

Non-Gaussianity from Compositeness

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Abstract

By assuming the field seeding the curvature perturbations ζ is a dynamically arising condensate, we are able to derive the relation $f_{\text{NL}}^2 \simeq 10^8 H/M_c$ between the non-Gaussianity parameter f_{NL} and the ratio of the inflationary scale H to the cutoff scale M_c of the effective theory describing the condensate, thus relating the experimental bound on f_{NL} to a bound on M_c .

1 Introduction

The origin of the structures in the Universe seems to be the primordial curvature perturbation ζ , present already a few Hubble times before cosmological scales enter the horizon and come into causal contact. ζ is Gaussian within the observational uncertainty and has a practically scale-independent spectrum. Future observation, though, may find a non-Gaussian component. The usual assumption is that ζ originates from the vacuum fluctuations of a light scalar field in an inflationary environment and these fluctuations are promoted to practically Gaussian classical perturbations around the time of horizon exit. By expanding ζ in powers of the field perturbations, one can see the linear term is the source of a Gaussian spectrum and the quadratic term can adequately account for non-Gaussianity [1].

In this article, we will rephrase, within the framework of non-Gaussianity, the results of Ref. [2], where it was assumed that the field at the origin of the curvature perturbations is not fundamental but is instead a dynamically arising condensate. Explicit examples of the possible realization of the condensation mechanism in the context of inflation may be found in Refs. [3, 4]. In the following we will only assume that such a mechanism is at work. In this model the correlation functions of a scalar field, usually employed in the derivation the CMB power spectrum, should therefore be replaced by the correlation functions for a composite scalar field operator. The leading behavior of these correlation functions is assumed to reproduce that of a fundamental scalar field in a de Sitter space-time, leading to a scale invariant spectrum for the two-point correlation. The deviations from a scale-invariant power spectrum, which we interpret as a sign of the compositeness of the seeding field, can then be consistently related to

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a quadratic term in the expansion of the curvature perturbation ζ in terms of the seeding field, thus providing a link with the literature on non-Gaussianity [5].

By identifying the corrections obtained in these two different approaches, one can find a relation between H/M_c (the ratio of the Hubble scale H during inflation to the cutoff M_c of the effective theory which describes the scalar condensate) which describes the amplitude of the correction to the two-point function and the parameter f_{NL} which is usually employed to describe non-Gaussianity.

In the next Section, we briefly review the corrections arising from a fermion condensate to the spectral index and, in Section 3, we estimate the size of possible non-Gaussian contributions. Finally, we summarize and comment our main results in Section 4.

2 The Condensate Corrections

Let us assume that the condensation mechanism described in Refs. [3, 4] is realized in the inflationary expansion of the Universe, and that the dynamically generated composite scalar field is at the origin of the curvature perturbations. We expect the composite nature of this field to become manifest in its correlation functions, and thus in the CMB spectrum. The equal-time two-point correlation function for a scalar field $\langle \hat{\sigma}(\eta, \mathbf{x}) \hat{\sigma}(\eta, \mathbf{x}') \rangle$ should be replaced by the two-point function for the condensate $\bar{\psi}\psi$, so that the power spectrum is defined by

$$\langle (\bar{\psi}\psi)(\eta, \mathbf{x}) (\bar{\psi}\psi)(\eta, \mathbf{x}') \rangle \equiv \int \frac{dk}{k} \frac{\sin k|\mathbf{x} - \mathbf{x}'|}{k|\mathbf{x} - \mathbf{x}'|} P(k, \eta) . \quad (1)$$

We next assume that the leading behavior of the two-point function is not modified and produces an almost scale invariant spectrum of perturbations. For this reason, we factorize the Gaussian leading behavior and take the Fourier transform of the two-point function $\tilde{\xi}(k)$ of the form

$$\tilde{\xi}(k) \simeq \frac{1}{k^3} \left(1 + \delta\tilde{\xi} \right) . \quad (2)$$

Given the relation

$$\mathcal{P}(k) = \frac{k^3}{2\pi} \tilde{\xi}(k) \simeq k^{n_s-1} , \quad (3)$$

one obtains

$$\log \left(1 + \delta\tilde{\xi} \right) \simeq (n_s - 1) \log k , \quad (4)$$

where n_s is the spectral index.

