Generalizing the Cosmic Energy Equation

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We generalize the cosmic energy equation to the case when massive particles interact via a modified gravitational potential of the form $\phi(a, r)$, which is allowed to explicitly depend on the cosmological time through the expansion factor $a(t)$. Using the nonrelativistic approximation for particle dynamics, we derive the equation for the cosmological expansion which has the form of the Friedmann equation with a renormalized gravitational constant. The generalized Layzer–Irvine cosmic energy equation and the associated cosmic virial theorem are applied to some recently proposed modifications of the Newtonian gravitational interaction between dark-matter particles. We also draw attention to the possibility that the cosmic energy equation may be used to probe the expansion history of the universe thereby throwing light on the nature of dark matter and dark energy.

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I. INTRODUCTION

Almost half a century ago, Layzer & Irvine [1], and independently Zeldovich & Dmitriev [2], demonstrated that in an expanding universe, the *peculiar* kinetic (K) and potential (U) energies of a large system of pressureless particles interacting via the Newtonian potential $\phi \equiv -G/r$ satisfy the *cosmic energy equation*

$$\dot{E} = -(2K + U)H\,, \qquad E = K + U\,, \tag{1}$$

where (see also [3, 4])

$$K = \frac{1}{2}\sum_i m_i v_i^2\,, \tag{2}$$

and

$$U = -\frac{G}{2}\int \frac{[\rho(\boldsymbol{r}_1) - \varrho]\,[\rho(\boldsymbol{r}_2) - \varrho]}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}\, d^3\boldsymbol{r}_1 d^3\boldsymbol{r}_2 \tag{3}$$

are, respectively, the peculiar kinetic and potential energies of a system of particles m_i having coordinates $\boldsymbol{r}_i$ and peculiar velocities v_i. Here, $\rho(\boldsymbol{r})$ is the mass density of these particles, ϱ is its cosmological background average, and $H = \dot{a}/a$ is the Hubble parameter describing the cosmic expansion. For the discrete distribution $\rho(\boldsymbol{r}) = \sum_i m_i\delta(\boldsymbol{r} - \boldsymbol{r}_i)$, the integral in (3) should avoid the configuration $\boldsymbol{r}_1 = \boldsymbol{r}_2$.

One can introduce the correlation function $\xi(r)$:

$$\langle[\rho(\boldsymbol{r}_0 + \boldsymbol{r}) - \varrho]\,[\rho(\boldsymbol{r}_0) - \varrho]\rangle = \varrho^2\xi(r) \tag{4}$$

with the obvious property

$$\int \xi(r)\, d^3\boldsymbol{r} = 4\pi \int \xi(r)\, r^2 dr = 0\,. \tag{5}$$

Then Eq. (1) can be averaged to yield the corresponding equation per unit mass

$$\langle\dot{\mathcal{E}}\rangle = -\left[2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle\right] H \tag{6}$$

with $\mathcal{E} = E/M$, $\mathcal{U} = U/M$, and

$$\langle\mathcal{U}\rangle = -2\pi G\varrho \int \xi(r) r dr\,. \tag{7}$$

Here, $M = \varrho V$ is the total mass of the system in a large volume V.

Two important limiting cases of Eq. (1) need mention [3, 4]:

(i) For noninteracting particles, we have $U = 0$, and the relation $\dot{K} = -2HK$ corresponds to the kinematic decay of the peculiar velocities with time $v_i \propto 1/a(t)$.

(ii) Once the clustering has entered a stationary regime, the condition $\dot{E} = 0$ results in the virial relation

$$2K + U = 0\,. \tag{8}$$

A breakdown of (8) for galaxies belonging to the Coma cluster led Zwicky [5] to suggest that a large amount of dark matter might dominate the dynamics of Coma. While dark matter appears to be ubiquitous, its nature remains elusive. Indeed it is now believed that, to properly account for the energy budget of the universe, dark matter (DM) must be supplemented by an even more enigmatic 'substance' called dark energy (DE), which, on account of its large negative pressure, causes the universe to accelerate instead of decelerating.

