Future deceleration due to effect of event horizon on cosmic backreaction

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The present acceleration of the universe leads to the formation of a cosmological future event horizon. We explore the effects of the event horizon on cosmological backreaction due to inhomogeneities in the universe. Beginning from the onset of the present accelerated era, we show that backreaction in presence of the event horizon causes acceleration to slow down in the subsequent evolution. Transition to deceleration occurs eventually, ensuring avoidance of a big rip.

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A. Introduction.— There exists overwhelming observational evidence for the present acceleration of the Universe [1]. The accelerating universe leads to a future event horizon from beyond which it is not possible for any signal to reach us. On the other hand, observations also tell us that our Universe is inhomogeneous up to at least the scales of super clusters of galaxies. The idea that backreaction originating from the density inhomogenities could lead to modifications in evolution of the universe as described by the background Friedmann-Robertson-Walker (FRW) metric at large scales has gained popularity in recent years [2–10]. Here we show that backreaction in the presence of the cosmological event horizon could have a remarkable consequence of ushering in another decelerated era beyond the present accelerating epoch.

In spite of numerous creative ideas proposed for the present acceleration [11], there is still a lack of convincing explanation of this phenomenon. The simplest possible explanation provided by a cosmological constant is endowed with conceptual problems [12]. Alternative mechanisms based on either modifications of the gravitational theory, or invoking extra fields with tailored dynamics mostly suffer from the coincidence problem, as to why the era of acceleration begins around the same era when the Universe becomes structured. The ultimate fate of our Universe remains clouded in considerable mystery. Backreaction from inhomogeneities provides an interesting platform for investigating this issue without invoking additional physics, since the effects of backreaction gain stength as the inhomogeneities develop into structures around the present era.

Approaches have been developed to calculate the effect of inhomegeneous matter distibution on the evolution of the Universe [2, 4, 5]. Though there exists some debate in the literaure on the impact of inhomogeneities on observables of an overall homogeneous FRW model [6, 13], arguments in favour of the viability of backreaction seem rather compelling [8]. Using the framework fomulated by Buchert [2] it has been shown [7] that backreaction could lead to an accelerated expansion during the present epoch. A notable application of the formalism has been developed by Wiltshire showing an apparent volume acceleration of the universe based on the different lapse of time between the underdense and overdense regions [9]. Further, gauge invariant averages in the Buchert framework have also been constructed recently [10].

While upcoming observations may ultimately decide whether backreaction from density inhomogeneities drives the present acceleration, the above studies [2, 3, 5, 7–10] have highlighted that backreaction could be a crucial ingredient of the present evolution and future fate of our Universe. Here we explore this issue with a fresh perspective, *viz.*, the impact of the event horizon on cosmological backreaction. The presently accelerating epoch dictates the existence of an event horizon since the transition from the previously matter dominated decelerating expansion. Since backreaction is evaluated from the global distribution of matter inhomogeneities, the event horizon demarcates the spatial regions which are causally connected to us and hence impact the evolution of our part of the Universe. Any contribution from inhomogenieties of scales which cross outside the event horizon due to accelerated expansion, needs to be excluded while computing the overall impact of backreaction. Such an approach has remained unexplored in previous studies of backreaction. We show here that backreaction with the event horizon could lead to a surprising possibility of transition to another decelerated future era.

B. Backreaction Framework.— In the framework developed by Buchert [2, 3] for the Universe filled with an irrotational fluid of dust the spacetime is foliated into flow–orthogonal hypersurfaces featuring the line–element $ds^2 = -dt^2 + g_{ij}dX^i dX^j$, where the proper time t labels the hypersurfaces and X^i are Gaussian normal coordinates (locating free–falling fluid elements or generalized fundamental observers) in the hypersurfaces, and g^{ij} is the full inhomogeneous three metric of the hypersurfaces of constant proper time. For a compact spatial domain $\mathcal{D}$ whose volume is given by $|\mathcal{D}|_g = \int_{\mathcal{D}} d\mu_g$ where $d\mu_g = \sqrt{^{(3)}g(t, X^1, X^2, X^3)}dX^1 dX^2 dX^3$, the scale fac-

