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Lattice QCD Study for Gluon Propagator and Gluon Spectral Function

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We study the gluon propagator in the Landau gauge in SU(3) lattice QCD at β=5.7, 5.8 and 6.0 at the quenched level. The Euclidean Landau-gauge gluon propagator $D(r) \equiv D^{aa}_{\mu\mu}(x)/24$ is found to be well described by four-dimensional Yukawa-type function e^{-mr}/r in the infrared and intermediate region of $r \equiv (x_\mu x_\mu)^{1/2} = 0.1 \sim 1.0$fm. The infrared effective gluon mass is obtained as $m \simeq 600$MeV. Associated with the 4D Yukawa-type gluon propagator, we derive analytical expressions for the zero-spatial-momentum propagator $D_0(t)$, the effective mass $M_{\rm eff}(t)$, and the spectral function $\rho(\omega)$ of the gluon field. Remarkably, the obtained gluon spectral function $\rho(\omega)$ is almost negative definite, except for a positive δ-functional peak at $\omega = m$. Since the Yukawa-type propagation indicates a three-dimensional space-time character, we consider a hypothesis of an effective dimensional reduction by generalized Parisi-Sourlas mechanism in a stochastic color-magnetic vacuum of infrared QCD.

Lattice Field Theories 2010
June 12-19, 2010
Tanka, Sardinia, Italy

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1. Introduction

The analysis of gluon properties is an important key point to clarify the nonperturbative aspects of QCD [1, 2, 3]. In particular, the gluon propagator, *i.e.*, the two-point Green function is one of the most basic quantities in QCD, and has been investigated with much interests [1, 4, 5]. Dynamical gluon-mass generation is also an important subject related to the infrared gluon propagation. While gluons are perturbatively massless, they are conjectured to acquire a large effective mass as the self-energy through their self-interaction in a nonperturbative manner [3, 6]. Actually, glueballs, color-singlet bound states of gluons, are theoretically predicted to be fairly massive, *e.g.*, about 1.5GeV for the lowest 0^{++} and about 2GeV for the lowest 2^{++}, in lattice QCD calculations [7].

For the direct investigation of the gluon field, gauge fixing is to be done. Among gauges, the Landau gauge is one of the most popular gauges in QCD, and it keeps Lorentz covariance and global SU(N_c) symmetry. In Euclidean QCD, the Landau gauge has a global definition to minimize the global quantity $R \equiv \int d^4x \, \mathrm{Tr}\{A_\mu(x)A_\mu(x)\} = \frac{1}{2}\int d^4x A^a_\mu(x)A^a_\mu(x)$ by gauge transformation. The local condition $\partial_\mu A_\mu(x) = 0$ is derived from the minimization of R. The global quantity R can be regarded as "total amount of the gluon-field fluctuation" in Euclidean space-time. In the global definition, the Landau gauge has a clear physical interpretation that it maximally suppresses artificial gauge-field fluctuations relating to gauge degrees of freedom [1].

In lattice QCD, the Landau gauge is defined by the maximization of $R_{\mathrm{latt}} \equiv \sum_x \sum_\mu \mathrm{ReTr} U_\mu(x)$, with the link-variable $U_\mu(x) \equiv e^{iagA_\mu(x)}$ (a: lattice spacing, g: QCD gauge coupling). The gluon field is defined as $A_\mu(x) \equiv \frac{1}{2iag}\{U_\mu(x) - U^\dagger_\mu(x)\} - (\mathrm{trace\ part})$. In the Landau gauge, the minimization of gluon-field fluctuations justifies the expansion by small lattice spacing a. In Euclidean metric, the gluon propagator is defined by the two-point function as $D^{ab}_{\mu\nu}(x-y) \equiv \langle A^a_\mu(x)A^b_\nu(y)\rangle$. Here, owing to the symmetries and the transverse property, the color and Lorentz structure of the gluon propagator is uniquely determined in the Landau gauge.

