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Investigating the normal structure of certain subgroups and computing their induced characters*

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Abstract: The normal structure of the maximal parabolic subgroups and Borel subgroups of the finite general linear groups were investigated, and the induced characters of the finite general linear group $GL(n,q)$ from those subgroups were computed. A computer program system GAP was used to check the results and generalize them.

Key words: representation theory; maximal parabolic subgroups; Borel subgroups; chief series; GAP

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某些子群的正则结构的研究及其诱导特征标的计算

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摘要: 研究了有限生成线性群的极大抛物子群和Borel子群的正则结构. 通过上述子群来计算一般有限线性群 $GL(n,q)$ 的诱导特征标; 并用计算机编程系统GAP来检验和推广我们的结果.

关键词: 表示理论; 极大抛物子群; Borel子群; 主群列; GAP

0 Introduction

One of the most effective methods for studying the normal structure of a given group is to consider a specific kind of descending series subgroups of that group. In order to investigate the normal structure of the maximal parabolic subgroups and Borel subgroups of a finite general linear group, we have adapt this approach by using one of such series, namely, chief series, and then exploiting the fact that the chief factors are a direct product of mutually isomorphic simple groups. The later groups are completely classified in 1980.

Seeking for a better understanding for the representation of the considered subgroups (maximal parabolic and Borel), an attempt has been made to compute their induced characters with two different methods.

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Through out our work was used a computer program system called GAP[14] was used to check the results and generalize them.

1 Preliminaries

Definition 1.1 Let F be a finite field, a parabolic subgroup P^{η} of the general linear group $GL(n,q)$ is the multiplicative subgroup of the block triangular algebra

$$\begin{pmatrix} M_{\eta_1}(F) & M_{\eta_1,\eta_2}(F) & \cdots & M_{\eta_1,\eta_t}(F) \\ 0 & M_{\eta_2}(F) & \cdots & M_{\eta_2,\eta_t}(F) \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & M_{\eta_t}(F) \end{pmatrix},$$

where $M_{i,j}(F), 1\leqslant i,j\leqslant t$, is the $i\times j$ matrix over F, $\eta=(\eta_1, \eta_2, \cdots, \eta_t)$ is a sequence of positive integers such that $n=|\eta|$.

A maximal parabolic subgroup MP of the finite general linear group $GL(n,q)$ takes the form $P^{(k,h)}$. A Borel subgroup B of the finite general linear group $GL(n,q)$ takes the form $P^{(1^n)}$.

The following code was essentially designed to compute the standard parabolic subgroups, however, we will make a simple modifications on it to get those subgroups of our concern, and then make our investigation and computations on them.

```
Main Program:
gap> G :=GL(n,q);;
gap> lambdas := OrderedPartitions(n);;
gap> DirectProductMatGroup := function(G, H)
> local gens;
>   gens := Concatenation(
>     List(GeneratorsOfGroup(G),
          x -> DirectSumMat(x,One(H))),
>     List(GeneratorsOfGroup(H),
          y -> DirectSumMat(One(G),y)));
>   return Group(gens);
> end;
function(G, H)…end

gap> StandardLevi := function(lambda, q)
> local G, i;
>   G := GL(lambda[1],q);
>   for i in [2..Length(lambda)] do
>     G :=
       DirectProductMatGroup(G, GL(lambda[i],q));
>   od;
>   return G;
> end;
function(lambda, q)…end

gap> StandardParabolic := function(lambda, q)
> local L, gens, idx, g, i, j;
>
>     L := StandardLevi(lambda, q);
>   gens := ShallowCopy(GeneratorsOfGroup(L));
>   for idx in [1..Length(lambda)-1] do
>     g := MutableCopyMat(One(L));
>     j := Sum(lambda{[1..idx-1]}) + 1;
>     i := Sum(lambda{[1..idx]}) + 1;
>     g[i][j] := One(GF(q));
>     Add(gens, Immutable(g));
>
>   od;
>   return Group(gens);
> end;
function(lambda, q)…end

gap> # for example the standard parabolic subgroups
        of GL(3,2) are:
gap> n := 3;; q := 2;;
gap> groups :=
List(lambdas,lambda->
                         StandardParabolic(lambda,q));
[<matrix group with 8 generators>,
 <matrix group with 5 generators>,
 <matrix group with 5 generators>,
 Group([ <an immutable 3×3 matrix over GF2>,
      <an immutable 3×3 matrix over GF2>])]
```