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tor $a_{\mathcal{D}}(t) = \left(\frac{|\mathcal{D}|_g}{|\mathcal{D}_i|_g}\right)^{1/3}$ encodes the average stretch of all directions of the domain. The Einstein equations then lead to [2, 3]

$$\begin{aligned} 3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= -4\pi G \langle\rho\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} + \Lambda \\ 3H_{\mathcal{D}}^2 &= 8\pi G \langle\rho\rangle_{\mathcal{D}} - \frac{1}{2}\langle\mathcal{R}\rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}} + \Lambda \\ 0 &= \partial_t \langle\rho\rangle_{\mathcal{D}} + 3H_{\mathcal{D}} \langle\rho\rangle_{\mathcal{D}} \end{aligned} \tag{1}$$

where the average of the scalar quantities on the domain $\mathcal{D}$ is defined as $\langle f\rangle_{\mathcal{D}}(t) = \frac{\int_{\mathcal{D}} f(t,X^1,X^2,X^3)d\mu_g}{\int_{\mathcal{D}} d\mu_g} = |\mathcal{D}|_g^{-1}\int_{\mathcal{D}} f d\mu_g$, and where ρ, $\mathcal{R}$ and $H_{\mathcal{D}}$ denote the local matter density, the Ricci-scalar of the three-metric g_{ij}, and the domain dependent Hubble rate $H_{\mathcal{D}} = \dot{a}_{\mathcal{D}}/a_{\mathcal{D}}$ respectively. The kinematical backreaction $\mathcal{Q}_{\mathcal{D}}$ is defined as $\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left(\langle\theta^2\rangle_{\mathcal{D}} - \langle\theta\rangle_{\mathcal{D}}^2\right) - 2\sigma_{\mathcal{D}}^2$, where θ is the local expansion rate and $\sigma^2 = 1/2\sigma_{ij}\sigma^{ij}$ is the squared rate of shear. $\mathcal{Q}_{\mathcal{D}}$ encodes the departure from homogeneity.

The "global" domain $\mathcal{D}$ is assumed to be separated into subregions $\mathcal{F}_\ell$ which themselves consist of elementary space entities $\mathcal{F}_\ell^{(\alpha)}$ that may be associated with some averaging length scale, i.e., $\mathcal{D} = \cup_\ell \mathcal{F}_\ell$, where $\mathcal{F}_\ell = \cup_\alpha \mathcal{F}_\ell^{(\alpha)}$ and $\mathcal{F}_\ell^{(\alpha)} \cap \mathcal{F}_m^{(\beta)} = \emptyset$ for all $\alpha \neq \beta$ and $\ell \neq m$. The average of the scalar valued function f on the domain $\mathcal{D}$, may then be split into the averages of f on the subregions $\mathcal{F}_\ell$ in the form, $\langle f\rangle_{\mathcal{D}} = \sum_\ell |\mathcal{D}|_g^{-1} \sum_\alpha \int_{\mathcal{F}_\ell^{(\alpha)}} f d\mu_g = \sum_\ell \lambda_\ell \langle f\rangle_{\mathcal{F}_\ell}$, where $\lambda_\ell = |\mathcal{F}_\ell|_g/|\mathcal{D}|_g$, is the volume fraction of the subregion $\mathcal{F}_\ell$. Due to the $\langle\theta\rangle_{\mathcal{D}}^2$ term, the expression for the backreaction $\mathcal{Q}_{\mathcal{D}}$ is given by

$$\mathcal{Q}_{\mathcal{D}} = \sum_\ell \lambda_\ell \mathcal{Q}_\ell + 3\sum_{\ell\neq m} \lambda_\ell\lambda_m \left(H_\ell - H_m\right)^2 \tag{2}$$

where, $\mathcal{Q}_\ell$ is defined in terms of $\mathcal{F}_\ell$ in the same way as $\mathcal{Q}_{\mathcal{D}}$ is defined in terms of $\mathcal{D}$. The shear part $\langle\sigma^2\rangle_{\mathcal{F}_\ell}$ is completely absorbed in $\mathcal{Q}_\ell$ whereas the variance of the local expansion rates $\langle\theta^2\rangle_{\mathcal{D}} - \langle\theta\rangle_{\mathcal{D}}^2$ is partly contained in $\mathcal{Q}_\ell$ but also generates the extra term $3\sum_{\ell\neq m}\lambda_\ell\lambda_m \left(H_\ell - H_m\right)^2$. Analogous to the scale factor for the global domain, a scale factor a_ℓ for each of the subregions $\mathcal{F}_\ell$ can be defined such that $|\mathcal{D}|_g = \sum_\ell |\mathcal{F}_\ell|_g$, and hence $a_{\mathcal{D}}^3 = \sum_\ell \lambda_{\ell_i} a_\ell^3$, where $\lambda_{\ell_i} = |\mathcal{F}_{\ell_i}|_g/|\mathcal{D}_i|_g$ is the initial volume fraction of the subregion $\mathcal{F}_\ell$. Now from Eq.(1) one gets