We now need to determine the form of the modification of the Fourier transform of the two-point function $\delta\tilde{\xi}(k)$ which corresponds to the four-point function for the fermion field ψ in the particular case where two fermions form a composite of ingoing momentum k and two form a composite of outgoing momentum k . This way the four-point function for the fermion fields reduces to the two-point function for the composite scalars. If we look at the flat space case (see Ref. [8]), we see that, in the momentum representation, the inverse propagator for a composite scalar particle is modified as (see Fig. 1)

$$\Gamma(k) = \delta Z (\Gamma_0(k) + \delta\Gamma(k)) \equiv \delta Z (m^2 + k^2) I(k) , \quad (5)$$

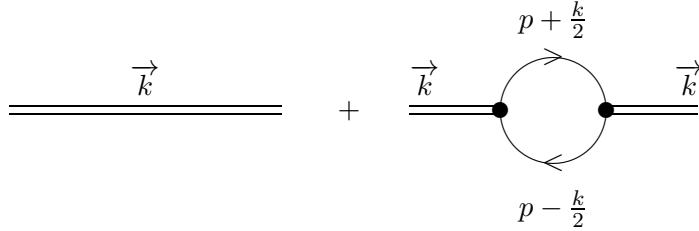


Figure 1: Diagrammatic representation of Eq. (6). Double lines correspond to composite scalars of momentum k .

where $\Gamma_0(k)$ is the free scalar propagator, δZ the wave-function renormalization, $I(k)$ the form factor, and

$$\delta \Gamma(k) \simeq \int \frac{d^4 p}{(2\pi)^4} \text{tr} [(\not{p} + \not{k}/2 + im)^{-1} (\not{p} - \not{k}/2 + im)^{-1}] . \quad (6)$$

The form factor $I(k)$ is related to the non-local component of the propagator which is given by the fermion loop. We also note that a wave function renormalization for the composite field is required in order to get a nonsingular behavior of the two point function. On the other hand, the assumption that the leading order of the two-point function for the composite field reproduces the two-point function of a scalar, is equivalent to assuming that the only nonlocal term describing the modification of the two-point function comes from the fermion loop. By looking at Ref. [9], one can see that loop diagrams were evaluated in the cosmological context in order to describe the influence of higher order quantum effects on the CMB spectrum. A conclusion one can draw from Refs. [9] is that a logarithmic correction to the Fourier transform of the two-point function is expected from the fermion loop. As in any effective theory, details which depend on the high-energy physics enter only through non-renormalizable operators, which will here be suppressed by the compositeness scale. Therefore, in the absence of a special argument to the contrary, we must expect that the leading effects from compositeness are proportional to $A \equiv H/M_c$ where M_c is the cutoff of the effective theory above which the compositeness effects become unobservable. If the condensation scale is the Planck mass M_{P} , as implied by the models described in Refs. [3, 4], then $A \sim 10^{-5}$, with lower values of M_c implying bigger values for A .

To summarize, our ansatz for the correction to $\tilde{\xi}$ in Eq. (2) is of the form

$$\delta \tilde{\xi} = A \log \left(\frac{B}{k} \right) , \quad (7)$$

where B is, in general, the unit k is measured (or renormalization scale) and which we will relate to the Hubble horizon in the following Section. Note also that we here allow the coupling constant for the dynamically generated condensate to be more general than that described in Refs. [3, 4], where the factor A would naturally be of the order H/M_{P} . Eq. (7) leads to a spectral index of the form

$$n_s \simeq 1 + \frac{\log [1 + A \log (\frac{B}{k})]}{\log k} , \quad (8)$$

whose behavior is shown in Fig. 2 for positive and negative A . In the first case the spectral index is smaller than one for all k and a red spectrum is generated, whereas in the second case it is larger than one and a blue spectrum results.

