Cosmological Constraints on the Sign-Changeable Interactions

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ABSTRACT

Recently, Cai and Su [Phys. Rev. D **81**, 103514 (2010)] argued that the sign of interaction Q in the dark sector changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$, by using a model-independent method to deal with the observational data. In fact, this result raises a remarkable problem, since most of the familiar interactions cannot change their signs in the whole cosmic history. Motivated by the work of Cai and Su, we have proposed a new type of interaction in a previous work [arXiv:1008.4968]. The key ingredient is the deceleration parameter q in the interaction Q. Therefore, the interaction Q can change its sign when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). In the present work, we consider the cosmological constraints on this type of sign-changeable interactions, by using the latest observational data. We find that the constraints on the model parameters are fairly tight. In particular, the key parameter β can be constrained to a very narrow range.

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I. INTRODUCTION

In the dark energy cosmology [1], the well-known cosmological coincidence problem has an important position. This problem is asking why are we living in an epoch in which the densities of dark energy and matter are comparable? Since their densities scale differently with the expansion of our universe, there should be some fine-tunings. To alleviate this coincidence problem, it is natural to consider the possible interaction between dark energy and dark matter in the literature (see e.g. [2–12]). In fact, since the nature of both dark energy and dark matter are still unknown, there is no physical argument to exclude the possible interaction between them. On the contrary, some observational evidences of the interaction in the dark sector have been found recently [13, 14]. In the literature, it is usual to assume that dark energy and dark matter interact through a coupling term Q, according to

$$\dot{\rho}_m + 3H\rho_m = Q\,, \tag{1}$$

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q\,, \tag{2}$$

where ρ_m and ρ_{de} are densities of dark matter and dark energy (we assume that the baryon component can be ignored); p_{de} is the pressure of dark energy; $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1+z)^{-1}$ is the scale factor (we have set $a_0 = 1$; the subscript "0" indicates the present value of corresponding quantity; z is the redshift); a dot denotes the derivative with respect to cosmic time t. Note that Eqs. (1) and (2) preserve the total energy conservation equation $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$, where $\rho_{tot} = \rho_m + \rho_{de}$. Since there is no natural guidance from fundamental physics on the interaction Q, one can only discuss it to a phenomenological level.

The most familiar interactions extensively considered in the literature (see e.g. [2–12]) include $Q = 3\alpha H\rho_m$, $Q = 3\beta H\rho_{tot}$, and $Q = 3\eta H\rho_{de}$. It is easy to see that these interactions are always positive or negative and hence cannot give the possibility to change their signs. However, recently Cai and Su [15] argued that the sign of interaction Q changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$, by using a model-independent method to deal with the observational data. In fact, this result raises a remarkable problem. Motivated by the work of Cai and Su, we have proposed a new type of interaction in a previous work [16], which is given by

$$Q = q(\alpha\dot{\rho} + 3\beta H\rho)\,, \tag{3}$$

where α and β are both dimensionless constants; the energy density ρ could be ρ_m, ρ_{tot} and ρ_{de} for examples; the deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}\,. \tag{4}$$

Obviously, this new type of interaction Q can change its sign when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). In fact, the deceleration parameter q in Q is the key ingredient of this new interaction, which makes our proposal different from the previous ones considered in the literature. Note that the term $\alpha\dot{\rho}$ in Q is introduced from the dimensional point of view. One can remove this term by simply setting $\alpha = 0$, and then Q becomes $Q = 3\beta qH\rho$.

In [16], we have investigated the cosmological evolution of quintessence and phantom with this new type of sign-changeable interactions, and found some interesting results. In the present work, we would like to consider the cosmological constraints on this new type of sign-changeable interactions, by using the latest observational data. To be simple, in this work, we restrict ourselves to the decaying Λ model (see e.g. [17] and references therein), namely, the role of dark energy is played by the decaying vacuum energy. In this case, Eq. (2) becomes

$$\dot{\rho}_\Lambda = -Q\,. \tag{5}$$

The Friedmann and Raychaudhuri equations are given by

$$H^2 = \frac{\kappa^2}{3}\rho_{tot} = \frac{\kappa^2}{3}(\rho_\Lambda + \rho_m)\,, \tag{6}$$

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa^2}{2}\rho_m\,, \tag{7}$$

where $\kappa^2 \equiv 8\pi G$. Notice that we consider a flat Friedmann-Robertson-Walker (FRW) universe throughout this work. In Sec. II, we briefly introduce the latest observational data which will be used in this work. In Sec. III, we consider the cosmological constraints on three particular sign-changeable interactions, i.e.,

$$Q = q(\alpha\dot{\rho}_m + 3\beta H\rho_m)\,, \tag{8}$$

$$Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot})\,, \tag{9}$$

$$Q = q(\alpha\dot{\rho}_\Lambda + 3\beta H\rho_\Lambda)\,. \tag{10}$$

Finally, some concluding remarks are given in Sec. IV.