Our current theoretical understanding of DM and DE can be broadly divided into ideas that are mainstream and those that are radical. Mainstream notions suggest that DM is formed of nonbaryonic particles of relic origin while DE consists of the cosmological constant or a relic scalar field such as quintessence. Radical notions suggest that the purported existence of DM/DE may be pointing to a breakdown of Newtonian/Einsteinian gravity on large scales. Theoretical models which incorporate this latter set of ideas include Braneworld and $f(R)$ gravity theories in the case of DE, and Modified Newtonian Dynamics (MOND) and the screened gravitational interaction model in the case of DM (see also [6, 7]).

Ever since its discovery, the cosmic energy equation (1) has been one of the bulwarks of modern cosmology, and its many applications include estimates of the matter density and its gravitational binding energy [3, 4, 8]. In this paper, we show how the cosmic energy equation can be generalized to incorporate more flexible forms of the gravitational interaction of dark matter, some of which have been suggested in the literature.

Our assumption is that nonrelativistic dark-matter particles interact with each other via the two-particle potential $\phi(a, r)$ so that the potential energy between two particles is

$$m_1 m_2 \phi\left(a, |\boldsymbol{r}_1 - \boldsymbol{r}_2|\right)\,. \tag{9}$$

As indicated in (9), we allow the two-particle potential to depend explicitly on the cosmological time through the scale factor $a(t)$, which is characteristic of some class of the models to be considered below. We consider the universe dominated by these dark-matter particles and by the uniform dark energy.

In Sec. II, using the nonrelativistic theory, we obtain the law of cosmological expansion for our universe, which turns out to have the Friedmannian form with a renormalized gravitational constant. In Sec. III, we then derive the generalized cosmic energy equation and the corresponding virial relations, which are analogs of the corresponding Layzer–Irvine equation (1) and virial relation (8). In Sec. IV, we comment on the results obtained.

II. COSMOLOGICAL EXPANSION

In this section, we will derive the law of cosmological expansion given the interaction potential (9). The universe model can be taken to be spatially flat since we are dealing with spatial scales much smaller than the Hubble length. We describe particle dynamics within a large volume using the nonrelativistic approximation with respect to comoving particle velocities.

A. Expansion law in the ΛCDM model

The Lagrangian for a nonrelativistic particle is given by the expression [3]

$$\mathcal{L} = \frac{1}{2} m \dot{\boldsymbol{r}}^2 - m\Phi(\boldsymbol{r}, t)\,, \tag{10}$$

where $\Phi(\boldsymbol{r}, t)$ is the full potential acting on the particle. In the ΛCDM (cosmological constant Λ + cold dark matter) model, in which dark-matter particles interact via Newtonian potential, for a pointlike matter distribution $\rho(\boldsymbol{r}, t) = \sum_i m_i \delta(\boldsymbol{r} - \boldsymbol{r}_i)$, the potential $\Phi(\boldsymbol{r}, t)$ is given by

$$\Phi(\boldsymbol{r}, t) = -G \sum_j \frac{m_j}{|\boldsymbol{r} - \boldsymbol{r}_j|} - \frac{\Lambda}{6} r^2\,. \tag{11}$$

Proceeding to the comoving coordinates $\boldsymbol{x} = \boldsymbol{r}/a$ and making a canonical transformation

$$\mathcal{L} \to \mathcal{L} - \frac{d}{dt}\left(\frac{1}{2} m a \dot{a} x^2\right)\,, \tag{12}$$

one transforms the one-particle Lagrangian (10) to

$$\mathcal{L} = \frac{1}{2} m a^2 \dot{\boldsymbol{x}}^2 - m\varphi\,, \tag{13}$$

where the new potential is given by

$$\varphi = \Phi + \frac{1}{2} a \ddot{a} x^2\,. \tag{14}$$

In the Newtonian case, with account taken of (11), the new potential is

$$\varphi = -\frac{G}{a} \sum_j \frac{m_j}{|\boldsymbol{x} - \boldsymbol{x}_j|} + \frac{1}{2}\left(a\ddot{a} - \frac{\Lambda a^2}{3}\right) x^2\,. \tag{15}$$

The law of cosmological expansion can be derived directly from (15) under the requirement that the potential φ should not exert force on a test particle in the limit of a homogeneous distribution of matter $\rho(\boldsymbol{r}) \equiv \varrho$. To see this, we note that, in particular, the Laplacian of the potential should vanish in this limit. Now, we have

$$\begin{aligned}\nabla^2\varphi &= 4\pi G a^2 \sum_j m_j\, \delta(\boldsymbol{r}-\boldsymbol{r}_j) + 3a\ddot{a} - \Lambda a^2 \\ &\to 4\pi G a^2 \varrho + 3a\ddot{a} - \Lambda a^2 \,. \end{aligned} \tag{16}$$