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \sum_\ell \lambda_\ell \frac{\ddot{a}_\ell(t)}{a_\ell(t)} + \sum_{\ell\neq m} \lambda_\ell\lambda_m \left(H_\ell - H_m\right)^2 \tag{3}$$

Following the simplifying assumption of Ref.[3], (which captures the essential physics) we work with only two subregions. Clubbing those parts of $\mathcal{D}$ which consist of initial overdensity as $\mathcal{M}$ (called "wall"), and those with initial underdensity as $\mathcal{E}$ (called "void"), such that $\mathcal{D} = \mathcal{M}\cup\mathcal{E}$, one obtains $H_{\mathcal{D}} = \lambda_{\mathcal{M}} H_{\mathcal{M}} + \lambda_{\mathcal{E}} H_{\mathcal{E}}$, with similar expressions for $\langle\rho\rangle_{\mathcal{D}}$ and $\langle\mathcal{R}\rangle_{\mathcal{D}}$, and

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \lambda_{\mathcal{M}} \frac{\ddot{a}_{\mathcal{M}}}{a_{\mathcal{M}}} + \lambda_{\mathcal{E}} \frac{\ddot{a}_{\mathcal{E}}}{a_{\mathcal{E}}} + 2\lambda_{\mathcal{M}}\lambda_{\mathcal{E}}(H_{\mathcal{M}} - H_{\mathcal{E}})^2 \tag{4}$$

Here $\lambda_{\mathcal{M}} + \lambda_{\mathcal{E}} = 1$, with $\lambda_{\mathcal{M}} = |\mathcal{M}|/|\mathcal{D}|$ and $\lambda_{\mathcal{E}} = |\mathcal{E}|/|\mathcal{D}|$. Since the global domain $\mathcal{D}$ is large enough for a scale of homogeneity to be associated with it, one can write $|\mathcal{D}|_g = \int_{\mathcal{D}} \sqrt{-g}\, d^3X = f(r) a_F^3(t)$, where $f(r)$ is a function of the FRW comoving radial coordinate r. It then follows that $a_{\mathcal{D}} = \left(\frac{f(r)}{|\mathcal{D}_i|_g}\right)^{1/3} a_F$, and hence, the volume average scale factor $a_{\mathcal{D}}$ and the FRW scale factor a_F are related by $a_{\mathcal{D}} = c_F a_F$, where c_F is constant in time. Thus, $H_F = H_{\mathcal{D}}$, where H_F is the FRW Hubble parameter associated with $\mathcal{D}$.

C. Effect of event horizon.— We now come to the central issue of the paper, as to what happens to the evolution of the universe once the present stage of acceleration sets in. Note henceforth, we do not need to necessarily assume that the acceleration is due to backreaction [3, 7]. For the purpose of our present analysis, it suffices to consider the observed accelerated phase of the universe that could occur due to any of a variety of mechanisms [11]. Given that we are undergoing a stage of acceleration since transition from an era of structure formation, our aim here is to explore the subsequent evolution of the Universe due to the effects of backreaction in presence of the cosmic event horizon is defined by

$$r_h = a_{\mathcal{D}} \int_t^\infty \frac{dt'}{a_{\mathcal{D}}(t')} \tag{5}$$

which forms at the onset of acceleration.