In this paper, using SU(3) lattice QCD Monte Carlo calculations, we study the functional form of the Landau-gauge gluon propagator, $D(r) \equiv \frac{1}{3(N_c^2-1)} D^{aa}_{\mu\mu}(x) = \frac{1}{3(N_c^2-1)}\langle A^a_\mu(x)A^a_\mu(0)\rangle$, as a function of 4D Euclidean distance $r \equiv (x_\mu x_\mu)^{1/2}$. We mainly deal with the coordinate-space propagator $D(r)$ for the infrared and intermediate region of $r = 0.1 \sim 1.0$fm, which is relevant for quark-hadron physics. Based on the obtained function form of the gluon propagator, we aim at a nonperturbative description of gluon properties,

2. Functional form of Landau-gauge gluon propagator

The SU(3) lattice QCD Monte Carlo calculations are performed at the quenched level using the standard plaquette action with $\beta \equiv 2N_c/g^2$=5.7, 5.8, and 6.0, on the lattice size of $16^3 \times 32$, $20^3 \times 32$, and 32^4, respectively. The lattice spacing a is found to be $a = 0.186, 0.152$, and 0.104fm, at β = 5.7, 5.8, and 6.0, respectively, when the scale is determined so as to reproduce the string tension as $\sqrt{\sigma} = 427$MeV from the static $\mathrm{Q\bar{Q}}$ potential [8]. Here, we choose the renormalization scale at $\mu = 4$GeV for $\beta = 6.0$, and make corresponding rescaling for β=5.7 and 5.8 [1].

Figure 1(a) and (b) show the coordinate-space gluon propagator $D(r)$ and the momentum-space gluon propagator $\tilde{D}(p^2) \equiv \int d^4x \, e^{ip\cdot x} D(r)$, respectively. Our lattice QCD result of $\tilde{D}(p^2)$ is consistent with that obtained in previous lattice studies, although recent huge-volume lattice studies [4] indicate a suppression of the gluon propagator in the Deep-IR region ($p < 0.5$GeV).

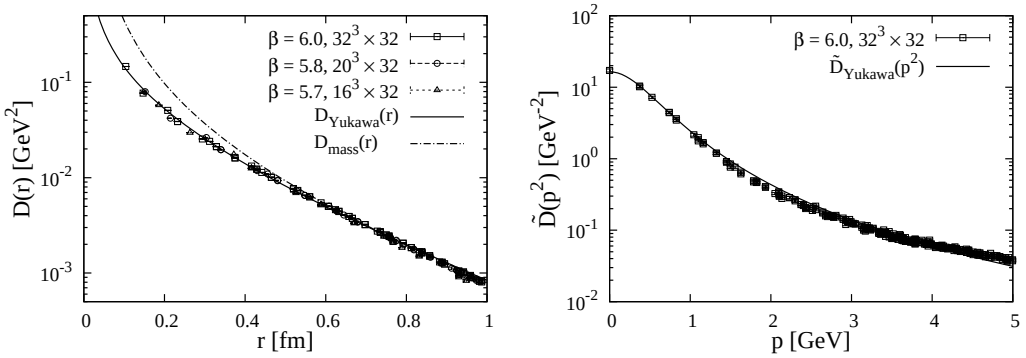

Figure 1: (a) Lattice QCD results (symbols) of the Landau-gauge gluon propagator $D(r) \equiv D^{aa}_{\mu\mu}(x)/24$ in coordinate space, and 4D Yukawa-type function $D_{\rm Yukawa}(r) = Ame^{-mr}/r$ (solid line) with $m = 0.624$GeV and $A = 0.162$. The dash-dotted line denotes a typical example of the massive-vector propagator $D_{\rm mass}(r)$. (b) The Landau-gauge gluon propagator $\tilde{D}(p^2)$ in the momentum space of $p_\mu = \frac{2}{a}\sin(\frac{\pi n_\mu}{L_\mu})$. The solid line denotes 4D Fourier-transformed Yukawa-type propagator, *i.e.*, $\tilde{D}_{\rm Yukawa}(p^2) = 4\pi^2 Am(p^2+m^2)^{-3/2}$.