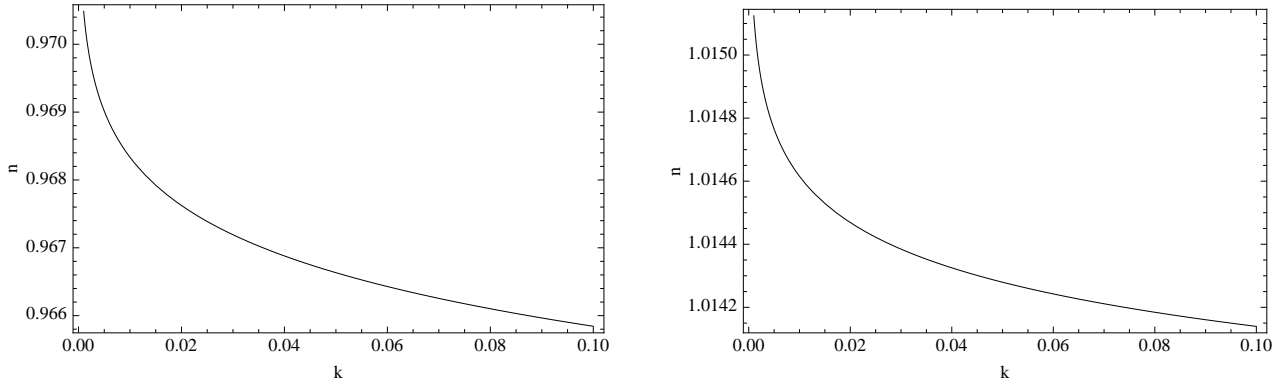


Figure 2: Spectral index n_s for $A = 10^{-2}$ and $B = 10^{-6} \text{ Mpc}^{-1}$ (left panel) and for $A = -10^{-2}$ and $B = 10^{-6} \text{ Mpc}^{-1}$ (right panel).

3 Non-Gaussianity

We now consider the previous results in the framework of the formalism used to extract the non-Gaussian features of the CMB spectrum. Following Ref. [6], we assume the curvature perturbation ζ is the sum of two terms

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \zeta_\sigma(\mathbf{x}) \quad (9)$$

where ζ_g and ζ_σ are respectively its Gaussian and non-Gaussian component. One can consistently express the non-Gaussian part as the sum of a linear and a quadratic term in the field σ as

$$\zeta_\sigma(\mathbf{x}) = \sigma(\mathbf{x}) - \alpha f_{\text{NL}} (\sigma^2(\mathbf{x}) - \langle \sigma^2(\mathbf{x}) \rangle) , \quad (10)$$

where $\sigma(\mathbf{x})$ now represents the condensate field and α is a coefficient of order one (which depends on the convention adopted). There are in principle two possibilities: *(i)* the field σ which describes the scalar condensate is the dominating field in the generation of the curvature perturbations. In this case the linear part in (10) accounts for the Gaussian part ζ_g of the spectrum and the quadratic term accounts for the non-Gaussian part; *(ii)* the field σ is sub-dominant in the generation of the curvature perturbation ζ (in this case ζ_g is generated by another field), so that we can neglect the linear part in (10) and concentrate on the quadratic term. In any case we have

$$P_\zeta(k) = P_{\zeta_g}(k) + P_{\zeta_\sigma}(k) . \quad (11)$$

We first evaluate the non-Gaussian part of the spectrum coming from the quadratic term in Eq. (10). Fourier components of σ^2 are given by

$$(\sigma^2)_{\mathbf{k}} = \frac{1}{(2\pi)^3} \int d^3q \sigma_{\mathbf{q}} \sigma_{\mathbf{k}-\mathbf{q}} . \quad (12)$$

For non vanishing \mathbf{k} and \mathbf{k}'

$$\begin{aligned} \langle (\sigma^2)_{\mathbf{k}} (\sigma^2)_{\mathbf{k}'} \rangle &= \frac{1}{(2\pi)^6} \int d^3p d^3p' \langle \sigma_{\mathbf{p}} \sigma_{\mathbf{k}-\mathbf{p}} \sigma_{\mathbf{p}'} \sigma_{\mathbf{k}'-\mathbf{p}'} \rangle \\ &= 2\delta^{(3)}(\mathbf{k} + \mathbf{k}') \int d^3p P_\sigma(p) P_\sigma(|\mathbf{k} - \mathbf{p}|) , \end{aligned}$$

and, taking \mathcal{P}_σ scale-independent [7],

$$\mathcal{P}_{\sigma^2}(k) = \frac{k^3}{2\pi} \mathcal{P}_\sigma^2 \int_{L^{-1}} \frac{d^3 p}{p^3 |\mathbf{p} - \mathbf{k}|^3} . \quad (13)$$

The subscript L^{-1} indicates that the integrand is set equal to zero in a sphere of radius L^{-1} around each singularity, and the discussion makes sense only for $L^{-1} \ll k \ll k_{\max}$. In this regime one finds [7]

$$\mathcal{P}_{\sigma^2}(k) = 4\mathcal{P}_\sigma^2 \ln(kL) . \quad (14)$$

If we plug this result back into Eq. (11) we obtain

$$P_\zeta(k) = P_{\zeta_g}(k) + P_{\sigma^2}(k) = P_{\zeta_g}(k) [1 + 8\alpha^2 f_{\text{NL}}^2 \mathcal{P}_\sigma \log(kL)] . \quad (15)$$

We can now compare this result with our discussion of compositeness effects in the spectrum of perturbations. The first thing to notice by looking at Eqs. (7) and (15) is that they coincide in producing a logarithmic correction to the Gaussian part.

