

Propagation of thermonuclear flame in SNIa

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Abstract

The propagation of thermonuclear flame in presupernovae Ia is considered. Front parameters are obtained, some speculations on front stability are presented.

1 Introduction

The problem of thermonuclear flame propagation in supernovae Ia still stands. Full hydrodynamic simulation requires knowledge of small scale parameters of flame: its normal propagation velocity, instability regimes. In this paper we show instability manifestations using a toy model. In literature there is no clear understanding whether flame front is stable or pulsates under instability [1]. In the second part we carry out full hydrodynamical simulations of flame and obtain flame parameters for the range of densities, a similar analysis was performed in [2].

2 Toy Model

Let us consider a simple model for evolution of temperature T and reagent fraction c ¹:

$$\partial_t T = \kappa \partial_x^2 T + \omega_0 c \Theta(T - T_0), \quad \partial_t c = -\omega_0 c \Theta(T - T_0), \quad (1)$$

where Θ is a theta-function (a step-function). The system models deflagration burning in solid propellants because two main physical processes that drive slow front are presented in it: thermoconductivity and burning itself. Medium in supernovae is gaseous, but when flame propagates in the centre

¹The model was proposed by P.V. Sasorov (ITEP)

of the white dwarf, in dense matter ($\rho \sim 10^8 \div 10^9 \text{ g/cm}^3$) density jump is low, so hydrodynamical effects are small and evolution matches burning of solid medium. Moreover, at Lewis number $\text{Le} \gg 1$, the process of burning in supernovae in general is similar to that described to our system. The choice of burning rate function is explained below.

A stationary wave must obey boundary conditions:

$$t = 0, x \rightarrow \infty : T = 0, c = 1, \partial_x T = 0, \quad t = 0, x \rightarrow -\infty : c = 0. \quad (2)$$

The system can be simplified by redefinition of x to put $\kappa = 1$. We are searching for the wave front, so every quantity depends only on $\xi = x - vt$. Due to translation invariance we put $\xi = 0 : T = T_0$ (the point of center of flame). The system can be easily solved:

$$\begin{aligned} \xi > 0 : \quad & c = 1, \quad T = T_0 e^{-v\xi}, \\ \xi < 0 : \quad & c = e^{\omega_0 \xi / v}, \quad T = 1 - \frac{\omega_0}{(\omega_0/v)^2 + \omega_0} e^{\omega_0 \xi / v}, \\ & v = \sqrt{\frac{1-T_0}{T_0}} \omega_0. \end{aligned} \quad (3)$$

For more simplification we put $v = 1$, that means $\omega_0 = T_0/(1 - T_0)$. Let us finally write down the simplified system and its solution:

$$\partial_t T = \partial_x^2 T + \omega_0 c \Theta(T - T_0), \quad \partial_t c = -\omega_0 c \Theta(T - T_0), \quad (4)$$

$$\begin{aligned} \xi > 0 : \quad & c = 1, \quad T = T_0 e^{-\xi}, \\ \xi < 0 : \quad & c = e^{\omega_0 \xi}, \quad T = 1 - \frac{1}{\omega_0 + 1} e^{\omega_0 \xi}. \end{aligned} \quad (5)$$

The stability of such a system under small perturbations can be easily considered analytically:

$$T = T_{\text{n.p.}} + T_{\text{p}}, \quad c = c_{\text{n.p.}} + c_{\text{p}}, \quad (6)$$

$$T_{\text{p}} = e^{pt} f(\xi), \quad c_{\text{p}} = e^{pt} g(\xi). \quad (7)$$

After some calculations [3] the following result could be obtained: the system is stable when $\omega_0 < 6$, and perturbations grow exponentially when $\omega_0 > 6$.

Such a system can be easily numerically simulated and full evolution of unstable regime could be obtained. The task is set as follows:

$$c|_{t=0} = c_{\text{theor}}, \quad T|_{t=0} = T_{\text{theor}}, \quad (8)$$

where c_{theor} and T_{theor} are defined in (5) with center at x_c and bound conditions:

$$c|_{x=0} = 0, \quad c|_{x=L} = 1, \quad T|_{x=0} = 1, \quad T|_{x=L} = 0. \quad (9)$$

ω_0	v	comm.
1.0	1.000	flame
4.0	0.996	flame
5.5	1.006	flame
5.8	1.010	flame
6.0	1.019	flame
6.1	–	therm
7.0	–	therm
8.0	–	therm
9.0	–	therm

Table 1: Numerical simulation. v – measured front velocity.

