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# Confidence Level Estimator for cosmological models *(Research Note)*

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**ABSTRACT**

Models of the Universe like the Concordance Model today used to interpret cosmological observations give expectation values for many cosmological observables so accurate that frequently peoples speak of Precision Cosmology. The quoted accuracies however do not include the effects of the priors used in optimizing the Model nor allow to evaluate the confidence one can attach to the Model. We suggest an estimator of the Confidence Level for Models and the accuracies of the expectation values of the Model observables.

**Key words.** cosmology: cosmological parameters – cosmology: observations – cosmology: miscellaneous

## 1. Introduction

After the discovery of the Cosmic Microwave Background (CMB) in 1964 (Penzias and Wilson 1964), cosmological observation went through a very rapid expansion. Measurements confirmed to very high degree of accuracy the expected first order (planckian spectrum, isotropic distribution, absence of polarization) and second order (angular power spectra of tiny temperature anisotropies and even smaller level of residual polarization) features of the CMB (COBE 2000, WMAP 2010). Complementary information then arrived from no-CMB channels of information, namely: i)peculiarities in the rotation curves of galaxies and clusters of galaxies, (which suggest the existence of ”invisible” or ”dark” matter), ii)discovery of BAO (Barion Acoustic Oscillations) in the spatial distribution of the luminous matter, similar to the oscillations observed in the CMB angular power spectra, iii)Supernovae Ia used to get the distance of objects at very large Z, iv)N-body simulations of the formation of matter condensations from isotropic distribution of matter particles (LSS 2010).

The whole set of CMB and no-CMB information, besides confirming the general layout of the Standard Big Bang Model (Peebles 1993), suggests that the Universe is probably suffering re-acceleration and its geometry substantially flat (euclidean). To close the Universe it was therefore assumed the existence, beside barionic and dark matter, of a third component of the matter-energy mixture, Dark Energy. Moreover besides the Hubble constant $H_o$, the ratio between the primordial abundances of helium and hydrogen and the Universe average matter density $\Omega$ (in units of the critical density $\rho_c = 3H_o^2/(8\pi G)$), new observables were introduced: the abundances (still in unit of critical density) of barionic matter $\Omega_b$, dark matter $\Omega_{dm}$ and dark energy $\Omega_\Lambda$, the optical thickness $\tau$ of the Universe at reionization (when at $10 \leq Z_{ion} \leq 1300$ stable matter condensation emerged from the primordial uniform matter distribution), the amplitude and spectral index ( $A_s$ and $n_s$) of the fluctuations (seed for the birth of the above condensations), and $\sigma_8$ an indicator of the fluctuations of the galactic matter distribution.

To reconcile these new data and the old Big Bang model the Concordance Model or $\Lambda$-CDM Model gradually emerged (Ostriker and Steinhardt 1995, Kowalski et al. 2008). It has six independent parameters, representative of six observable chosen among the observables listed above, usually $H_o$, $\Omega_b$, $\Omega_{dm}$, $\tau$, $A_s$ and $n_s$, from which other observable (e.g age of the Universe $T_{univ} \simeq 1/H_o$, critical density $\rho_c$ of matter-energy, Dark Energy density, $\Omega_\Lambda$ (in unit of $\rho_c$), reionization red shift $Z_{ion}$ and $\sigma_8$) can be derived.

The observables-parameters and the observational data are entangled: different sets of data are produced by different parameter combinations. Disentangling the effects of the various parameters and getting the parameters values is extremely hard. A way out is obtained fitting the Model to the full set of CMB and no-CMB cosmological data and getting the parameter values by best fit methods. This is done by Montecarlo methods using Markov chains to implement the stochastic procedure which drives Montecarlo calculations (Landau and Binder 2009) with additional *a priori* assumptions, e.g. the Universe geometry ($\Omega = \Omega_b + \Omega_{dm} + \Omega_\Lambda = 1$ if the Universe is flat) and/or intervals of allowed variability for the parameters. The parameter values which give the best fit are interpreted as expectation values of the associated observables.

This procedure, which forces the model toward preferred solutions, is now well established. Frequently improved expectation values are published. They are obtained adding new observational data to the preexisting data base (e.g.Komatsu et al. 2009, Brown et al. 2009, Komatsu et al. 2010). Recently published values (Komatsu et al. 2010) of free and derived observable - parameters of the Concordance Model are shown in Table 1 and Table 2. The accuracy of the expectation values is so high that observers sometimes speak of Precision Cosmology (e.g. Primack 2005, Bridle et al. 2003).