Following the Buchert framework [2, 3] as discussed above, the global domain $\mathcal{D}$ is divided into a collection of overdense regions $\mathcal{M} = \cup_j \mathcal{M}^j$, with total volume $|\mathcal{M}|_g = \sum_j |\mathcal{M}^j|_g$, and underdense regions $\mathcal{E} = \cup_j \mathcal{E}^j$ with corresponding volume $|\mathcal{E}|_g = \sum_j |\mathcal{E}^j|_g$. Assuming that the scale factors of the regions $\mathcal{E}^j$ and $\mathcal{M}^j$ are respectively given by $a_{\mathcal{E}_j} = c_{\mathcal{E}_j} t^\alpha$ and $a_{\mathcal{M}_j} = c_{\mathcal{M}_j} t^\beta$, where α, β, $c_{\mathcal{E}_j}$ and $c_{\mathcal{M}_j}$ are constants, one has

$$a_{\mathcal{E}}^3 = c_{\mathcal{E}}^3 t^{3\alpha}; \quad a_{\mathcal{M}}^3 = c_{\mathcal{M}} t^{3\beta} \tag{6}$$

where $c_{\mathcal{E}}^3 = \frac{\sum_j c_{\mathcal{E}_j}^3 |\mathcal{E}_i^j|_g}{|\mathcal{E}_i|_g}$ is a constant, and similarly for $c_{\mathcal{M}}$. Note here that we do not assume any specific form for the curvature and the backreaction in the individual sub-domains. Below we consider two cases of accelerated expansion of the global domain separately. Keeping our analysis close to observations, we first model the onset of the present acceleration of the Universe by an exponential expansion. Next, in order to show that our conclusions are not contingent to any specific mechanism or model responsible for origin of the present acceleration, we also consider the other extreme of a simple power law for the accelerating global scale factor.

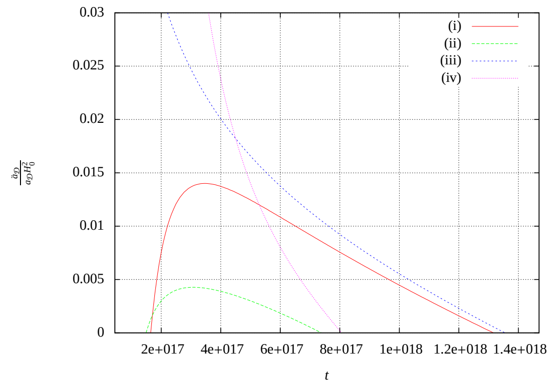

FIG. 1: The dimensionless global acceleration parameter $\frac{\ddot{a}_D}{a_D H_0^2}$ is plotted versus time for various values of the parameters (i) $\alpha = 0.995, \beta = 0.5$, (ii) $\alpha = 0.999, \beta = 0.6$, (iii) $\alpha = 1.0, \beta = 0.5$, and (iv) $\alpha = 1.02, \beta = 0.66$.

We first consider the case of exponential expansion, i.e., $a_{\mathcal{D}} \propto e^{H_{\mathcal{D}} t}$. The event horizon in this case is given by $r_h = H_D^{-1}$ which is a constant. The volume fraction of the subdomain $\mathcal{M}$ can be written in terms of the corresponding scale factors as $\lambda_{\mathcal{M}} = \frac{a_{\mathcal{M}}^3 |\mathcal{M}_i|_g}{a_{\mathcal{D}}^3 |\mathcal{D}_i|_g}$. Since an event horizon forms, only those regions of $\mathcal{D}$ that are within the event horion are accessible to us. Hence we will be able to measure an apparent volume fraction $\lambda_{\mathcal{M}_h}$ given by $\lambda_{\mathcal{M}_h} = \frac{a_{\mathcal{M}}^3 |\mathcal{M}_i|_g}{\frac{4}{3}\pi (H_{\mathcal{D}}^{-1})^3}$. From eqn. (6) it follows that

$$\lambda_{\mathcal{M}_h} = \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3}; \quad \lambda_{\mathcal{E}_h} = 1 - \lambda_{\mathcal{M}_h} \tag{7}$$

where $c_{\mathcal{M}_h}^3 = c_{\mathcal{M}}^3 |\mathcal{M}_i|_g / \frac{4}{3}\pi$ is a constant, and $\lambda_{\mathcal{E}_h}$ the apparent volume fraction for the subdomain $\mathcal{E}$ is obtained by normalizing the total accessible volume in the presence of the event horizon. Using Eqns. (6) and (7) we can now write (4) for the global evolution as

$$\begin{aligned} \frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3} \frac{\beta(\beta-1)}{t^2} + \left(1 - \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3}\right) \frac{\alpha(\alpha-1)}{t^2} \\ &+ 2\frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3} \left(1 - \frac{c_{\mathcal{M}_h}^3 t^{3\beta}}{r_h^3}\right) \left(\frac{\beta}{t} - \frac{\alpha}{t}\right)^2 \end{aligned} \tag{8}$$