We find that the lattice gluon propagator $D(r)$ cannot be described by the free massive Euclidean propagator $D_{\rm mass}(r) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\frac{1}{p^2+m^2} = \frac{1}{4\pi^2}\frac{m}{r}K_1(mr)$ ($K_\nu(z)$: modified Bessel function) [5] in the whole region of $r = 0.1 \sim 1.0$fm, as shown in Fig.1(a).

By the functional-form analysis, we find that the Landau-gauge gluon propagator $D(r)$ in the coordinate space is well described by the 4D Yukawa-type function [1]

$$D(r) \equiv \frac{1}{24}D^{aa}_{\mu\mu}(r) = Am\frac{e^{-mr}}{r}, \tag{2.1}$$

with $m = 0.624(8)$GeV and $A = 0.162(2)$ in the range of $r = 0.1 \sim 1.0$fm, as shown in Fig.1(a). The gluon propagator $\tilde{D}(p^2)$ in the momentum space is also well described by 4D Fourier-transformed Yukawa-type function as $\tilde{D}(p^2) = \frac{1}{24}\tilde{D}^{aa}_{\mu\mu}(p^2) = \frac{4\pi^2 Am}{(p^2+m^2)^{3/2}}$ for $0.5\text{GeV} \le p \le 3\text{GeV}$ [1].

3. Analytical applications

In this section, as applications of the Yukawa-type gluon propagator, we derive analytical expressions for the zero-spatial-momentum propagator $D_0(t)$, the effective mass $M_{\rm eff}(t)$, and the spectral function $\rho(\omega)$ of the gluon field [1]. All the derivations can be analytically performed, starting from the Yukawa-type gluon propagator $D_{\rm Yukawa}(r)$. Although the real gluon propagator has some deviation from the Yukawa-type in UV region, this method is found to be workable to reproduce lattice QCD results, as shown below.

3.1 Zero-spatial-momentum propagator of gluons

First, we consider zero-momentum gluon propagator $D_0(t) \equiv \frac{1}{24}\sum_{\vec{x}}\langle A^a_\mu(\vec{x},t)A^a_\mu(\vec{0},0)\rangle = \sum_{\vec{x}} D(r)$, where $r = \sqrt{\vec{x}^2+t^2}$ is the 4D Euclidean distance. For the simple argument, we here deal with the continuum formalism with infinite space-time. Starting from the Yukawa-type gluon propagator $D_{\rm Yukawa}(r)$, we derive the zero-spatial-momentum propagator as [1]

$$D_0(t) = \int d^3x\, D_{\rm Yukawa}(\sqrt{\vec{x}^2+t^2}) = 4\pi Am\int_0^\infty dx \frac{x^2}{\sqrt{x^2+t^2}} e^{-m\sqrt{x^2+t^2}} = 4\pi A t K_1(mt). \tag{3.1}$$

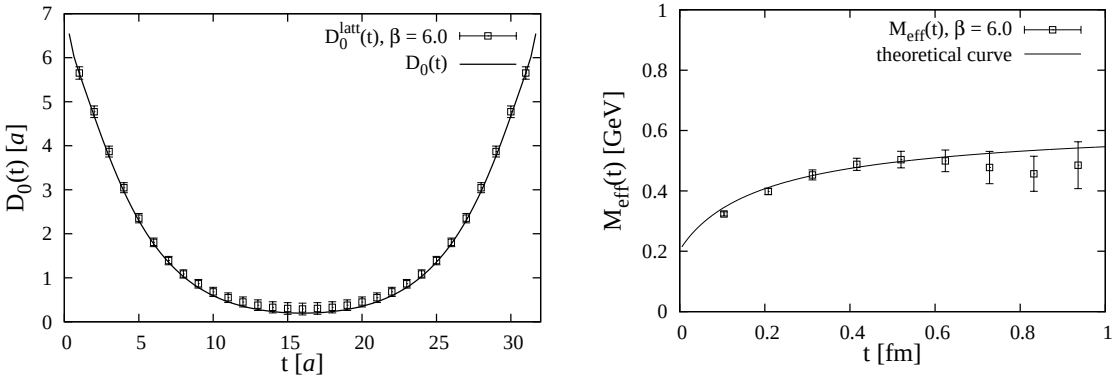

Figure 2: (a) The zero-spatial-momentum propagator $D_0(t)$ of gluons in the Landau gauge. (b) The effective mass $M_{\rm eff}(t)$ of gluons in the Landau gauge. The symbols are the lattice QCD data on 32^4 at $\beta = 6.0$, and the solid line is the theoretical curve derived from 4D Yukawa-type propagator with m=0.624GeV and A=0.162.