II. OBSERVATIONAL DATA

In the present work, we will consider the latest cosmological observations, namely, the 557 Union2 Type Ia Supernovae (SNIa) dataset [18], the shift parameter R from the Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [19], and the distance parameter A of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies [20, 21].

The data points of the 557 Union2 SNIa compiled in [18] are given in terms of the distance modulus $\mu_{obs}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{th}(z_i) \equiv 5\log_{10} D_L(z_i) + \mu_0\,, \tag{11}$$

where $\mu_0 \equiv 42.38 - 5\log_{10} h$ and h is the Hubble constant H_0 in units of 100 km/s/Mpc, whereas

$$D_L(z) = (1+z)\int_0^z \frac{d\tilde{z}}{E(\tilde{z};\mathbf{p})}\,, \tag{12}$$

in which $E \equiv H/H_0$, and $\mathbf{p}$ denotes the model parameters. Correspondingly, the χ^2 from the 557 Union2 SNIa is given by

$$\chi^2_\mu(\mathbf{p}) = \sum_i \frac{\left[\mu_{obs}(z_i) - \mu_{th}(z_i)\right]^2}{\sigma^2(z_i)}\,, \tag{13}$$

where σ is the corresponding 1σ error. The parameter μ_0 is a nuisance parameter but it is independent of the data points. One can perform an uniform marginalization over μ_0. However, there is an alternative way. Following [22, 23], the minimization with respect to μ_0 can be made by expanding the χ^2_μ of Eq. (13) with respect to μ_0 as

$$\chi^2_\mu(\mathbf{p}) = \tilde{A} - 2\mu_0\tilde{B} + \mu_0^2\tilde{C}\,, \tag{14}$$

where

$$\tilde{A}(\mathbf{p}) = \sum_i \frac{\left[\mu_{obs}(z_i) - \mu_{th}(z_i;\mu_0=0,\mathbf{p})\right]^2}{\sigma^2_{\mu_{obs}}(z_i)}\,,$$

$$\tilde{B}(\mathbf{p}) = \sum_i \frac{\mu_{obs}(z_i) - \mu_{th}(z_i;\mu_0=0,\mathbf{p})}{\sigma^2_{\mu_{obs}}(z_i)}\,, \qquad \tilde{C} = \sum_i \frac{1}{\sigma^2_{\mu_{obs}}(z_i)}\,.$$

Eq. (14) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\tilde{\chi}^2_\mu(\mathbf{p}) = \tilde{A}(\mathbf{p}) - \frac{\tilde{B}(\mathbf{p})^2}{\tilde{C}}\,. \tag{15}$$

Since $\chi^2_{\mu,\,min} = \tilde{\chi}^2_{\mu,\,min}$ obviously, we can instead minimize $\tilde{\chi}^2_\mu$ which is independent of μ_0.

There are some other relevant observational data, such as the observations of cosmic microwave background (CMB) anisotropy [19] and large-scale structure (LSS) [20]. However, using the full data of CMB and LSS to perform a global fitting consumes a large amount of computation time and power. As an alternative, one can instead use the shift parameter R from the CMB, and the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies. In the literature, the shift parameter R and the distance parameter A have been used extensively. It is argued that they are model-independent [24], while R and A contain the main information of the observations of CMB and BAO, respectively.

As is well known, the shift parameter R of the CMB is defined by [24, 25]

$$R\equiv\Omega_{m0}^{1/2}\int_0^{z_*}\frac{d\tilde{z}}{E(\tilde{z})}\,, \tag{16}$$

where Ω_{m0} is the present fractional energy density of pressureless matter; the redshift of recombination $z_*=1091.3$ which has been updated in the Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [19]. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [24, 25]. The value of R has been updated to 1.725 ± 0.018 from the WMAP7 data [19]. On the other hand, the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies is given by [20]

$$A\equiv\Omega_{m0}^{1/2}E(z_b)^{-1/3}\left[\frac{1}{z_b}\int_0^{z_b}\frac{d\tilde{z}}{E(\tilde{z})}\right]^{2/3}\,, \tag{17}$$

where $z_b=0.35$. In [21], the value of A has been determined to be $0.469\,(n_s/0.98)^{-0.35}\pm 0.017$. Here the scalar spectral index n_s is taken to be 0.963, which has been updated from the WMAP7 data [19]. So, the total χ^2 is given by