2 Investigating the normal structure

Definition 2.1 A series of subgroups

$$G=G_0>G_1>\cdots>G_r=1$$

of a group G is called a chief series if for each i:

(Ⅰ) $G_{i+1}\lhd G_i$,

(Ⅱ) $G_i\lhd G$,

(Ⅲ) $\nexists H\lhd G$ which satisfy $G_{i+1}<H<G_i$.

The successive quotients G_i/G_{i+1} are called the chief factors of G.

Remark 2.2 In GAP, according to Definition

1.1, we can obtain the maximal parabolic subgroups and Borel subgroups from the main program as follows:

```
lambdas := OrderedPartitions(n, 2); # to get the maximal parabolic subgroups of GL(n, q) for a specific values of n and q;
lambdas :=OrderedPartitions(n,n); # to get the Borel subgroup of GL(n,q) for a specific values of n and q.
```

We will take the group

$$G = GL(3,2) = SL(3,2)$$

as example, however, the same procedure can be used for a different finite general linear group.

```
# Using Remark 2.1, G has two maximal parabolic
  subgroups:
gap> MP1 :=groups[1];
<matrix group with 5 generators>
gap> MP2 := groups[2];
<matrix group with 5 generators>
#G has one Borel subgroup:
gap> B :=groups[1];
<matrix group with 8 generators>
```

The following function is used to compute the minimal normal subgroups.

```
gap> MinimalNormalSubgroups :=function (G)
>      local minimal, normal, n;
>      normal  :=
                ShallowCopy(NormalSubgroups(G));
>      Sort(normal, function (x, y)
>            return Size(x)<Size(y);
>        end);
>      minimal  := [ ];
>      for n in normal {[ 2 .. Length(normal) ]} do
>          if ForAll(minimal, function (x)
>                    return not IsSubset(n, x);
>               end) then
>              Add(minimal, n);
>          fi;
>      od;
>      return minimal;
> end;
function(G)…end
gap>#for example the Borel subgroup B
gap> # has only one minimal normal subgroup which
       is:
gap> MinimalNormalSubgroups(B);
[<group of 3×3 matrices of size 2 in characteristic 2>]
```

The following function tests whether or not a subgroup H is a minimal normal subgroup of a group G.

```
gap> IsMinimalNormalSubgroup := function (G,H)
> local M, value;
> if IsSubgroup(G,H) then
> M  := MinimalNormalSubgroups(G);
> fi;
> if H in M then
> value  := true;
> else
> value := false;
> fi;
> return value;
> end;
function(G, H)…end
```

The chief factor G_i/G_{i+1} is a minimal normal subgroup of the quotient group G/G_{i+1} where

$$G = G_1 > G_2 > \cdots > G_r = 1$$

is the chief series of G. In GAP the following function shows that this fact was achieved for arbitrary group G as follows.

```
gap> TestCFisMinimalNormal := function (S)
>   local hom,x,f,g,i,k;
>    if IsGroup(S) then
>    S :=ChiefSeries(S);
>    fi;
>    x :=[];
>    for i in [2..Length(S)] do
>    hom  :=
  NaturalHomomorphismByNormalSubgroup(S[1],S[i]);
>    g :=Image(hom,S[1]); # the quotient group
>    f  := Image (hom,S[i-1]); # the chief factor
>    k  := IsMinimalNormalSubgroup(g,f);
>    Add(x,k);
>    od;
>    return x;
>    end;
function(S) … end
```

Take as example $MP1$, we have:

```
gap> TestCFisMinimalNormal(MP1);
[ true, true, true ]
```

