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Quark masses and strong CP violation

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Abstract. Two flavor QCD involves three independent mass parameters for which non-perturbative effects are not universal. This precludes matching lattice and perturbative results for non-degenerate quarks and eliminates a vanishing up quark mass as a viable solution to the strong CP problem.

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In massless two-flavor QCD, chiral symmetry breaking gives rise to three massless Goldstone pions. In contrast, the two flavor analog of the eta prime meson acquires a mass from the anomaly. Thus, as shown schematically in Fig. 1, meson exchange will contribute to a hypothetical quark spin-flip scattering experiment.

Now turn on a small d quark mass. This allows connecting the ingoing and outgoing d quark lines in Fig. 1, and gives a mixing between the left and right handed u quark. The presence of a non-zero d quark mass creates an effective mass for the u quark, even if the latter initially vanishes. Non-perturbative effects renormalize m_u/m_d. If this ratio is zero at some scale, it cannot remain so for all scales. This cross talk between the masses of different quark species has been noted several times in the past [1] and contradicts the lore that mass renormalization is flavor blind. The practice of matching lattice calculations to $\overline{MS}$ is problematic when $m_u \neq m_d$.

A general mass term is an electrically neutral quadratic form that transforms as a Lorentz singlet. This leaves four candidates $m_1\overline{\psi}\psi + m_2\overline{\psi}\tau_3\psi + im_3\overline{\psi}\gamma_5\psi + im_4\overline{\psi}\gamma_5\tau_3\psi$. The massless limit should have the flavored chiral symmetry under $\psi \longrightarrow e^{i\gamma_5\tau_\alpha\phi_\alpha}\psi$. With the masses present, this mixes m_1 with m_4 and m_2 with m_3. The four mass terms are not independent and one can select any one of the m_i to vanish and a second to be positive. The chiral anomaly is responsible for the singlet rotation $\psi \longrightarrow e^{i\gamma_5\phi}\psi$ not being a valid symmetry [2]. This rotation does, however, allow one to remove any topological term from the gauge part of the action. Assume this has been done.

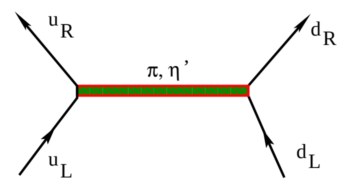

FIGURE 1. Both pion and eta prime exchange contribute towards spin flip scattering between up and down quarks. Because these mesons are non-degenerate, this scattering is not helicity suppressed.

Adopt the common choice $m_4 = 0$ and m_1 as the average quark mass. Then m_2 is the quark mass difference and m_3 is CP violating. The possible presence of m_3 represents the strong CP problem.

Strong interactions preserve CP to high accuracy. With the above conventions, it is natural to ask why is m_3 so small? One proposed solution is that the up quark mass might vanish, allowing a flavored chiral rotation to remove any phases from the quark mass matrix.

Why is this not a sensible approach? From the above, one can define the up quark mass as $m_u \equiv m_1 + m_2 + im_3$. But the quantities $\{m_1, m_2, m_3\}$ are independent parameters with different symmetry properties. As discussed earlier, the combination $m_1 + m_2 = 0$ is scale and scheme dependent. While it may be true that $m_1 + m_2 + im_3 = 0$ implies $m_3 = 0$, this would depend on scale and should be regarded as "not even wrong."

REFERENCES

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