In principle all the observables are measurable. Agreement between measured and expected values is a proof that the model described by the free parameters is a good one. Unfortunately for five of the six free observables - parameters of the Concordance Model measurements are very poor or not yet available. And no significant improvement is expected in the near future. For these observable in literature there are only large intervals inside which the measured values are expected. The same intervals are usually assumed as the maximum variability range of the parameters used in Monte Carlo studies of the Concordance Model (Brown et al. 2009). They are shown in Table 1. The only exception is the Hubble constant for which almost direct measurements are now very accurate (Riess et al. 2009).

## 2. Expected and measured values

Measured values $M^i$ and their uncertainties $\delta_{M^i}$ are extracted using classical statistical methods (see for instance Bevington and Robinson 1992) from sets of repeated measurements of a parameter $i$ or from combinations of measured quantities directly linked to $i$. The distribution of the measured values around their average $\overline{M^i}$ is usually gaussian.

Expected values $E^i_v$ are the values of the model parameters which gives the best fit of the Model to the full set of observations assuming *a priori* conditions or *priors*. The uncertainties $\delta_{E_v}$ of $E_v$ are the widths of the distributions (not always gaussian) which encompass 68 % of the parameter values obtained repeating the calculation with a random initial choice inside *a priori* chosen intervals of free parameter variabilities.

In doing it, sometimes unintentionally, a transition from classical to bayesian statistics occurred (Stanford 2003). Bayesian probabilities, no longer precisely defined as frequencies or ratios of measured quantities, give the (0-1) confidence level we associate to an event occurrence and are obtained by *a priori* assumptions. The Bayes Theorem allows to evaluate the effects new observations have on preexisting knowledge:

$$P_E(H) = [P(H)/P(E)]P_H(E) \tag{1}$$

where

i)H is a set of measured values of a parameter and $P(H)$ its probability distribution, derived for instance from previous observations, assumed *ab initio*, i.e. before new observations are made;

ii)E is a new set of measurements of the same parameter and $P(E)$ its distribution assumed *a priori*, or *prior*, without preexisting information;

iii) $P_H(E)$ is the conditional or "direct" probability of getting E given H;

iv) $P_E(H)$ is the *a posteriori* conditional probability of getting H given E, or "inverse" probability.

The ratio $0 \leq [P(H)/P(E)] \leq 1$ is the Confidence level $C_l(E)$ we can associate to the prior used to improve our knowledge.

For each parameter $i$ of a multi parameter model we can write

$$P^i(H) = P^i_{me}(X^i_{me}, \sigma^i_{me}) \tag{2}$$

$$P^i(E) = P^i_{ex}(X^i_{ex}, \sigma^i_{ex}) \tag{3}$$

where $P^i_{me}(X^i_{me}, \sigma^i_{me})$ and $P^i_{ex}(X^i_{ex}, \sigma^i_{ex})$ are the probabilities distributions of the measured (pedix $_{me}$) and expected (pedix $_{ex}$) values $X^i$, with dispersion $\sigma^i$. Assuming gaussian distributions the Confidence level of the expected value of parameter $i$ can be written:

$$\begin{aligned} C^i_l &= [P^i(H)/P^i(E)] \\ &= P^i_{me}(X^i_{me}, \sigma^i_{me})/P^i_{ex}(X^i_{ex}, \sigma^i_{ex}) \qquad (4) \\ &= \frac{\sigma^i_{ex}}{\sigma^i_{me}}\, exp\{-[(\Delta X^i_{me}/\sigma^i_{me})^2 - (\Delta X^i_{ex}/\sigma^i_{ex})^2]/2\} \end{aligned}$$

where $\Delta X^i = |X^i - \overline{X^i}|$.