$$\chi^2=\tilde{\chi}_\mu^2+\chi^2_{CMB}+\chi^2_{BAO}\,, \tag{18}$$

where $\tilde{\chi}_\mu^2$ is given in Eq. (15), $\chi^2_{CMB}=(R-R_{obs})^2/\sigma_R^2$ and $\chi^2_{BAO}=(A-A_{obs})^2/\sigma_A^2$. The best-fit model parameters are determined by minimizing the total χ^2. As in [26, 27], the 68.3% confidence level is determined by $\Delta\chi^2\equiv\chi^2-\chi^2_{min}\leq 1.0$, 2.3 and 3.53 for $n_p=1$, 2 and 3, respectively, where n_p is the number of free model parameters. Similarly, the 95.4% confidence level is determined by $\Delta\chi^2\equiv\chi^2-\chi^2_{min}\leq 4.0$, 6.17 and 8.02 for $n_p=1$, 2 and 3, respectively.

III. COSMOLOGICAL CONSTRAINTS ON THE SIGN-CHANGEABLE INTERACTIONS

A. The case of $Q=q(\alpha\dot{\rho}_m+3\beta H\rho_m)$

Firstly, we consider the case of $Q=q(\alpha\dot{\rho}_m+3\beta H\rho_m)$ given in Eq. (8). Substituting it into Eq. (1), one can find that

$$\dot{\rho}_m=\frac{\beta q-1}{1-\alpha q}\cdot 3H\rho_m\,. \tag{19}$$

Then, substituting into Eq. (8), we can finally obtain

$$Q=\frac{\beta-\alpha}{1-\alpha q}\cdot 3qH\rho_m\,. \tag{20}$$

From Eq. (7), we have

$$\rho_m=-\frac{2}{\kappa^2}\dot{H}\,. \tag{21}$$

Substituting into Eq. (19), we find that

$$\ddot{H}=\frac{\beta q-1}{1-\alpha q}\cdot 3H\dot{H}\,, \tag{22}$$

which is in fact a second-order differential equation for $H(t)$. We can change the time t to scale factor a with the help of the universal relation $\dot{f}=Haf'$ for any function f (where a prime denotes the derivative with respect to scale factor a), and recast Eq. (22) as

$$aH''+\frac{a}{H}H'^{\,2}+H'=\frac{\beta q-1}{1-\alpha q}\cdot 3H'\,, \tag{23}$$

which is a second-order differential equation for $H(a)$. Note that the deceleration parameter

$$q=-1-\frac{\dot{H}}{H^2}=-1-\frac{a}{H}H'\,, \tag{24}$$

is also a function of H and H'. Unfortunately, if $\alpha\neq 0$, there is *no* analytical solution for the second-order differential equation (23), because one will encounter a transcendental equation. Therefore, we consider only the case of $\alpha=0$ in this work. In this case, the sign-changeable interaction reads

$$Q=3\beta qH\rho_m\,. \tag{25}$$

By solving the second-order differential equation (23) with $\alpha=0$, we find that

$$H(a)=C_{12}\left[3C_{11}(1+\beta)-(2+3\beta)\,a^{-3(1+\beta)}\right]^{1/(2+3\beta)}\,, \tag{26}$$

where C_{11} and C_{12} are both integral constants, which can be determined in the following. From Eq. (21), we find that the fractional energy density of dark matter is given by

$$\Omega_m\equiv\frac{\kappa^2\rho_m}{3H^2}=-\frac{2\dot{H}}{3H^2}=-\frac{2aH'}{3H}\,. \tag{27}$$

Substituting Eq. (26) into Eq. (27), we have

$$\Omega_m=\frac{2\,(1+\beta)}{2+3\beta-3C_{11}\,(1+\beta)\;a^{3(1+\beta)}}\,. \tag{28}$$

Requiring $\Omega_m(a=1)=\Omega_{m0}$, we obtain

$$C_{11}=\frac{\Omega_{m0}(2+3\beta)-2(1+\beta)}{3\Omega_{m0}(1+\beta)}\,. \tag{29}$$

On the other hand, requiring $H(a=1)=H_0$, from Eq. (26) we can find that

$$C_{12}=H_0\left[3C_{11}(1+\beta)-(2+3\beta)\right]^{-1/(2+3\beta)}\,. \tag{30}$$

Substituting Eqs. (29) and (30) into Eq. (26), we finally obtain

$$E\equiv\frac{H}{H_0}=\left\{1-\frac{2+3\beta}{2(1+\beta)}\,\Omega_{m0}\left[1-(1+z)^{3(1+\beta)}\right]\right\}^{1/(2+3\beta)}\,. \tag{31}$$

There are two free model parameters, namely Ω_{m0} and β. Note that when $\beta=0$, Eq. (31) reduces to $E(z)=\left[\Omega_{m0}(1+z)^3+(1-\Omega_{m0})\right]^{1/2}$, i.e., the one of ΛCDM model.