So we set the exact analytical solutions as initial conditions and watch their evolution. The Table 1 shows results of simulations. Solutions could be split into two groups: “flame” and “therm”. “flame” – is the evolution as stationary flame front with constant velocity, this regime exists when $\omega_0 < 6$ (according to the table of results). “therm” describes flame decay, like evolution under thermoconductivity without burning (example of such evolution is shown in Fig. 1). Zero burning rate when $T < T_0$ forbids flame ignition after “therm” regime, so it clearly cuts only unstable evolution.

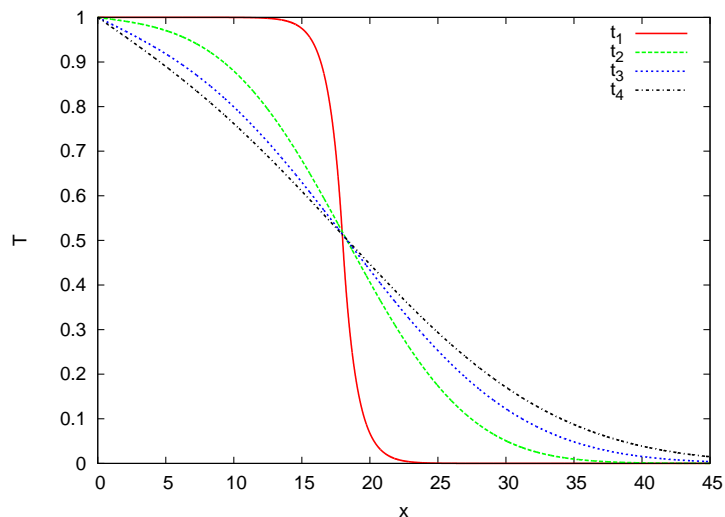


Figure 1: Front positions for $\omega_0 = 8$ at different time moments: $t_0 < t_1 < t_2 < t_3$.

The analytical predictions are in a very good agreement with numerical simulation of the model. So this model and its modifications may be used for theoretical study of unstable flame fronts.

3 One-dimensional flame properties

Let us consider full hydrodynamical evolutions of flame in presupernova Ia. Our goal is to study flame acceleration and deflagration to detonation transition [4]. Typical parameters of medium in the center of WD are the following [5]: $T \sim 10^9$ K, $\rho \sim 10^9$ g/cm³, chemical composition – ¹²C+¹⁶O. Let us suppose that only ¹²C remains in chemical composition. For given conditions the following relations hold: $Pr \ll 1$, $Le \gg 1$. It means that thermoconductivity is the only diffusion mechanism that matters in this case. The coefficient of thermoconductivity is the sum of two parts: electron conductivity [6] and radiative conductivity [7]. We consider the only one nuclear reaction for approximation (also this approach gives ability to study physical effects):



Here Mg^* means the excitation state, it decays through 3 channels: with n, with p, with α ; and we take into account the sum of rates, which could be found in [8]. Its caloricity is $q_1 = 5.5 \cdot 10^{17}$ erg/g. The reaction is the first in the network, it occurs between two highly charged nuclei (it is one of the slowest), so we could suppose that it determines the whole reaction rate. Further burning could be introduced in our model by changing caloricity. For burning up to ⁵⁶Ni it will be $q_2 = 9.2 \cdot 10^{17}$ erg/g. Nuclear screening should be taken into account because $\Gamma = \langle E_{\text{coul}} \rangle / kT \sim 1$.

The problem is formulated in the following way: a full one-dimensional hydrodynamical system of equations with thermoconductivity and nuclear reactions is considered. It is solved by our numerical code FRONT (where an implicit numerical scheme with a Newton iterations solver is used [9]). Initial region of calculation is filled uniformly with ¹²C at given T_0 and ρ_0 . Right wall should be free for stream. Left wall is heated by the linear law $T = T_0 + t(T_1 - T_0)/\tau$. Where τ obeys $\tau \gg L/c_s$ (L – the size of region of interest, c_s – sound speed). Such conditions lead to deflagration wave ignition by the hot wall. The sequential flame positions are shown on Fig. 2. Table 2 shows results of normal flame speed determination for different initial density ρ_0 .