Whence, we obtain one of the Friedmann laws of expansion

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\varrho + \frac{\Lambda}{3}\,. \tag{17}$$

B. Expansion law in the case of general potential

In the case of a general two-particle potential (9), the derivation that led to the cosmological potential (15) for a particle can be repeated and generalized to

$$\varphi = \sum_j m_j \phi(a, a|\boldsymbol{x}-\boldsymbol{x}_j|) + \frac{1}{2}\left(a\ddot{a} - \frac{\Lambda_{\rm eff} a^2}{3}\right) x^2\,, \tag{18}$$

where $\Lambda_{\rm eff}$ is the time-dependent ingredient in the universe which describes dark energy and replaces the cosmological constant and, in particular, whose interaction with dark matter makes the potential (9) time-dependent. Our equations below will be general and will not require the specific knowledge of this term; we will only assume that it does not cluster, remaining homogeneous in space. One such field-theoretic example is considered in Sec. II C below (example a).

The class of theories under consideration in this paper could, in fact, be defined by the property that the potential acting on a single dark-matter particle has the form (18). This complicated potential already includes the response of the universe, with all its field-theoretic ingredients, to the presence of dark-matter particles at specified spatial positions. In this subsection, we are going to derive the modified expansion law by taking the Laplacian of (18) and demanding that it vanish in the case of a spatially uniform distribution of matter.

Suppose that the gravitational two-particle potential has the form (we suppress the dependence on a as it is not relevant here)

$$\phi(r) = -\frac{G}{r} f(r)\,, \tag{19}$$

with a sufficiently regular function $f(r)$ [in particular, $\lim_{r\to 0} r f'(r) = 0$], as will be the case in all our concrete examples. Then, calculating its Laplacian, we have

$$\begin{aligned}\nabla^2_{\boldsymbol{r}}\phi(r) &= -G f(r) \nabla^2_{\boldsymbol{r}} \frac{1}{r} - 2G \nabla_{\boldsymbol{r}} \frac{1}{r}\cdot \nabla_{\boldsymbol{r}} f(r) - \frac{G}{r}\nabla^2_{\boldsymbol{r}} f(r) \\ &= 4\pi G f(0)\, \delta(\boldsymbol{r}) + \frac{2G}{r^2} f'(r) - \frac{G}{r}\left[f''(r) + \frac{2}{r} f'(r)\right] \\ &= 4\pi G f(0)\, \delta(\boldsymbol{r}) - \frac{G}{r} f''(r)\,. \end{aligned} \tag{20}$$

Applying this to (18) and requiring that $\nabla^2\varphi \equiv 0$ for uniformly distributed matter, we obtain the equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\rm eff}}{3}\varrho + \frac{\Lambda_{\rm eff}}{3}\,, \tag{21}$$

where the effective gravitational constant is given by

$$G_{\rm eff} = G \lim_{r\to\infty}\left[f(r) - rf'(r)\right]\,. \tag{22}$$

The derivation works as long as the limit in (22) exists and is finite, which we assume to be the case. For the Newtonian potential $\phi = -G/r$, we have $f \equiv 1$, and Eq. (22) returns the usual gravitational constant G. However, if the potential decays faster than $1/r$ at large distances, Eq. (22) may imply $G_{\rm eff} = 0$, so that the gravity of matter will not influence the rate of cosmic expansion. Note that the effective gravitational constant will be time-dependent if the function f in (19) depends on time.

C. Examples

To give examples to which our results can be applied, we consider some simple modifications of the gravitational interaction between dark-matter particles:

(a) The screened potential discussed in [9] has the form

$$\phi(a,r) = -\frac{G}{r}\left(1 + \beta e^{-r/r_s}\right)\,, \tag{23}$$

where β is a dimensionless constant of order unity, and the time-dependent screening length $r_s(t)$ is a comoving constant, i.e., $r_s(t) \propto a(t)$, such that $r_s(t_0) \simeq 1$ Mpc. The potential arises in the field-theoretic model of interaction of dark matter with dark energy via the scalar field [10] in the version with two dark-matter families. A subdominant relativistic family is used to stabilize the value of the scalar field; then the dominant nonrelativistic dark-matter particles have constant mass and interact via gravity as well as via the scalar field so that the two-particle interaction potential (23) is generated. A dark-matter particle then moves in potential (18) with ϕ given by (23), so that our analysis is applicable to this situation. One arrives at the Friedmann equation (21) with constant dark-energy term $\Lambda_{\rm eff}$, just as in the field-theoretic approach of [9, 10]. The potential (23) leads to faster evacuation of matter from voids and to an earlier epoch of structure formation, which could be perceived as an advantage for this model over ΛCDM [9].