By minimizing the corresponding total χ^2 in Eq. (18), we find the best-fit parameters $\Omega_{m0}=0.2738$ and $\beta=-0.010$, whereas $\chi^2_{min}=542.725$. In Fig. 1, we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0}-\beta$ plane. Obviously, the current observational data slightly prefer a

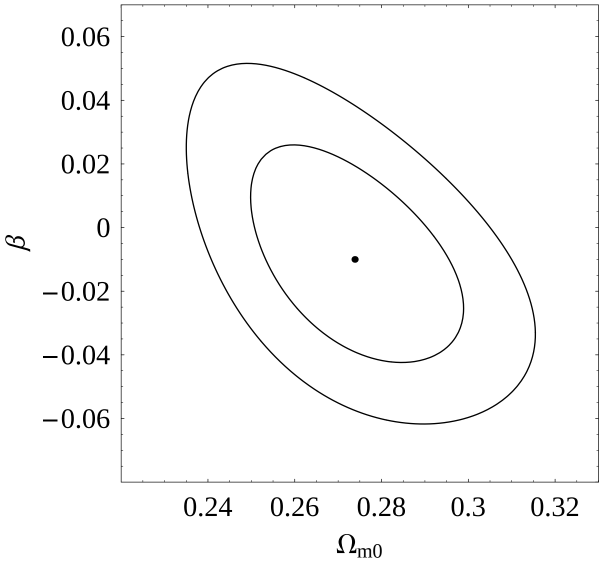

FIG. 1: The 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \beta$ plane for the case of $Q = 3\beta q H \rho_m$. The best-fit parameters are also indicated by a black solid point.

negative β. We are also interested to the fractional energy densities Ω_m given in Eq. (28) and $\Omega_\Lambda = 1 - \Omega_m$, the deceleration parameter q given in Eq. (24), and the effective equation-of-state parameter (EoS) $w_{\rm eff} \equiv p_{tot}/\rho_{tot} = (2q-1)/3$. We present them as functions of redshift z with the best-fit model parameters in Fig. 2. It is easy to find the transition redshift $z_t = 0.7489$ where the universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). Since the best-fit β is negative, dark matter decays into dark energy ($Q < 0$) when $z > z_t$, and dark energy decays into dark matter ($Q > 0$) when $z < z_t$. The interaction Q crosses the non-interacting line ($Q = 0$) at z_t.

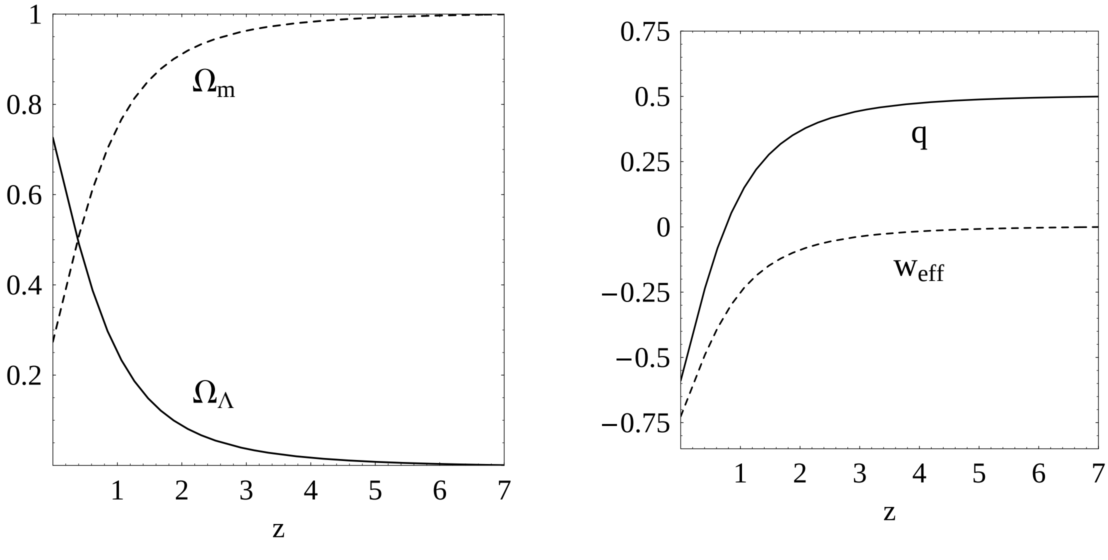

FIG. 2: Ω_m, Ω_Λ, q and $w_{\rm eff}$ as functions of redshift z with the best-fit parameters for the case of $Q = 3\beta q H \rho_m$.

