left(1-\left(\frac{\partial f}{\partial\eta}\right)^2\right)+\left(\frac{\beta_i+1}{2}\right)f'\frac{\partial^2 f}{\partial\eta^2}=0 \qquad (25)$$

With the boundary conditions:

$$f'(0)=f(0)=0$$

$$f'(\eta\to\infty)=1$$

$\frac{\partial f'}{\partial\eta}=0$ (Point of detachment)

2.3.1.2 Numerical resolution of the equations of the laminar boundary layer

Typically, solving non-linear differential equations governing physical problems appealed to the appropriate numerical methods. In this case, previous studies show that finite difference is more suitable [2] with the use of the scheme CRANCK - NICOLSON. Thus the transformation of differential equations with algebraic equations is obtained, commonly referred to the modelling.

2.4 Modelling of differential equations

2.4.1 Modelling of the equation of momentum

The modelling of the equation of momentum is doing term by term, it leads to the final equation after reorganized terms of the equation of momentum, which is

$$A_{i,j}\, f'_{i+1,j+1} + B_{i,j}\, f'_{i+1,j} + C_{i,j}\, f'_{i+1,j-1} = D_{i,j} \tag{27}$$

With the coefficients, which are defined by:

$$A_{i,j} = \frac{\overline{V}_{i,j}}{4\Delta\eta} - \frac{1}{2(\Delta\eta)^2} \tag{28}$$

$$B_{i,j} = \xi_{i+\frac{1}{2}}\, f'_{i,j}\frac{1}{\Delta\xi} + \beta_{i+\frac{1}{2}}\, f'_{i,j} + \frac{1}{(\Delta\eta)^2} \tag{29}$$

$$C_{i,j} = \xi_{i+\frac{1}{2}}\, f'_{i,j}\frac{1}{\Delta\xi} + \beta_{i+\frac{1}{2}}\, f'_{i,j} + \frac{1}{(\Delta\eta)^2} \tag{30}$$

2.4.4.2 Modelling of the equation of continuity

Similarly, the equation of continuity is

$$V_{i+1,j} = V_{i+1,j-1} + V_{i,j-1} - V_{i,j} + 2\Delta\eta\left(A_j^c\, f'_{i+1,j} + B_j^c\, f'_{i+1,j-1} + C_j^c\, f'_{i,j} + D_j^c\, f'_{i,j-1}\right) \tag{31}$$

The coefficients are determined by

$$A_j^c = -\frac{1}{2\Delta\xi}\xi_{i+\frac{1}{2}} - \frac{1}{4}\beta_{i+\frac{1}{2}} - \frac{1}{4\Delta\eta}\eta_{j-\frac{1}{2}}\left(\beta_{i+\frac{1}{2}} - 1\right) \tag{32}$$

$$B_j^c = -\frac{1}{2\Delta\xi}\xi_{i+\frac{1}{2}} - \frac{1}{4\beta_{i+\frac{1}{2}}} + \frac{1}{4\Delta\eta}\eta_{j-\frac{1}{2}}\left(\beta_{i+\frac{1}{2}} - 1\right) \tag{33}$$

$$C_j^c = \frac{1}{2\Delta\xi}\xi_{i+\frac{1}{2}} - \frac{1}{4\beta_{i+\frac{1}{2}}} - \frac{1}{4\Delta\eta}\eta_{j-\frac{1}{2}}\left(\beta_{i+\frac{1}{2}} - 1\right) \tag{34}$$

$$D_j^c = \frac{1}{2\Delta\xi}\xi_{i+\frac{1}{2}} - \frac{1}{4\beta_{i+\frac{1}{2}}} + \frac{1}{4\Delta\eta}\eta_{j-\frac{1}{2}}\left(\beta_{i+\frac{1}{2}} - 1\right) \tag{35}$$

2.5 Algorithm solving equations of the laminar boundary layer

The sorting system diagonal of the equation modelled of momentum is determined by the algorithm Thomas [3].

2 .6 Modelling of the original profile

The initial profile taxed at ($\xi = 0$) is determined from the solution of the equation of Falkner-Skan **[10, 11]**

2.7.1 Method Principle Shooting

This method allows the passage of a differential problem with the boundary conditions with a problem to initial conditions. Indeed, it is to adjust the initial conditions so that the boundary conditions are met, i.e., corrections to achieve target **[7.10]**

2.8 Algorithm resolution of the mathematical model of the laminar boundary layer

The trouble shooting steps are **[7]**

2.8.1 The choice of the barrier sets through its leading edge parameter of the initial form. This allows the resolution of the equation Falkner-Skan i.e. obtaining all values according to the initial vertical.
2.8.2 The determination of the speed outside.
2.8.3 The calculation of the shape parameter for all « i » stations along the profile.
2.8.4 The calculation of the values for all of the original station.
2.8.5 The resolution of the equation of momentum by the algorithm Thomas.
2.8.6 The calculation of values and speeds for the new station.
2.8.7 The test, if the detachment occurs, we stop the calculations, if you go back to the stage (2.8.5).