Since the chief factors are minimal normal subgroups and hence a direct product of mutually isomorphic simple groups which are completely classified, we can write the following function that displays the normal structure of a given group in details.

```
gap> DisplayChiefSeries := function (S)
>   local f, i,C,T,numb;
> if IsGroup(S) then
> S :=ChiefSeries(S);
> fi;
> Print(GroupString(S[1], "G"), "\n");
> for i in [ 2 .. Length(S) ] do
> f :=
Image(NaturalHomomorphismByNormalSubgroup
        (S[i-1],S[i]));
>   C := CompositionSeries(f);
> numb := Length(C)-1;
> Print("\n");
> Print(" | ",
    IsomorphismTypeInfoFiniteSimpleGroup(T). name,
    " : ","number of direct factors=", numb,"\n");
> Print("\n");
> if i<Length(S) then
> Print(GroupString(S[i], "S"), "\n");
> else
>      Print(GroupString(S[i], "1"), "\n");
> fi;
> od;
> return;
> end;
function(S) ... end

gap> # for example MP1 has the following structure:
gap> DisplayChiefSeries(MP1);
G (size 24)
   | Z(2): number of direct factors=1
S (3 gens, size 12)
   | Z(3): number of direct factors=1
S (2 gens, size 4)
   | Z(2): number of direct factors=2
1 (size 1)
```

Remark 2.3 When the number of the direct factors equals 1, it means that the corresponding chief factor is a simple group. For full exposition for the isomorphism type of finite simple groups see Ref. [14, p. 350～351]. The same function can be used to view the normal structure of $MP2$, B and other different groups.

3 Computing the induced characters

As we have seen in the previous section, $G = GL(3,2)$ has two maximal parabolic subgroups, $MP1$ and $MP2$, and one Borel subgroup B, moreover, we have some information about the subgroups listed in Tab. 1.

Tab. 1 Some information about the subgroups

group	size	number of conjugacy classes
G	168	6
$MP1$	24	5
$MP2$	24	5
B	8	5

To save space we proceed only for $MP1$. The corresponding representatives of the conjugacy classes $C_1, \cdots, C_5$ of $MP1$ are:

$$\boldsymbol{h}_1 = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \boldsymbol{h}_2 = \begin{pmatrix}1&0&0\\0&0&1\\0&1&1\end{pmatrix},$$

$$\boldsymbol{h}_3 = \begin{pmatrix}1&0&0\\0&0&1\\1&1&0\end{pmatrix}, \boldsymbol{h}_4 = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},$$

$$\boldsymbol{h}_5 = \begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix}.$$

The corresponding representatives of the conjugacy classes $K_1, \cdots, K_6$ of G are:

$$\boldsymbol{r}_1 = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \boldsymbol{r}_2 = \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},$$

$$\boldsymbol{r}_3 = \begin{pmatrix}0&0&1\\1&0&1\\0&1&1\end{pmatrix}, \boldsymbol{r}_4 = \begin{pmatrix}0&0&1\\1&0&0\\0&1&1\end{pmatrix},$$

$$r_5=\begin{pmatrix}0&0&1\\1&0&1\\0&1&0\end{pmatrix},\ r_6=\begin{pmatrix}1&0&0\\0&0&1\\0&1&1\end{pmatrix}$$

In GAP, one may use the following function to get these representatives.

```
gap> Representatives := function (G)
>  local i, cc,L;
>   if IsGroup(G) then
>  cc := ConjugacyClasses(G);
>  fi;
>  L := List(cc,Representative);
>
>  for i in [1..Length(L)] do PrintArray(L[i]);
>  Print("\n");
>   od;
>    end;
function(G)…end
```

The character tables of $MP1$ and G are:

```
gap> tbl1 :=CharacterTable(MP1);;Display(last);
CT1
     2  3  .  2  2  3
     3  1  1  .  .  .

        1a  3a  4a  2a  2b
X.1      1   1   1   1   1
X.2      1   1  -1  -1   1
X.3      2  -1   .   .   2
X.4      3   .   1  -1  -1
X.5      3   .  -1   1  -1
gap> tbl :=CharacterTable(G);;Display(last);
CT2

     2  3  3  2  .  .  .
     3  1  .  .  .  .  1
     7  1  .  .  1  1  .

        1a  2a  4a  7a  7b  3a
X.1      1   1   1   1   1   1
X.2      3  -1   1   A  /A   .
X.3      3  -1   1  /A   A   .
X.4      6   2   .  -1  -1   .
X.5      7  -1  -1   .   .   1
X.6      8   .   .   1   1  -1

A =E(7)+E(7)^2+E(7)^4
  =(-1+ER(-7))/2=b7
```

In what follows we give two different methods for computing the induced characters of G from $MP1$.