B. The case of $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot})$

Secondly, we consider the case of $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot})$ given in Eq. (9). From Eq. (6), it is easy to find $\rho_{tot} = 3H^2/\kappa^2$. Substituting into Eq. (9), we can finally obtain

$$Q = \frac{3qH^3}{\kappa^2}\left(2\alpha\frac{\dot{H}}{H^2} + 3\beta\right). \tag{32}$$

Substituting Eqs. (21) and (32) into Eq. (1), we have

$$\ddot{H} + 3H\dot{H}\left(1+\alpha q\right) + \frac{9}{2}\beta q H^3 = 0\,. \tag{33}$$

Similarly, we recast it as

$$aH'' + \frac{a}{H}H'^{\,2} + \left(4+3\alpha q\right)H' + \frac{9\beta q H}{2a} = 0\,, \tag{34}$$

which is a second-order differential equation for $H(a)$. Note that the deceleration parameter q is also a function of H and H' [cf. Eq. (24)]. Similar to the case of $Q = q(\alpha\dot{\rho}_m + 3\beta H\rho_m)$, we consider only the case of $\alpha = 0$ in this work. In this case, the sign-changeable interaction reads

$$Q = 3\beta q H\rho_{tot}\,. \tag{35}$$

By solving the second-order differential equation (34) with $\alpha = 0$, we find that

$$H(a) = C_{22}\cdot a^{-3(2-3\beta+r_1)/8}\cdot\left(a^{3r_1/2} + C_{21}\right)^{1/2}, \tag{36}$$

where C_{21}, C_{22} are both integral constants, and

$$r_1 \equiv \sqrt{4+\beta\left(4+9\beta\right)}\,. \tag{37}$$

Substituting Eq. (36) into Eq. (27), we have

$$\Omega_m = \frac{1}{4}\left[2-3\beta+\left(\frac{2C_{21}}{a^{3r_1/2}+C_{21}}-1\right)r_1\right]. \tag{38}$$

Requiring $\Omega_m(a=1) = \Omega_{m0}$, we obtain

$$C_{21} = -1 + \frac{2\,r_1}{2-3\beta-4\Omega_{m0}+r_1}\,. \tag{39}$$

On the other hand, requiring $H(a=1) = H_0$, from Eq. (36) we can find that

$$C_{22} = H_0\left(1+C_{21}\right)^{-1/2}. \tag{40}$$

From Eqs. (36) and (40), it is easy to obtain

$$E \equiv \frac{H}{H_0} = (1+z)^{3(2-3\beta+r_1)/8}\cdot\left[\frac{(1+z)^{-3r_1/2}+C_{21}}{1+C_{21}}\right]^{1/2}, \tag{41}$$

where C_{21} and r_1 have been given in Eqs. (39) and (37), respectively. There are two free model parameters, namely Ω_{m0} and β. Note that when $\beta = 0$, Eq. (41) reduces to $E(z) = \left[\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})\right]^{1/2}$, i.e., the one of ΛCDM model.

Imposing the condition $0 \leq \Omega_m \leq 1$ when $a \to 0$, we have $\beta \geq 0$ from Eq. (38). Under this condition, by minimizing the corresponding total χ^2 in Eq. (18), we find the best-fit parameters $\Omega_{m0} = 0.2701$ and $\beta = 0.0$, whereas $\chi^2_{min} = 542.919$. In Fig. 3, we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \beta$ plane. In Fig. 4, we also present the Ω_m given in Eq. (38), $\Omega_\Lambda = 1 - \Omega_m$, q given in Eq. (24) and $w_{\rm eff} \equiv p_{tot}/\rho_{tot} = (2q-1)/3$ as functions of redshift z with the best-fit model parameters. The universe changes from deceleration ($q > 0$) to acceleration ($q < 0$) at the transition redshift $z_t = 0.7549$.