3 Discussion of Results

The tests of the numeric code developed, were conducted on a flat plate as well as a profile NACA0012. Distributions of pressure on these profiles are determined by searching the influence of the angle of attack and Mach number of flow undisturbed on the latter, the point of detachment is determined from the speed profiles on the surface profile. A technique for controlling the boundary layer by suction is applied to test the influence of several parameters such as the scope of aspiration, the Mach number, the direction of the angle of ejection, and the amount pumped. To validate the calculation code, we conducted a comparison of the results of the profile with solid surface and those obtained by the method of BLASUIS plane to the plate **[1, 12].**

3.1 Controlling detachment depending on the scope of suction

By applying the technique to control the boundary layer by suction, we can see from the figure (2) that the decline from the point of detachment towards the trailing edge increases. By increasing the scope of aspiration, this development is slower beyond a range of 30% suction of the rope **[1, 12].**

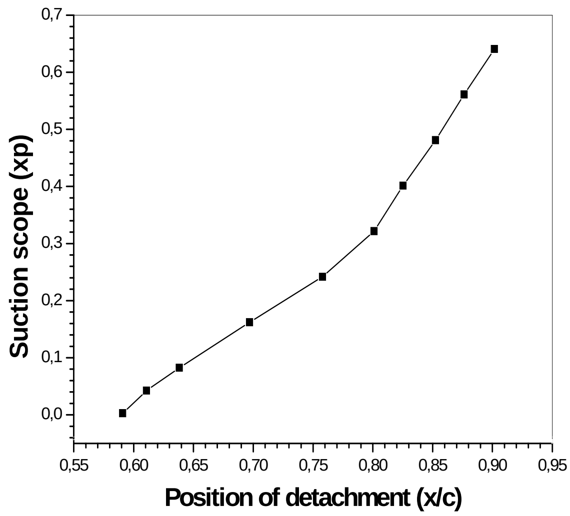

Fig. 2: Effect of the scope of suction on the verge of detachment on NACA 0012.
M = 0.23, α = 0, and vo / U = ∞ 0.01.

3.2 Monitoring detachment depending on the angle of suction

For angles intake below 87 ° taken on a profile NACA0012, their values have no positive effect on the verge of loosening, on the contrary, they encourage detachment, but from this angle, the point of detachment toward the rearward edge of a remarkable manner, the angle is high over the current detachment is delayed towards the trailing edge up to a value of 180, but 180 ° angle is not practical, we so just to the angle of 170 °.

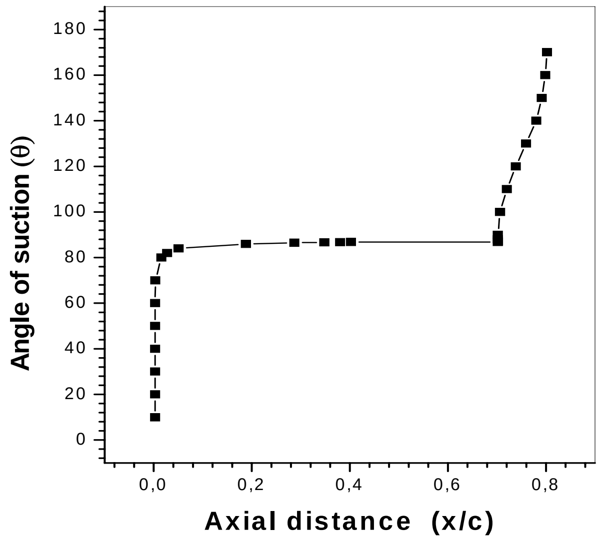

Fig. 3:The effect of the angle of suction on the detachment of a profile NACA0012.
M = ∞ 0.23 and α = 0, vo / U = ∞ 0.01.

3.3 Controlling detachment depending on the amount pumped

For small amounts of fluid sucked on a range of xp = 40٪ and an angle of 90° ejection, the point of detachment remains unmoved. Then, the more we aspire as their point of detachment moves to the edge, but when you reach a value of 10٪ point detachment becomes insensible to any quantity and more. So, we should not aspire too, because the excess flow is not useful.