3.1 First method

Proposition 3.1 Let $H\leqslant G,\varphi$ be a character of H and $g\in G$. Then

$$\varphi^G(g)=\begin{cases}\sum_{i=1}^{m}\dfrac{O\,(C_G(g))}{O\,(C_H(h_i))}\varphi(h_i) & \text{if } m>0,\\ 0 & \text{if } m=0,\end{cases}$$

where, m is the number of conjugacy classes of H whose members are conjugate in G to g, $\boldsymbol{h}_1,\cdots,\boldsymbol{h}_m$ are the representatives of the m classes, $O(C_G(g))$ is the order of the centralizer of g in G and $O(C_H(h_i))$ is the order of the centralizer of $\boldsymbol{h}_i$ in H.

Proposition 3.2 The induced characters of G from $MP1$ are listed in Tab. 2, where $\varphi_i\,(1\leqslant i\leqslant 6)$ are the irreducible characters of $MP1$.

Tab. 2 Induced characters of G from $MP1$

	r_1	r_2	r_3	r_4	r_5	r_6
φ_1^G	7	3	1	0	0	1
φ_2^G	7	−1	−1	0	0	1
φ_3^G	14	2	0	0	0	−1
φ_4^G	21	−3	1	0	0	0
φ_5^G	21	1	−1	0	0	0

Proof Since $C_1=K_1$, $C_2\subset K_6$, $C_3\subset K_3$ & $\{C_4,C_5\}\subset K_2$, moreover, there are no elements of $MP1$ which belong to the classes K_4, K_5, together with proposition 3.1 we have:

$$\forall\, 1\leqslant i\leqslant 6;$$

$$\varphi_i^G(\boldsymbol{r}_1)=\frac{O(C_G(\boldsymbol{r}_1))}{O(C_{MP1}(\boldsymbol{h}_1))}\varphi_i(\boldsymbol{h}_1)=\frac{168}{24}\varphi_i(\boldsymbol{h}_1),$$

$$\varphi_i^G(\boldsymbol{r}_2)=\frac{O(C_G(\boldsymbol{r}_2))}{O(C_{MP1}(\boldsymbol{h}_4))}\varphi_i(\boldsymbol{h}_4)+\frac{O(C_G(\boldsymbol{r}_2))}{O(C_{MP1}(\boldsymbol{h}_5))}\varphi_i(\boldsymbol{h}_5)=\frac{8}{4}\varphi_i(\boldsymbol{h}_4)+\frac{8}{8}\varphi_i(\boldsymbol{h}_5),$$

$$\varphi_i^G(\boldsymbol{r}_3)=\frac{O(C_G(\boldsymbol{r}_3))}{O(C_{MP1}(\boldsymbol{h}_3))}\varphi_i(\boldsymbol{h}_3)=\frac{4}{4}\varphi_i(\boldsymbol{h}_3),$$

$$\varphi_i^G(\boldsymbol{r}_4) = \varphi_i^G(\boldsymbol{r}_5) = 0,$$

$$\varphi_i^G(\boldsymbol{r}_6) = \frac{O(C_G(\boldsymbol{r}_6))}{O(C_{MP1}(\boldsymbol{h}_2))}\varphi_i(\boldsymbol{h}_2) = \frac{3}{3}\varphi_i(\boldsymbol{h}_2).$$

Substituting the values of the irreducible characters φ_i of $MP1$ in the above equations we have done. □