In Fig.2(a), we show the theoretical curve of $D_0(t)$ in Eq.(3.1) with m=0.624GeV and A=0.162, together with the lattice QCD result of $D_0(t)$ in the Landau gauge. For the actual comparison with the lattice data, we take account of the temporal periodicity [1]. The lattice QCD data are found to be well described by the theoretical curve, associated with the Yukawa-type gluon propagator.

3.2 Effective mass of gluons

Second, we investigate the effective mass $M_{\rm eff}(t)$ of gluons. The effective mass plot is often used for hadrons as a standard mass measurement in lattice QCD. For the simple notation, we use the lattice unit of $a = 1$ in this subsection. In the case of large temporal size, the effective mass is defined as $M_{\rm eff}(t) = \ln\{D_0(t)/D_0(t+1)\}$.

In Fig.2(b), we show the lattice result of $M_{\rm eff}(t)$, where we take account of the temporal periodicity. The effective gluon mass exhibits a significant scale-dependence, and it takes a small value at short distances. Quantitatively, the effective gluon mass is estimated to be about $400 \sim 600$MeV in the infrared region of about 1fm [1]. This value seems consistent with the gluon mass suggested by Cornwall [3], from a systematic analysis of nonperturbative QCD phenomena.

Now, we consider the consequence of 4D Yukawa-type propagator $D_{\rm Yukawa}(r)$ of gluons. For simplicity, we here treat the three-dimensional space as a continuous infinite-volume space, while the temporal variable t is discrete. We obtain an analytical expression of the effective mass [1],

$$M_{\rm eff}(t) = \ln \frac{D_0(t)}{D_0(t+1)} = \ln \frac{tK_1(mt)}{(t+1)K_1(m(t+1))}, \tag{3.2}$$

when the temporal periodicity can be neglected. In Fig.2(b), we add by the solid line the theoretical curve of $M_{\rm eff}(t)$ in Eq.(3.2) with m=0.624GeV. The lattice QCD data of $M_{\rm eff}(t)$ are found to be well described by the theoretical curve derived from the Yukawa-type gluon propagator. From the asymptotic form $K_1(z) \propto z^{-1/2}e^{-z}$, the effective mass of gluons is approximated as [1]

$$M_{\rm eff}(t) \simeq m - \frac{1}{2}\ln\left(1+\frac{1}{t}\right) \simeq m - \frac{1}{2t} \qquad (\text{for large } t). \tag{3.3}$$

This functional form indicates that $M_{\rm eff}(t)$ is an increasing function and approaches m from below, as t increases. Then, the mass parameter $m \simeq 600$MeV in the Yukawa-type gluon propagator has a definite physical meaning of the effective gluon mass in the infrared region.

Note that the simple analytical expression reproduces the anomalous "increasing behavior" of the effective mass $M_{\rm eff}(t)$ of gluons. Thus, this framework with the Yukawa-type gluon propagator gives an analytical and quantitative method, and is found to well reproduce lattice QCD results.

3.3 Spectral function of gluons in the Landau gauge

As a general argument, an increasing behavior of the effective mass $M_{\rm eff}(t)$ means that the spectral function is not positive-definite [1]. More precisely, the increasing property of $M_{\rm eff}(t)$ can be realized, only when there is some suitable coexistence of positive- and negative-value regions in the spectral function $\rho(\omega)$ [1]. However, the functional form of the spectral function of the gluon field is not yet known.