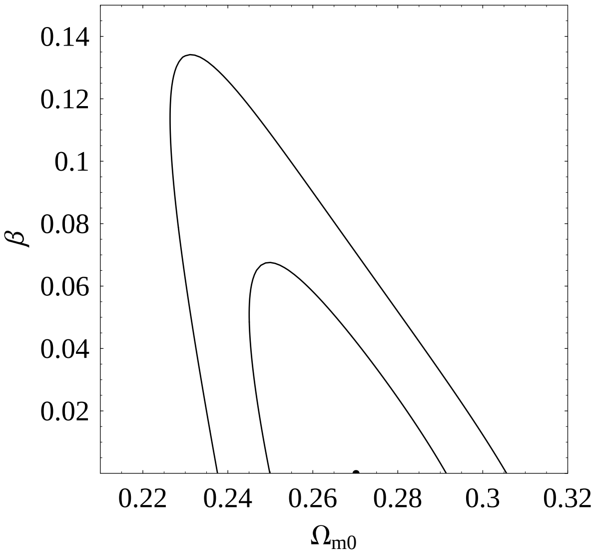

FIG. 3: The same as in Fig. 1, but for the case of $Q = 3\beta q H \rho_{tot}$ with the condition $\beta \geq 0$.

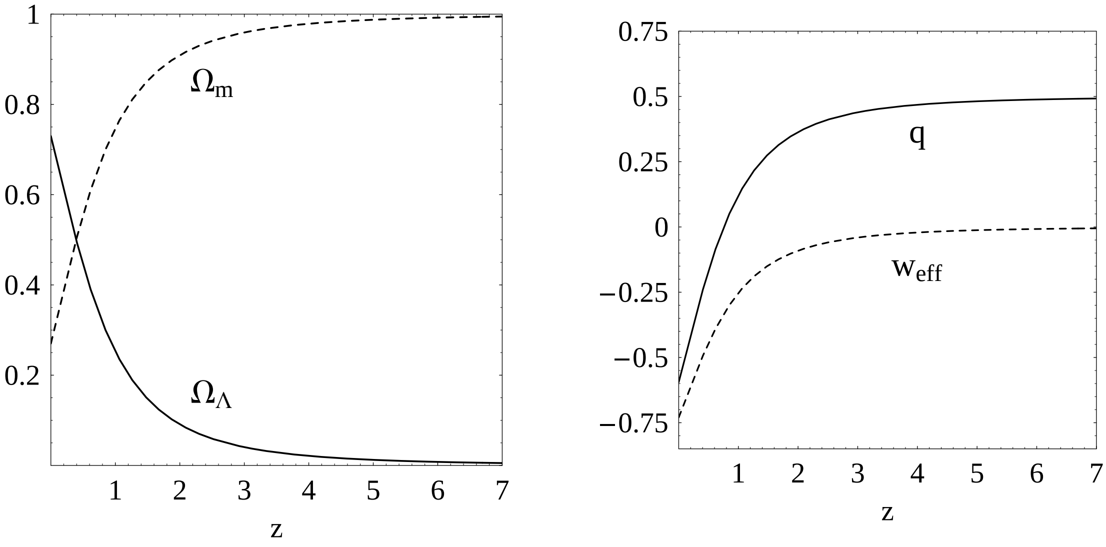

FIG. 4: The same as in Fig. 2, but for the case of $Q = 3\beta q H \rho_{tot}$ with the condition $\beta \geq 0$.

However, the above best-fit model with $\beta = 0$ is in fact the ΛCDM model without interaction between dark energy and dark matter. So, we would like to give up the condition $\beta \geq 0$. This means that in the early universe we have $\Omega_m \geq 1$ and then $\Omega_\Lambda \leq 0$, namely, ρ_Λ might be negative. Since the negative energy density can arise in quantum field theory (see e.g. [28] for a good review), it is reasonable to consider this possibility. Without the condition $\beta \geq 0$, by minimizing the corresponding total χ^2 in Eq. (18), we find the best-fit parameters $\Omega_{m0} = 0.2764$ and $\beta = -0.0247$, whereas $\chi^2_{min} = 542.711$. In Fig. 5, we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \beta$ plane. Obviously, the current observational data slightly prefer a negative β. In Fig. 6, we also present the Ω_m given in Eq. (38), $\Omega_\Lambda = 1 - \Omega_m$, q given in Eq. (24) and $w_{\rm eff} \equiv p_{tot}/\rho_{tot} = (2q - 1)/3$ as functions of redshift z with