Remark 3.3 (Ⅰ) In GAP we can show the relations between the conjugacy classes of group G and its subgroup H as follows.

```
gap> RelationsBetweenConjClasses :=
                                   function (G, H)
>      local C, K, i, j;
>      if IsGroup(G) and IsGroup(H) then
>         C := ConjugacyClasses(H);
>         K := ConjugacyClasses(G);
>      fi;
>      for i in [ 1 .. Length(C) ] do
>         for j in [ 1 .. Length(K) ] do
>            if C[i]=K[j] then
>    Print("C",i,"is equal","K",j,"\n");
>            elif IsSubset(K[j], C[i]) then
>   Print("C",i,"is proper subset of ","K",j,"\n");
>            else
>               ;
>            fi;
>         od;
>      od;
>      return;
> end;
function(G, H)…end
gap> # using this function we have:
gap> RelationsBetweenConjClasses(G,MP1);
C1 is equal K1
C2 is proper subset of K6
C3 is proper subset of K3
C4 is proper subset of K2
C5 is proper subset of K2
```

(Ⅱ) The orders of the centralizers of the representatives in G and $MP1$ can be obtain easily in GAP as follows.

```
gap> ccG := ConjugacyClasses(G);;
gap> L :=List(ccG,Representative);;
gap> OCeG :=
          List(L,x -> Order(Centralizer(G,x)));
[ 168, 8, 4, 7, 7, 3 ]
gap> # for MP1 we have:
gap> ccMP1 := ConjugacyClasses(MP1);;
gap> L :=List(ccMP1,Representative);;
gap> OCeMP1 :=
        List(L,x -> Order(Centralizer(MP1,x)));
[ 24, 3, 4, 4, 8 ]
```

3.2 Second method

Definition 3.4 Let $H \leqslant G, \varphi_1, \cdots, \varphi_w$ and $\chi_1, \cdots, \chi_z$ be the irreducible characters of H and G, respectively. The induction-restriction table of (G,H) is the $w \times z$ matrix whose (i,j)-entry is the common value of $(\varphi_i^G, \chi_j)_G$ and $(\varphi_i, \chi_j|_H)_H$.

The i' th row in the induction-restriction table represent the multiplicities with which the χ_j appears in the decomposition of φ_i^G.

The j' th column in the induction-restriction table represents the multiplicities with which the φ_i appears in the decomposition of $\chi_j|_H$.

Proposition 3.5 The restriction characters of G on $MP1$ are listed in Tab. 3.

Tab. 3 Restriction characters of G on $MP1$

	C_1	C_2	C_3	C_4	C_5
$\chi_1\|_{MP1}$	1	1	1	1	1
$\chi_2\|_{MP1}$	3	0	1	−1	−1
$\chi_3\|_{MP1}$	3	0	1	−1	−1
$\chi_4\|_{MP1}$	6	0	0	2	2
$\chi_5\|_{MP1}$	7	1	−1	−1	−1
$\chi_6\|_{MP1}$	8	−1	0	0	0

Proof Direct calculations using the relations between the conjugacy classes of $MP1$ and those of G stated in the proof of Proposition 3.2. □

Corollary 3.6 Let $\varphi_i (1 \leqslant i \leqslant 5)$, be the irreducible characters of $MP1$, then

$$\chi_1|_{MP1} = \varphi_1,$$
$$\chi_2|_{MP1} = \varphi_4,$$
$$\chi_3|_{MP1} = \varphi_4,$$
$$\chi_4|_{MP1} = \varphi_1 + \varphi_3 + \varphi_5,$$
$$\chi_5|_{MP1} = \varphi_2 + \varphi_4 + \varphi_5,$$
$$\chi_6|_{MP1} = \varphi_3 + \varphi_4 + \varphi_5.$$

Proposition 3.7 The induction-restriction table of $(G, MP1)$ is listed in Tab. 4.

Tab. 4 Induction-restriction table of $(G, MP1)$

	$\chi_1\mid_{MP1}$	$\chi_2\mid_{MP1}$	$\chi_3\mid_{MP1}$	$\chi_4\mid_{MP1}$	$\chi_5\mid_{MP1}$	$\chi_6\mid_{MP1}$
φ_1	1	0	0	1	0	0
φ_2	0	0	0	0	1	0
φ_3	0	0	0	1	0	1
φ_4	0	1	1	0	1	1
φ_5	0	0	0	1	1	1

Proof Using Corollary 3.6 and the inner product of irreducible characters. □

Corollary 3.8 The induction-restriction table (Tab. 4) could be written equivalently by means of induced characters of G from $MP1$ as seen in Tab. 5.