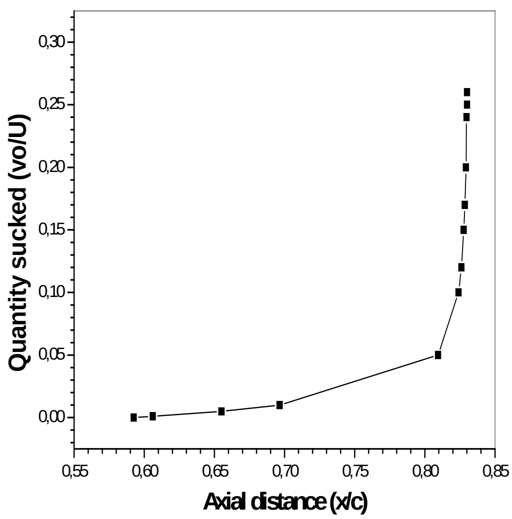

Fig 4 Effect of the amount pumped on the verge of separation.
M = 0.23, α = 0 ° and θ = 90°

3.4 Effect of control by suction on the thickness of the boundary layer

The molecular particles adhere better with the wall by applying control by suction, this is due to the absorption of these particles located just to the wall, allowing the acceleration of particles above, where the thickness of the layer limit becomes thinner.

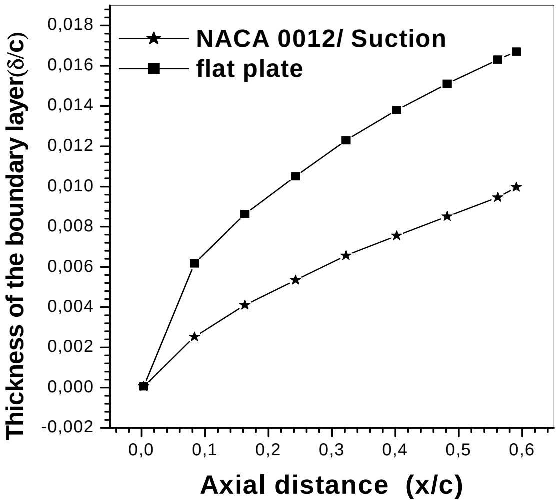

Fig. 5: Thickness of the laminar boundary layer on a porous profile NACA 0012 and on aflat plate solid. Xp=0.6, M=0.23, α=0, θ=10° and vo/U= ∞ 0.01.

3.5 Effect of control by suction on the coefficient of friction

Following the implementation of the suction control over the length of the profile, the particles in direct contact with the wall accelerating and the friction coefficient is lower than that obtained with the same profile solid.

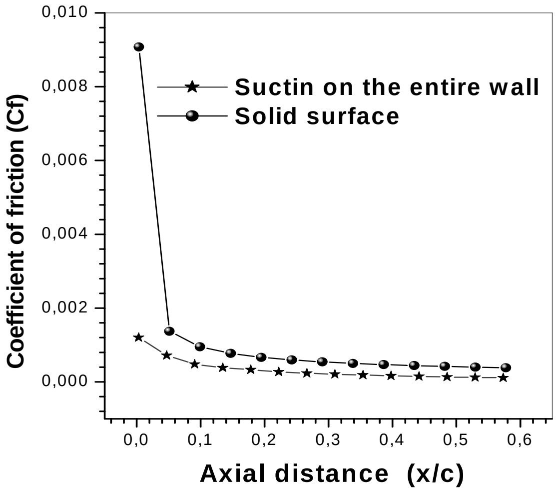

Fig. 6: Distribution of the coefficient of friction on a NACA 0012 and with control plane on a flat plate. Xp = 0.6, M = 0.23, θ = 10 ° and α = 0.

Conclusion

A numerical study is proposed to analyse the behaviour of a laminar boundary layer and two-dimensional incompressible around with profiles and porous solid surface. The profiles considered in this study are profiles NACA0012 and flat plate. It has formulated the mathematical model based on the equations of Prandtl, chosen for the study of the boundary layer. A change of variables is introduced to transform the system of differential equations with two variables into a single non-linear differential equation regular in one variable. The resolution may not be possible as numerical methods, therefore, it has appealed to the method of Shooting and the algorithm Thomas. The results were used to study the influence of the scope of aspiration, the angle of suction and the amount pumped. The results showed that the suction gives better results, which are a key objective sought by several research organizations aviation.

These positive results are consistent with the theoretical predictions and validated by the experimental results obtained by several agencies in the aviation industry. More scope suction is important, the angle and the amount siphoned are optimal, and the results are better. Indeed, resulting in a coefficient of friction and a lower thickness of the boundary layer thinner.

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