Using the potential (18) and the equations of motion for a dark-matter particle $\boldsymbol{x}(t)$,

$$\ddot{\boldsymbol{x}} + 2H\dot{\boldsymbol{x}} = -\frac{1}{a^2}\nabla\varphi\,, \tag{24}$$

and proceeding along the same lines as in [3], Sec. 27, one easily obtains the exact nonlinear evolution equation for the Fourier components $\delta_{\boldsymbol{k}}$ of the density contrast $\rho(\boldsymbol{x})/\varrho - 1$ in this theory:

$$\ddot{\delta}_{\boldsymbol{k}} + 2H\dot{\delta}_{\boldsymbol{k}} = 4\pi G\varrho \left[1 + \frac{\beta}{1 + (a/kr_s)^2}\right] \delta_{\boldsymbol{k}} + A_{\boldsymbol{k}} - C_{\boldsymbol{k}} \,, \tag{25}$$

where the nonlinear term $A_{\boldsymbol{k}}$ and the velocity term $C_{\boldsymbol{k}}$ are given by

$$A_{\boldsymbol{k}} = 4\pi G\varrho \sum_{\boldsymbol{k}' \neq 0, \boldsymbol{k}} \left[\frac{\boldsymbol{k}\boldsymbol{k}'}{k'^2} + \frac{\beta \boldsymbol{k}\boldsymbol{k}'}{k'^2 + (a/r_s)^2}\right] \delta_{\boldsymbol{k}-\boldsymbol{k}'} \delta_{\boldsymbol{k}'} \tag{26}$$

and

$$C_{\boldsymbol{k}} = \sum_i \frac{m_i}{M} (\boldsymbol{k}\dot{\boldsymbol{x}}_i)^2 \, e^{\mathrm{i}\boldsymbol{k}\boldsymbol{x}_i} \,, \tag{27}$$

respectively. The linear part of (25) reproduces equation (6) of [9] and leads to a more rapid development of gravitational instability than in ΛCDM.

(b) A power-law correction to the Newtonian potential on large scales

$$\phi(r) = -\frac{G}{r}\left[1 + \left(\frac{r_0}{r + r_0}\right)^n\right] \,, \qquad n \geq 1 \,, \tag{28}$$

which we have regularized on small scales to avoid a singularity. Potentials of this form with $n = 2$ arise in the Randall–Sundrum model [11] with a single large extra dimension. (However, in this case, $r_0 \ll 1$ mm, making the correction on cosmological scales extremely small.)

(c) A logarithmic correction to the Newtonian potential

$$\phi(r) = -\frac{G}{r} + \left(\frac{\alpha G}{r_0} \log \frac{r}{r_0}\right) e^{-r/r_c} \,. \tag{29}$$

The influence of the 'regularizing' exponent is confined to very large scales $r_c \gg r_0$.

For all these potentials the limit (22) gives

$$G_{\mathrm{eff}} = G \,, \tag{30}$$

implying that the background cosmological evolution remains unmodified.

III. GENERALIZED LAYZER–IRVINE EQUATION

The Lagrangian of the many particle system is obtained by the summation of (13) with account taken of (18), (21) and (22) and is equal to $L = K - U$, where the peculiar kinetic energy is [3, 4]

$$K = \frac{a^2}{2} \sum_i m_i \dot{x}_i^2 \,. \tag{31}$$

With consideration of Eq. (21), potential (18) can be written in the form

$$\varphi = \int [\rho(\boldsymbol{r}') - \varrho] \, \phi(a, |\boldsymbol{r} - \boldsymbol{r}'|) \, d^3\boldsymbol{r}' \,, \tag{32}$$