The relation between the spectral function $\rho(\omega)$ and the zero-spatial-momentum propagator $D_0(t)$ is given by the Laplace transformation, $D_0(t) = \int_0^\infty d\omega\, \rho(\omega)\, e^{-\omega t}$. When the spectral function is given by a δ-function such as $\rho(\omega) \sim \delta(\omega - \omega_0)$, which corresponds to a single mass spectrum, one finds a familiar relation of $D_0(t) \sim e^{-\omega_0 t}$. For the physical state, the spectral function $\rho(\omega)$ gives a probability factor, and is non-negative definite in the whole region of ω. This property is related to the unitarity of the S-matrix.

From the analytical expression of the zero-spatial-momentum propagator $D_0(t) = 4\pi A t K_1(mt)$, we can derive the spectral function $\rho(\omega)$ of the gluon field, associated with the Yukawa-type gluon propagator [1]. For simplicity, we take continuum formalism with infinite space-time. Using the inverse Laplace transformation of the modified Bessel function, we derive the spectral function $\rho(\omega)$ of the gluon field as [1]

$$\rho(\omega) = -\frac{4\pi A m}{(\omega^2 - m^2)^{3/2}}\theta(\omega - m - \varepsilon) + \frac{4\pi A/\sqrt{2m}}{(\omega - m)^{1/2}}\delta(\omega - m - \varepsilon), \tag{3.4}$$

with an infinitesimal positive ε, which is introduced for a regularization. Here, $m \simeq 600$MeV is the mass parameter in the Yukawa-type function for the Landau-gauge gluon propagator. The first term expresses a negative continuum spectrum, and the second term a δ-functional peak with the residue including a positive infinite factor as $\varepsilon^{-1/2}$ at $\omega = m + \varepsilon$.

We show in Fig.3 the spectral function $\rho(\omega)$ of the gluon field. As a remarkable fact, the obtained gluon spectral function $\rho(\omega)$ is negative-definite for all the region of $\omega > m$, except for the positive δ-functional peak at $\omega = m$. The negative property of the spectral function in coexistence with the positive peak leads to the anomalous "increasing behavior" of the effective mass $M_{\rm eff}(t)$ of gluons [1]. Actually, Eq.(3.4) leads to Eq.(3.2), which well describes the lattice result of the effective mass $M_{\rm eff}(t)$, as shown in Fig.2(b).

We note that the gluon spectral function $\rho(\omega)$ is divergent at $\omega = m + \varepsilon$, and there are *two* divergence structures: a δ-functional peak with a positive infinite residue and a negative wider power-damping peak. On finite-volume lattices, these singularities are to be smeared, and $\rho(\omega)$ is expected to take a finite value everywhere on ω. On the lattice, the spectral function $\rho(\omega)$ is conjectured to include a narrow "positive-valued peak" stemming from the δ-function in the vicinity of $\omega = m\ (+\varepsilon)$ and a wider "negative-valued peak" near $\omega \simeq m$ in the region of $\omega > m$ [1].

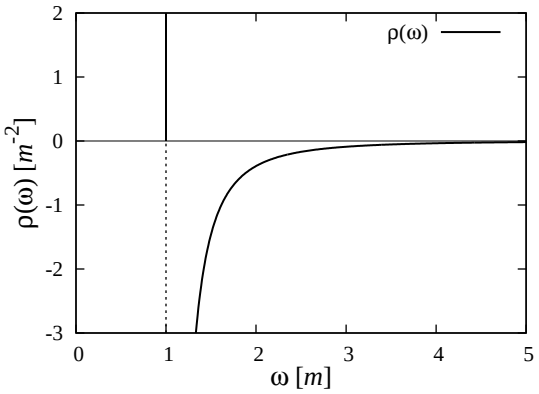

Figure 3: The spectral function $\rho(\omega)$ of the gluon field, associated with the Yukawa-type propagator. The unit is normalized by the mass parameter $m \simeq 600$MeV. $\rho(\omega)$ shows anomalous behaviors: it has a positive δ-functional peak with the residue of $+\infty$ at $\omega = m$, and takes negative values for all the region of $\omega > m$.

In this way, the Yukawa-type gluon propagator indicates an extremely anomalous spectral function of the gluon field in the Landau gauge. The obtained gluon spectral function $\rho(\omega)$ is negative almost everywhere, and includes a complicated divergence structure near the "anomalous threshold", $\omega = m$ $(+\varepsilon)$. Thus, this framework with the Yukawa-type gluon propagator gives an analytical and concrete expression for the gluon spectral function $\rho(\omega)$.