The global acceleration $\ddot{a}_{\mathcal{D}}$ vanishes at times corresponding to the zeroes of the above equation, given by

$$t^{3\beta} = \frac{r_h^3 \left[(3\beta - \alpha - 1) \pm \sqrt{(3\beta - \alpha - 1)^2 + 8\alpha(\alpha - 1)}\right]}{4(\beta - \alpha) c_{\mathcal{M}_h}^3} \tag{9}$$

The scale factor of the "wall" grows as t^β, where $\beta \leq 2/3$, (with the equality corresponding to a matter dominated FRW expansion when there is no effect of backreaction from any inhomogeneities comprising the sub-domains of the "wall"). Hence, (9) corresponds to real solutions for $\alpha \geq \frac{1}{3}\left[(\beta+1) + 2\sqrt{2\beta(1-\beta)}\right]$.

Now, let us consider the following two sub-cases separately: *Case I:* $\frac{1}{3}\left[(\beta+1) + 2\sqrt{2\beta(1-\beta)}\right] \leq \alpha < 1$. There exist two real solutions (9) corresponding to two values of time when the global acceleration vanishes. In Fig.1 we plot a dimensionless global acceleration parameter $\frac{\ddot{a}_D}{a_D H_0^2}$ with time using (8). The curves (i) and (ii) correspond to this case showing that the Universe first enters the epoch of acceleration due to backreaction, which subsequently slows down and finally vanishes at the onset of another decelerating era in the future. We use standard values of the parameters $H_{\mathcal{D}_0} \approx 2.29 \times 10^{-18} s^{-1}$, $r_h = H_{\mathcal{D}_0}^{-1}$ and $t_0 \approx 4.32 \times 10^{17} s$, while choosing the appropriate range for the parameters α and β, as given in the figure caption. Based on the N-body simulation values used in [3] we also take $\lambda_{\mathcal{M}_{h0}} = 0.09$. Using the relation $z_T = \exp\left[H_{\mathcal{D}_0}(t_0 - t_T)\right] - 1$, where t_T corresponds to the transition time in the past, the curve (i) corresponds to the transition redshift of $z_T \simeq 0.844$, and for curve (ii) we have $z_T \simeq 0.914$ (which are close to the ΛCDM value for the standard transition redshift [14]). *Case II:* $\alpha \geq 1$. From (9) it follows that there is only one real solution (plus sign for the square root) corresponding to a transition from acceleration to deceleration. This case models the Universe which accelerates due to some other mechanism (not backreaction), but subsequently enters an epoch of deceleration in future due to backreaction of inhomogeneities in the presence of the event horizon (see curves (iii) and (iv) of Fig.1).

The above results were obtained by considering a fixed event horizon which forms in the case of exponential expansion. However, as the expansion slows down, the event horizon becomes dynamic. In order to see what happens in the other extreme of a pure power law expansion, we now consider the case when $a_{\mathcal{D}} \propto t^\gamma$ (with $\gamma > 1$). From (5) the event horizon is given by $r_h = \frac{t}{\gamma - 1}$. Proceeding as earlier, we find the expression for the apparent "wall" and "void" volume fractions given by

$$\lambda_{\mathcal{M}_h} = g_{\mathcal{M}_h}^3 t^{3(\beta-1)}; \quad \lambda_{\mathcal{E}_h} = 1 - \lambda_{\mathcal{M}_h} \tag{10}$$

where $g_{\mathcal{M}_h}^3$ is a constant. It then follows that eq.(4) in this case takes the form

$$\begin{aligned} \frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= g_{\mathcal{M}_h}^3 t^{3(\beta-1)} \frac{\beta(\beta-1)}{t^2} + \left(1 - g_{\mathcal{M}_h}^3 t^{3(\beta-1)}\right) \frac{\alpha(\alpha-1)}{t^2} \\ &+ 2 g_{\mathcal{M}_h}^3 t^{3(\beta-1)} \left(1 - g_{\mathcal{M}_h}^3 t^{3(\beta-1)}\right) \left(\frac{\beta}{t} - \frac{\alpha}{t}\right)^2 \end{aligned} \tag{11}$$