using which, one obtains

$$U = \frac{1}{2} \int [\rho(\boldsymbol{r}_1) - \varrho] \, [\rho(\boldsymbol{r}_2) - \varrho] \, \phi(a, r_{12}) \, d^3\boldsymbol{r}_1 d^3\boldsymbol{r}_2 \,, \tag{33}$$

where, for the discrete distribution $\rho(\boldsymbol{r}) = \sum_i m_i \delta(\boldsymbol{r} - \boldsymbol{r}_i)$, the integral should avoid the configuration $\boldsymbol{r}_1 = \boldsymbol{r}_2$. Here and below, we use the notation $r_{ij} = |\boldsymbol{r}_i - \boldsymbol{r}_j|$ and $x_{ij} = |\boldsymbol{x}_i - \boldsymbol{x}_j|$.

The Hamiltonian of our system as a function of the canonical variables $(\boldsymbol{x}_i, \boldsymbol{p}_i)$ is given by

$$\mathcal{H} = K + U = \frac{1}{2a^2} \sum_i \frac{p_i^2}{m_i} + \frac{1}{2} \int [\rho(\boldsymbol{x}_1) - \varrho] \, [\rho(\boldsymbol{x}_2) - \varrho] \, a^6 \phi(a, a x_{12}) \, d^3\boldsymbol{x}_1 d^3\boldsymbol{x}_2 \,. \tag{34}$$

Using the fact that the product $a^3 \, [\rho(\boldsymbol{r}) - \varrho] = \sum_i m_i \delta(\boldsymbol{x} - \boldsymbol{x}_i) - a^3 \varrho$ does not depend explicitly on time, we immediately obtain the equation for the peculiar energy of the system:

$$\begin{aligned} \dot{E} &\equiv \frac{d}{dt} (K + U) = \frac{\partial \mathcal{H}}{\partial t} \\ &= -2HK + \frac{H}{2} \int [\rho(\boldsymbol{r}_1) - \varrho] \, [\rho(\boldsymbol{r}_2) - \varrho] \left[\frac{\partial \phi \, (a, r_{12})}{\partial r} \, r_{12} + \frac{\partial \phi \, (a, r_{12})}{\partial a} \, a \right] d^3\boldsymbol{r}_1 d^3\boldsymbol{r}_2 \,. \end{aligned} \tag{35}$$

Similarly to (6) and (7), we can write the averaged equations for quantities per unit mass:

$$\langle \dot{\mathcal{E}} \rangle = -2H \langle \mathcal{K} \rangle + 2\pi H \varrho \int \xi(r) \left[\frac{\partial \phi \, (a, r)}{\partial r} \, r + \frac{\partial \phi \, (a, r)}{\partial a} \, a \right] r^2 dr \,, \tag{36}$$

$$\langle \mathcal{U} \rangle = \frac{\varrho^2}{2M} \int \xi(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \phi(a, |\boldsymbol{r}_1 - \boldsymbol{r}_2|) \, d^3\boldsymbol{r}_1 d^3\boldsymbol{r}_2 = 2\pi\varrho \int \xi(r) \phi(a, r) \, r^2 dr \,. \tag{37}$$

Equations (35)–(37) form the main results of this letter.

In the Newtonian case, we have $\partial\phi/\partial r = -\phi/r$, $\partial\phi/\partial a = 0$, and Eqs. (35) and (36) reduce to the corresponding Layzer–Irvine Eqs. (1) and (6). In the general case, Eqs. (35) and (36) contain an unknown function $\phi(a, r)$. If one has a theory predicting the shape of ϕ, one can, in principle, use (35) and (36) to test it.

Once the system has decoupled from the Hubble expansion, its peculiar energy evolves mainly because of the time-dependence of the potential $\phi(a,r)$:

$$\langle\dot{\mathcal{E}}\rangle \approx 2\pi H\varrho \int \xi(r)\frac{\partial\phi\,(a,r)}{\partial a}\,ar^2 dr\,. \tag{38}$$

Taking into account (36), we obtain the generalized virial relation in the form

$$\langle\mathcal{K}\rangle = \pi\varrho \int \xi(r)\frac{\partial\phi\,(a,r)}{\partial r}\,r^3 dr\,. \tag{39}$$

Next, we apply the generalized cosmic energy equation to the modifications of the gravitational interaction between dark-matter particles that we listed in Sec. II C:

(a) Substituting the screened potential (23) into (35), one finds, quite remarkably, that the generalized cosmic energy equation reduces to its Layzer–Irvine form (1) with the *modified* total potential U determined by (23) and (33). For a system decoupled from the Hubble expansion, the energy evolution, according to (38), is given by

$$\langle\dot{\mathcal{E}}\rangle = -\frac{2\pi\beta GH\varrho}{r_s}\int \xi(r)e^{-r/r_s}r^2 dr\,, \tag{40}$$

and the generalized virial relation (39) in this case reads

$$2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle = \frac{2\pi\beta G\varrho}{r_s}\int \xi(r)e^{-r/r_s}r^2 dr\,, \tag{41}$$

On length scales $r \ll r_s(t) \leq 1$ Mpc, we recover the usual Newtonian relations:

$$-\frac{\langle\dot{\mathcal{E}}\rangle}{H} = 2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle_{\text{Newton}} = \frac{2\pi\beta G\varrho}{r_s}\int \xi(r)r^2 dr = 0\,, \tag{42}$$

where, in the last equality, we have used property (5), and

$$\langle\mathcal{U}\rangle_{\text{Newton}} = -2\pi(1+\beta)G\varrho\int \xi(r)r dr \tag{43}$$

is the Newtonian virial gravitational energy per unit mass with renormalized gravitational coupling.

It should be noted that, since $r_c \simeq 1$ Mpc roughly corresponds to the Abell radius associated with a cluster of galaxies, it is unlikely that (42) can be applied to galaxy clusters. On these scales, the full form of the virial relation (41) should be used.

(b) The power-law correction to the Newtonian potential (28) leads to the cosmic energy equation:

$$\langle\dot{\mathcal{E}}\rangle \approx -\left[2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle_{\text{Newton}}\right]H + 2\pi(1+n)GH\varrho\int \xi(r)\left(\frac{r_0}{r+r_0}\right)^n r dr\,, \tag{44}$$

and to the virial relation

$$2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle_{\rm Newton} = 2\pi(1+n)GH\varrho \int \xi(r) \left(\frac{r_0}{r+r_0}\right)^n r dr \,, \tag{45}$$

where $\langle\mathcal{U}\rangle_{\rm Newton}$ is the usual averaged Newtonian peculiar potential energy per unit mass given by (7).

(c) The logarithmic correction to the Newtonian potential (29) leads to the usual cosmic energy equation

$$\langle\dot{\mathcal{E}}\rangle \approx - \left[2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle_{\rm Newton}\right] H + \frac{2\pi\alpha GH\varrho}{r_0} \int \xi(r) r^2 e^{-r/r_c} dr \approx - \left[2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle_{\rm Newton}\right] H \,, \tag{46}$$

where the last equality is valid in view of (5) if the correlation length of the system is much smaller than the cutoff scale r_c. The virial relation in this case is

$$2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle_{\rm Newton} = \frac{2\pi\alpha G\varrho}{r_0} \int \xi(r) r^2 e^{-r/r_c} dr \approx 0 \,. \tag{47}$$

IV. DISCUSSION

We have investigated a cosmological theory which produces a modified and, perhaps, time-dependent gravitational potential between matter particles in the form (9). For such a theory, we have determined the background cosmological equation, which turned out to have the Friedmann form possibly with a modified gravitational constant. We have also derived the cosmic energy equation generalizing the Layzer–Irvine equation for the theory under investigation.

We also note that the cosmic energy equation (6) or its generalization (36) can be used to determine the cosmic expansion history. For instance, from (1) we obtain

$$H(t) = - \frac{\langle\dot{\mathcal{E}}\rangle}{2\langle\mathcal{K}\rangle + \langle\mathcal{U}\rangle} \,. \tag{48}$$

The Hubble parameter $H(z)$ determined in this manner could shed light on the nature of dark energy either through the Om diagnostic [12] or by means of the effective equation of state of dark energy [13]

$$w_{\rm DE}(x) = \frac{(2x/3)\, d\ln H/dx - 1}{1 - (H_0/H)^2\, \Omega_m\, x^3} \,, \qquad x = 1 + z \,. \tag{49}$$

This determination of the cosmic expansion history $H(z)$, via the energy equation, is complementary to usual methods which rely either on standard candles (supernovae of type Ia) or rulers (baryon acoustic oscillations). This possibility will be examined in detail elsewhere.

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