We note that the correction arising from Eq. (10) has a definite sign, since the factor in front of the logarithmic correction is given by $8\mathcal{P}_\sigma\alpha^2 f_{\text{NL}}^2 > 0$, and thus maps into the case $A < 0$ of Eq. (7), which gives a blue correction to the spectrum. However the size of this correction is very small, of the order of one part on 10^{-5} and its effect will not appear in the experimental measurements of the spectral index. Nonetheless we will see that its effects can be detectable in the measurements of the bispectrum. Let us now consider the two possibilities described earlier. The first one is that the linear term in the expansion (10) is responsible for the Gaussian part of the spectrum. As we have seen the correction coming from the non-Gaussian term provides a negligible correction and we have to assume that a potential is generated for the condensate in order to account for the deviation from $n_s = 1$. If this is the case we can take $\mathcal{P}_\sigma \simeq \mathcal{P}_{\zeta_g} \simeq 10^{-9}$. By identifying the amplitude of the logarithmic correction for the composite scalar case with the contribution due to the quadratic term we are able to make an estimate of the magnitude of the bispectrum B_g defined by

$$\langle g_{\mathbf{k}_1} g_{\mathbf{k}_2} g_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_g(k_1, k_2, k_3) . \quad (16)$$

In fact if the curvature perturbation has the form (10), its bispectrum is given to leading order by [10]

$$B_\zeta(k_1, k_2, k_3) = -2\alpha f_{\text{NL}} [P_\zeta(k_1) P_\zeta(k_2) + \text{cyclic}] . \quad (17)$$

(Only the term linear in f_{NL} is kept, which is justified because the second term of Eq. (10) is much smaller than the first term.) Current observations [12, 13] give $|f_{\text{NL}}| \lesssim 100$, which makes the non-Gaussian fraction of ζ less than $100 \mathcal{P}_\zeta^{1/2} \sim 10^{-3}$. Absent a detection, PLANCK [11] will bring this down to roughly $|f_{\text{NL}}| \lesssim 1$. Now, identifying the correction to the power spectrum due to the non-Gaussian term with that due to compositeness yields

$$A = 8\alpha^2 f_{\text{NL}}^2 \mathcal{P}_{\zeta_g} , \quad (18)$$

which means that

$$f_{\text{NL}}^2 \simeq 10^8 \frac{H}{M_c} . \quad (19)$$

This is our prediction for the value of f_{NL} , and by taking $A \lesssim 10^{-4}$, we obtain $f_{\text{NL}} \lesssim 100$ which is the current experimental bound. Conversely, the bound $|f_{\text{NL}}| \lesssim 100$ may be interpreted as a constraint on A in the form $A \lesssim 10^{-4} \alpha^2 \simeq 10^{-4}$. This yields a lower bound on the effective scale $M_c \gtrsim 10^{-1} M_P$.

The second possibility is that the non-Gaussian contribution is sub-dominant. In this case instead of equating \mathcal{P}_ζ with \mathcal{P}_σ in eq. (15) one can express $\mathcal{P}_\sigma = r\mathcal{P}_\zeta$ which describes the ratio between the non-Gaussian part and the Gaussian part of the spectrum, and is related to the so called non-Gaussian fraction $r_{\zeta_\sigma} \equiv (\mathcal{P}_{\sigma^2}/\mathcal{P}_\zeta)^{1/2}$. In this case the relation (18) becomes $A = 8r\alpha^2 f_{\text{NL}}^2 \mathcal{P}_{\zeta_g}$, and for $r = 10^{-1}$, one has that $f_{\text{NL}} \simeq 100$ implies $A = H/M_p$ as expected in the case where the mechanism of cosmological condensation described in [3, 4] is at work.

4 Conclusions

By assuming the field seeding the non-Gaussian component of curvature perturbations ζ is a dynamically generated condensate, we derived the relation (18) for the parameter f_{NL} in terms of the ratio of the inflationary scale H to the cutoff scale M_c of the effective theory describing the condensate. The condensate corrections affect the bispectrum according to Eq. (17), thus inducing the bound $M_c \gtrsim 10^{-1} M_P$. If the condensate field is sub-dominant it is shown that the mechanism of cosmological condensation of Refs. [3, 4] can produce $f_{\text{NL}} \simeq 100$.

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