Zeroes of the above equation (vanishing acceleration) correspond to the solution

$$t^{3(\beta-1)} = \frac{\left[(3\beta - \alpha - 1) \pm \sqrt{(3\beta - \alpha - 1)^2 + 8\alpha(\alpha - 1)}\right]}{4(\beta - \alpha) g_{\mathcal{M}_h}^3} \tag{12}$$

A similar analysis as in the exponential case, leads us to the following conclusions. For

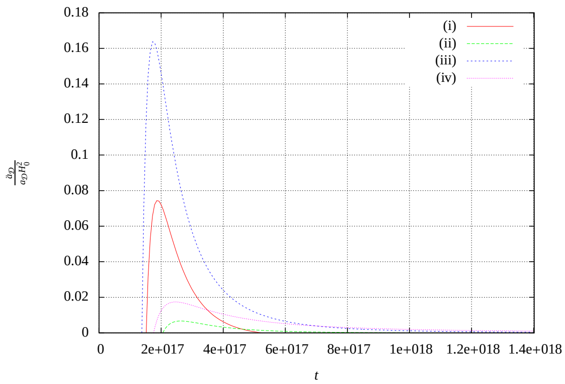

FIG. 2: The dimensionless global acceleration parameter $\frac{\ddot{a}_D}{a_D H_0^2}$ is plotted versus time for the values of the parameters (i) $\alpha = 0.986, \beta = 0.5$, (ii) $\alpha = 0.998, \beta = 0.6$, (iii) $\alpha = 1.0, \beta = 0.5$, and (iv) $\alpha = 1.01, \beta = 0.66$. Here the event horizon is taken to evolve corresponding to a power-law expansion.

$\frac{1}{3}\left[(\beta+1) + 2\sqrt{2\beta(1-\beta)}\right] \leq \alpha < 1$, the global acceleration first appears and then disappears in finite time (see the curves (i) and (ii) of Fig.2). However, for $\alpha \geq 1$, the only real solution of (12) corresponds to a transition from deceleration to acceleration. In this case subsequently the acceleration does slow down but vanishes again only asymptotically in time (see the curves (iii) and (iv) of Fig.2), as is evident from (11).

D. Conclusions.— To summarize, in this work we have explored the effect of backreaction due to inhomogeneities on the evolution of the Universe which is already in the present observed accelerating epoch. The cosmic event horizon which is an inevitable consequence of the global acceleration, impacts the role of inhomogeneities on the evolution in a non-trivial way, causing the acceleration to slow down significantly with time. Our results indicate the fascinating possibility of backreaction being responsible not only for the present acceleration as shown in earlier works [3, 7], but also leading to a subsequent transition to another decelerated era in the future. The other possibility following from our analysis is that of the Universe having entered the current accelerating era due to a different mechanism [11], but with the framework of backreaction [2, 3] in presence of the event horizon later causing acceleration to slow down and vanishing subsequently. The scenario of the currently slowing down acceleration leading to another transition to deceleration fits smoothly with the earlier era of structure formation and the first transition to acceleration in the standard ΛCDM model, as shown here (transiton redshift $z_T \approx 0.9$).

Before concluding, it may be relevant to make the following observations. First, our analysis (from (7) onwards) is valid only while the event horizon exists. Thus, as the acceleration vanishes at some epoch in the future, the above set of equations will no longer remain valid. Physically, the scales which crossed ouside the horizon earlier, will begin re-entering and the backreaction from their associated inhomogenieties will again start causally impacting the evolution of our part of the Universe. Such a scenario is somewhat reminiscent of the horizon crossing of modes during inflation in the early universe, and their subsequent reentry with rich cosmological consequences [15]. In the present context, the impact on the global evolution of the reentering scales needs to investigated further. Secondly, it may be noted here that though the concept of the event horizon is observer dependent, it follows from the symmetry of the equations (4) and (7) that our analysis should lead to similar conclusions for a "void" centric observer, as it does for a "wall" centric one. Finally, we wish to emphasize that numerical simulations of the integro-differential set of equations (4), (5), (6), and (7) would be required for making more accurate predictions of observational interest.

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