4. Effective dimensional reduction in gluonic vacuum by Parisi-Sourlas mechanism

We discuss the Yukawa-type gluon propagation and a possible dimensional reduction due to the stochastic behavior of the gluon field in the infrared region [1]. As shown before, the Landau-gauge gluon propagator is well described by the Yukawa function in *four*-dimensional Euclidean space-time. However, the Yukawa function e^{-mr}/r is a natural form in *three*-dimensional Euclidean space-time, since it is obtained by the three-dimensional Fourier transformation of the ordinary massive propagator $(p^2+m^2)^{-1}$. In fact, the Yukawa-type propagator has a "three-dimensional" property. In this sense, as an interesting possibility, we propose to interpret this Yukawa-type behavior of the gluon propagation as an "effective reduction of the space-time dimension".

Such a "dimensional reduction" sometimes occurs in stochastic systems, as Parisi and Sourlas pointed out for the spin system in a random magnetic field [9]. In fact, on the infrared dominant diagrams, the D-dimensional system coupled to the Gaussian-random external field is equivalent to the $(D-2)$-dimensional system without the external field, due to a hidden SUSY structure.

We note that the gluon propagation in the QCD vacuum resembles the situation of the system coupled to the stochastic external field. Actually, as is indicated by a large positive value of the gluon condensate $\langle G^a_{\mu\nu} G^{\mu\nu}_a \rangle = 2(\mathbf{H}^2_a - \mathbf{E}^2_a) > 0$ in the Minkowski space, the QCD vacuum is filled with a strong color-magnetic field [10], which can contribute spontaneous chiral-symmetry breaking [11], and the color-magnetic field is considered to be highly random at the infrared scale. Since gluons interact with each other, the propagating gluon is violently scattered by the other gluons in the randomly-oriented color-magnetic fields of the infrared QCD vacuum, as shown in Fig.4.

Actually at the infrared scale, the gluon field shows a strong randomness due to the strong interaction, and this infrared strong randomness is considered to be responsible for color confine-

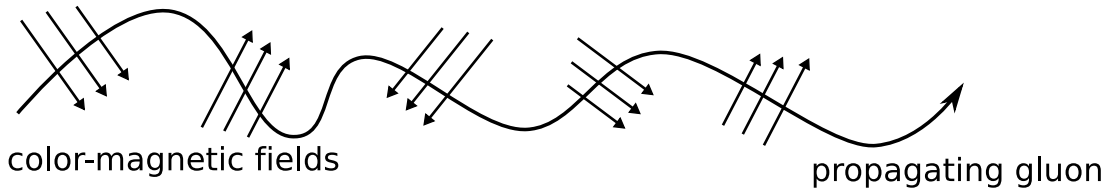

Figure 4: A schematic figure for a propagating gluon. The QCD vacuum is filled with color-magnetic fields which are stochastic at an infrared scale, and the gluon propagates in the random color-magnetic fields.

ment, as is indicated in strong-coupling lattice QCD. Even after the removal of fake gauge degrees of freedom by gauge fixing, the gluon field exhibits a strong randomness accompanying a quite large fluctuation at the infrared scale.

As a generalization of the Parisi-Sourlas mechanism, we conjecture that the infrared structure of a theory in the presence of quasi-random external fields in higher-dimensional space-time has a similarity to the theory without the external field in lower-dimensional space-time [1]. From this point of view, the Yukawa-type propagation of gluons may indicate an "effective reduction of space-time dimension" by one, reflecting the interaction between the propagating gluon and the other gluons in randomly-oriented color-magnetic fields in the infrared QCD vacuum.

In any case, it is an interesting and important subject to clarify the nonperturbative QCD vacuum structure in terms of gluonic properties [2] including the gluon propagation [1].

Acknowledgements

H.S. is supported by Grant-in-Aid for Scientific Research [(C) No. 19540287, Priority Areas "New Hadrons" (E01:21105006)]. The lattice calculations are done on SX-8R at Osaka University.

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