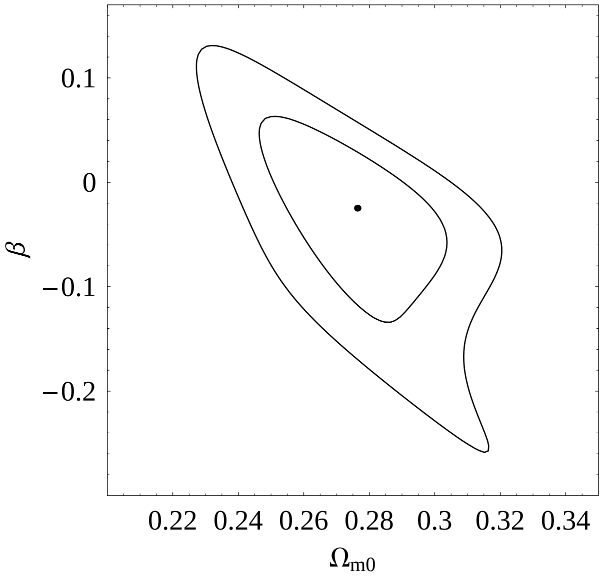

FIG. 5: The same as in Fig. 1, but for the case of $Q = 3\beta q H \rho_{tot}$ without the condition $\beta \geq 0$.

the best-fit model parameters. It is easy to find the transition redshift $z_t = 0.7688$ where the universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). Since the best-fit β is negative, dark matter decays into dark energy ($Q < 0$) when $z > z_t$, and dark energy decays into dark matter ($Q > 0$) when $z < z_t$. The interaction Q crosses the non-interacting line ($Q = 0$) at z_t.

C. The case of $Q = q(\alpha\dot{\rho}_\Lambda + 3\beta H \rho_\Lambda)$

Finally, we consider the case of $Q = q(\alpha\dot{\rho}_\Lambda + 3\beta H \rho_\Lambda)$ given in Eq. (10). Substituting it into Eq. (5), one can find that

$$\dot{\rho}_\Lambda = -\frac{3\beta q H \rho_\Lambda}{1+\alpha q}\,. \tag{42}$$

Then, substituting into Eq. (10), we can finally obtain

$$Q = \frac{3\beta q H \rho_\Lambda}{1+\alpha q}\,. \tag{43}$$

From Eqs. (6) and (7) [or equivalently Eq. (21)], we have

$$\rho_\Lambda = \frac{3}{\kappa^2}H^2 - \rho_m = \frac{1}{\kappa^2}\left(3H^2 + 2\dot{H}\right). \tag{44}$$

Substituting Eqs. (21), (43) and (44) into Eq. (1), we find that

$$\ddot{H} + 3H\dot{H}\left(1 + \frac{\beta q}{1+\alpha q}\right) + \frac{9\beta q H^3}{2(1+\alpha q)} = 0\,. \tag{45}$$

Similarly, we recast it as

$$aH'' + \frac{a}{H}H'^2 + \left(4 + \frac{3\beta q}{1+\alpha q}\right)H' + \frac{9\beta q H}{2a(1+\alpha q)} = 0\,, \tag{46}$$

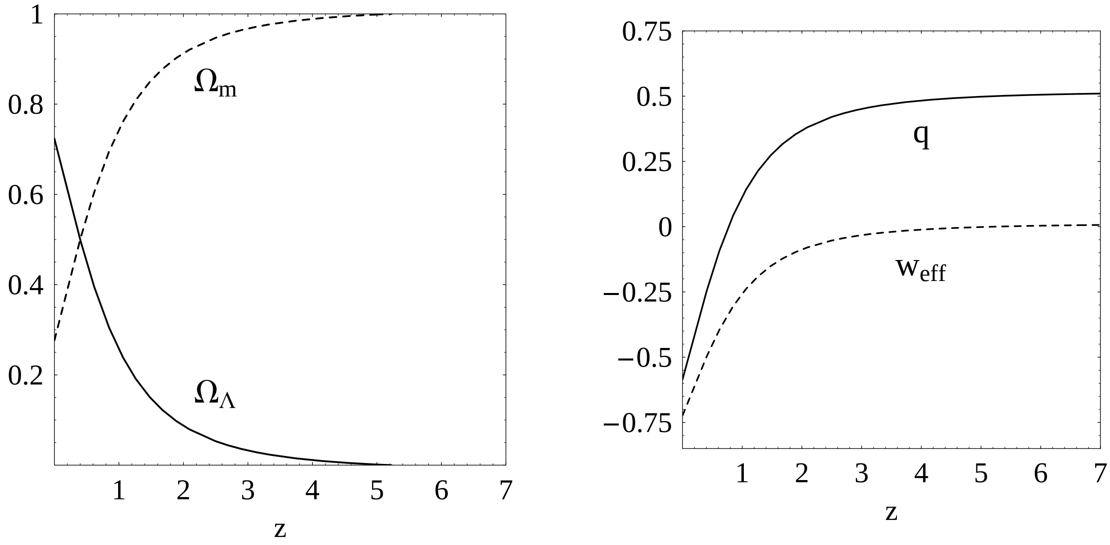

FIG. 6: The same as in Fig. 2, but for the case of $Q = 3\beta qH\rho_{tot}$ without the condition $\beta \geq 0$.