For a model not too far from the real world

$$< E^i > \; \longrightarrow \; M^i, \quad \Rightarrow \quad \Delta X^i_{me} \simeq \Delta X^i_{ex} \simeq \Delta X^i < \sigma^i$$

therefore

$$C^i_l = \frac{\sigma^i_{ex}}{\sigma^i_{me}}\Big[1 - \frac{\Delta X^i}{2}\,\frac{(\sigma^i_{me})^2 - (\sigma^i_{ex})^2}{(\sigma^i_{me})^2\,(\sigma^i_{ex})^2}\Big] \to \sigma^i_{ex}/\sigma^i_{me} \tag{5}$$

for parameter $i$ and for the whole model

$$C_l = \frac{1}{6}\sum_{i=1}^{6} C^i_l \tag{6}$$

The resulting Confidence Levels of the expected values of the free parameters of the Concordance Model are shown in the last column of Table 1. For the whole Model

$$C_l \sim 0.4$$

a value which leaves room for other models to be considered, obtained for instance relaxing priors, e.g. $\Omega_b + \Omega_c + \Omega_\Lambda = 1$.

## 3. Discussion

The numerical values of $C^i_l$ and $C_l$ presented above have been obtained for the Λ-CDM Concordance Model of the Universe. The difference between measured values and expectation values of the observable - parameters however hold for other Models. For each model we can evaluate expected value and Confidence Level $C^i_l$ of each observable and $C_l$ for the whole Model. The uncertainties of the calculated Expectation values can be assumed as uncertainties on the observable real values only when the model *priors* are such that $\sigma_{me} \simeq \sigma_{ex}$.

If not, systematic uncertainties introduced by the *priors* have to be added to the uncertainties of the observable.

A way to validate priors and model is therefore repeating direct (or almost direct) measurements of the parameters until their accuracies becomes comparable to the accuracy of the expected values.

Until $C_l < 1$ models based on different priors cannot be excluded.

## 4. Conclusion

The use of Montecarlo methods and Bayesian Statistics to analyze the enormous quantity of data of cosmological interest which are continously poured by ground and space observations is almost unavoidable. However Montecarlo and Bayesian Methods are based on *a priori* assumption whose statistical weight should be added, but rarely is added, to the the quoted accuracies of the parameter expected values.

Peoples who currently use these methods are aware of that and warning have been put forward (e.g. Bridle et al. 2003). Unfortunately general public and professionals not involved in cosmological observations may be unaware of it, misinterpret the results of model simulations and attribute weights above their real values to models. Forgetting it may stop or reduce support to studies of other models based on different *priors* still possible and not yet excluded by observation.

Similar situations occur in other fields of pure and applied research (e.g. unification of fundamental forces, string theories, elementary particle models, models of climate evolution and so on). To avoid misunderstandings and preserve possibilities of

**Table 1.** $\Lambda CDM$-Concordance Model: Expectation values, Measured Values and Confidence Level of Model Parameters - (adapted from Komatsu et al. 2010, Brown et al. 2009)

| Parameter | | $< E >$ Expec. value | M Meas. value | $C_l$ Conf. Level |
|---|---|---|---|---|
| Hubble Constant (km/sec Mpc) | $H_o$ | $70.4^{+1.3}_{-1.4}$ | $74.2 \pm 3.6$ | 0.38 |
| Barionic Matter Density | $\Omega_b$ | $0.0456 \pm 0.0016$ | $0.005 - 0.1$ | $< 10^{-2}$ |
| Dark Matter density | $\Omega_{dm}$ | $0.227 \pm 0.0014$ | $0.006 - 1$ | $< 10^{-2}$ |
| Optical thickness at reionization | $\tau$ | $0.087 \pm 0.0014$ | $0.01 - 0.80$ | $< 10^{-2}$ |
| Scalar fluctuations Amplitude | $A_s$ | $(2.441^{+0.088}_{-0.092}\ 10^{-9})$ | ? | ? |
| Scalar Spectral index | $n_s$ | $0.963 \pm 0.012$ | $0.5 - 1.5$ | $< 2\ 10^{-2}$ |

**Table 2.** $\Lambda CDM$-Concordance Model: Expectation values of Derived Parameters - (adapted from Komatsu et al. 2010, Brown et al. 2009)

| Parameter | | Expected value |
|---|---|---|
| Dark Energy Density | $\Omega_\Lambda$ | $0.728^{+0.015}_{-0.016}$ |
| Reionization Red Shift | $Z_{ion}$ | $10.4 \pm 1.2$ |
| galactic fluctuations amplitude | $\sigma_8$ | $0.809 \pm 0.024$ |
| Universe Age (years) | $t_o$ | $(13.75 \pm 0.11)\ 10^9$ |

pursuing alternatives lines of research, this point of view must be transmitted to the bodies who support research activities and to the general public.

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