Tab. 5 Equivalent view for table 4

	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6
φ_1^G	1	0	0	1	0	0
φ_2^G	0	0	0	0	1	0
φ_3^G	0	0	0	1	0	1
φ_4^G	0	1	1	0	1	1
φ_5^G	0	0	0	1	1	1

We can deduce from Corollary 3.8 and Definition 3.4 that:

$$\varphi_1^G = \chi_1 + \chi_4,$$
$$\varphi_2^G = \chi_5,$$
$$\varphi_3^G = \chi_4 + \chi_6,$$
$$\varphi_4^G = \chi_2 + \chi_3 + \chi_5 + \chi_6,$$
$$\varphi_5^G = \chi_4 + \chi_5 + \chi_6.$$

In GAP, we can use the following function to get the induction-restriction table.

```
gap> ResTab := function (tG, tU)
>      local r;
>      if IsGroup(tG) then
>          tG := CharacterTable(tG);
>      fi;
>      if IsGroup(tU) then
>          tU := CharacterTable(tU);
>      fi;
>      r := Restricted(tG, tU, Irr(tG));
>      return MatScalarProducts(tU, r, Irr(tU));
> end;
function(tG, tU) … end

gap> ResTab(G,MP1);
[ [ 1, 0, 0, 1, 0, 0 ], [ 0, 0, 0, 0, 1, 0 ],
  [ 0, 0, 0, 1, 0, 1 ], [ 0, 1, 1, 0, 1, 1 ],
  [ 0, 0, 0, 1, 1, 1 ] ]
```

Equivalently, but the outputs displayed as columns, we have:

```
gap> RestTab := function (tG,tU)
> local r;
>   if IsGroup(tG) then
>   tG := CharacterTable (tG);
> fi;
>   if IsGroup (tU) then
>   tU := CharacterTable (tU);
> fi;
>    r := InducedClassFunctions(Irr(tU), tG);
>   return MatScalarProducts (tG, r, Irr (tG));
> end;
function(tG, tU) … end

gap> RestTab (G,MP1);
[ [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 1, 0 ],
  [ 0, 0, 0, 1, 0 ], [ 1, 0, 1, 0, 1 ],
  [ 0, 1, 0, 1, 1 ], [ 0, 0, 1, 1, 1 ] ]
gap> # we can use the command TransposedMat
gap> # to display the result as required:
gap> TransposedMat(last);
[ [ 1, 0, 0, 1, 0, 0 ], [ 0, 0, 0, 0, 1, 0 ],
  [ 0, 0, 0, 1, 0, 1 ], [ 0, 1, 1, 0, 1, 1 ],
  [ 0, 0, 0, 1, 1, 1 ] ]
```

To check the computed induced characters in GAP we can do the following.

```
gap> I := Irr(MP1);;
                        # Irreducible Characters of MP1
gap> IndChars := InducedClassFunctions(I,G);
                        # the required induced characters
[Character(CharacterTable(SL(3,2)),
          [ 7, 3, 1, 0, 0, 1 ]),
 Character(CharacterTable(SL(3,2)),
          [ 7, -1, -1, 0, 0, 1 ]),
```

Character(CharacterTable(SL(3,2)),
[14, 2, 0, 0, 0, −1]),
Character(CharacterTable(SL(3,2)),
[21, −3, 1, 0, 0, 0]),
Character(CharacterTable(SL(3,2)),
[21, 1, −1, 0, 0, 0])]

The induced characters of G from $MP2$ are identical to those induced from $MP1$.

The induced characters of G from B are listed in Tab. 6, where ψ_i ($1 \leqslant i \leqslant 6$) are the irreducible characters of B.

Tab. 6 Induced characters of G from B

	r_1	r_2	r_3	r_4	r_5	r_6
ψ_1^G	21	5	1	0	0	0
ψ_2^G	21	−3	1	0	0	0
ψ_3^G	21	1	−1	0	0	0
ψ_4^G	21	1	−1	0	0	0
ψ_5^G	42	−2	0	0	0	0

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