It should be emphasized here that all velocities obtained are correct only for one reaction in network ¹²C+¹²C. Use of full nuclear network changes the speed radically [10].

$\rho_0, \text{g/cm}^3$	Cal.	$T_{\text{max}}, 10^9 \text{ K}$	$\rho_u, \text{g/cm}^3$	$\rho_b, \text{g/cm}^3$	$v_n, \text{km/s}$	$\Delta x_{\text{fr}}, \text{cm}$
$2 \cdot 10^8$	Mg	6.8	$2.05 \cdot 10^8$	$1.35 \cdot 10^8$	222	$5 \cdot 10^{-4}$
	Ni	7.9	$2.12 \cdot 10^8$	$1.16 \cdot 10^8$	460	$4 \cdot 10^{-4}$
$7 \cdot 10^8$	Mg	9.0	$7.27 \cdot 10^8$	$5.38 \cdot 10^8$	888	$4.6 \cdot 10^{-5}$
	Ni	10.8	$8.08 \cdot 10^8$	$5.07 \cdot 10^8$	1950	$5.3 \cdot 10^{-5}$
$2 \cdot 10^9$	Mg	11	$2.10 \cdot 10^9$	$1.67 \cdot 10^9$	1880	$1.1 \cdot 10^{-5}$
	Ni	13	$2.37 \cdot 10^9$	$1.62 \cdot 10^9$	3450	$2.8 \cdot 10^{-5}$

Table 2: Measured deflagration flame front parameters for reaction $^{12}\text{C}+^{12}\text{C}$.

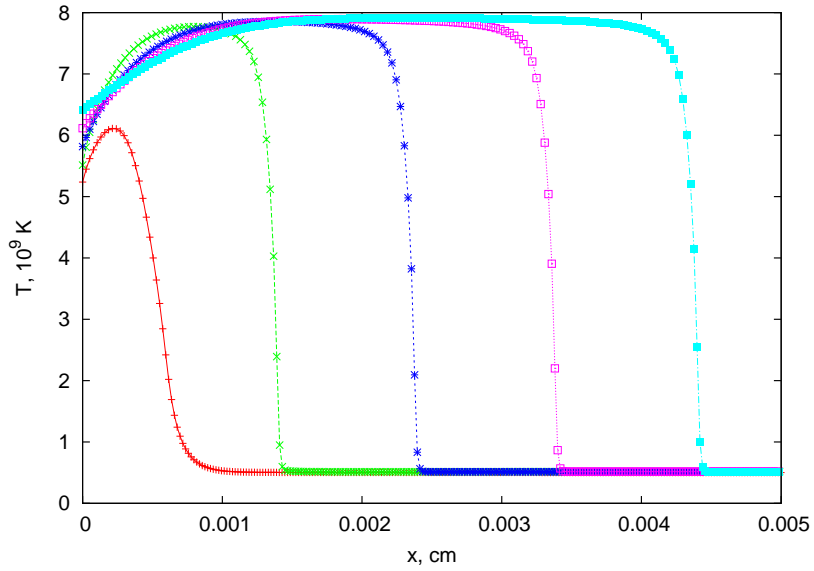


Figure 2: Sequential temperature distributions (for different physical time).

4 Conclusions

The toy model was presented for several purposes: first it clearly shows how instability of flame front manifests: it leads to front destruction, second, the model could be used in theoretical speculations and in academic studies. Full hydrodynamic simulations shows stable front propagations at all considered densities (with no evolution shown by “therm” regime in toy model), so we could state that thermonuclear flame front is stable in all density range from $2 \cdot 10^8 \text{ g/cm}^3$ to $2 \cdot 10^9 \text{ g/cm}^3$, the reason for that should be explored. Flame front parameters were obtained in hydrodynamical simulations. Front ve-

locities differs greatly from determined in [2]. The reason is in very simple nuclear network: only one reaction is taken into account (for details see [10]). The work is supported partly by Federal Program “Scientific and pedagogical specialists of innovation Russia” contract number 02.740.11.0250, partly by the Russian Foundation for Basic Research grant RFBR 10-02-00249-a, by SNSF SCOPES project No. IZ73Z0-128180/1, and “Dynasty” foundation.

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