which is a second-order differential equation for $H(a)$. Note that the deceleration parameter q is also a function of H and H' [cf. Eq. (24)]. Unfortunately, if $\alpha \neq 0$, there is *no* analytical solution for the second-order differential equation (46), because one will encounter a transcendental equation. Therefore, we consider only the case of $\alpha = 0$ in this work. In this case, the sign-changeable interaction reads

$$Q = 3\beta qH\rho_\Lambda \,. \tag{47}$$

By solving the second-order differential equation (46) with $\alpha = 0$, we find that

$$H(a) = C_{32} \cdot a^{-3(2-5\beta+r_2)/[4(2-3\beta)]} \cdot \left(a^{3r_2/2} + C_{31}\right)^{1/(2-3\beta)} \,, \tag{48}$$

where C_{31}, C_{32} are both integral constants, and

$$r_2 \equiv \sqrt{(2-\beta)^2} = |\, 2-\beta\,| \,. \tag{49}$$

Substituting Eq. (48) into Eq. (27), we have

$$\Omega_m = \frac{1}{2\,(2-3\beta)}\left[2 - 5\beta + \left(\frac{2C_{31}}{a^{3r_2/2}+C_{31}} - 1\right) r_2\right] \,. \tag{50}$$

Requiring $\Omega_m(a=1) = \Omega_{m0}$, we obtain

$$C_{31} = -1 + \frac{2\,r_2}{2 - 5\beta + r_2 + 2\Omega_{m0}\,(3\beta - 2)} \,. \tag{51}$$

On the other hand, requiring $H(a=1) = H_0$, from Eq. (48) we get

$$C_{32} = H_0\,(1 + C_{31})^{1/(3\beta-2)} \,. \tag{52}$$

From Eqs. (48) and (52), it is easy to obtain

$$E \equiv \frac{H}{H_0} = (1+z)^{3(2-5\beta+r_2)/[4(2-3\beta)]} \cdot \left[\frac{(1+z)^{-3r_2/2} + C_{31}}{1 + C_{31}}\right]^{1/(2-3\beta)} \,, \tag{53}$$

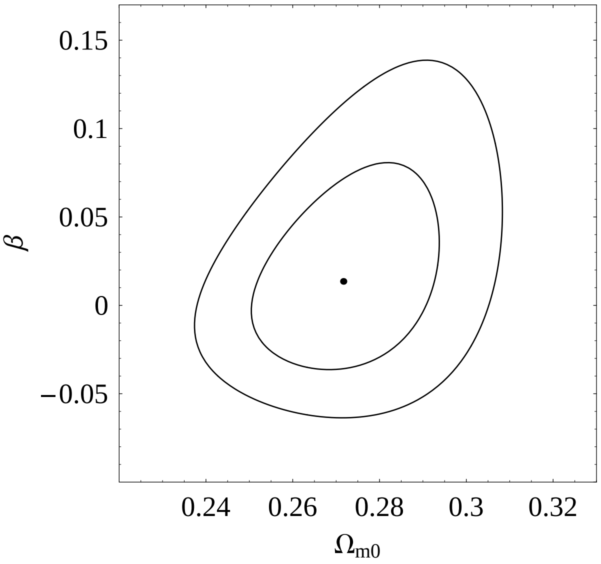

FIG. 7: The same as in Fig. 1, but for the case of $Q = 3\beta q H \rho_\Lambda$.

where C_{31} and r_2 have been given in Eqs. (51) and (49), respectively. There are two free model parameters, namely Ω_{m0} and β. Note that when $\beta = 0$, Eq. (53) reduces to $E(z) = \left[\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})\right]^{1/2}$, i.e., the one of ΛCDM model.

By minimizing the corresponding total χ^2 in Eq. (18), we find the best-fit parameters $\Omega_{m0} = 0.2717$ and $\beta = 0.0136$, whereas $\chi^2_{min} = 542.778$. In Fig. 7, we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \beta$ plane. Obviously, the current observational data slightly prefer a positive β. In Fig. 8, we also present the Ω_m given in Eq. (50), $\Omega_\Lambda = 1 - \Omega_m$, q given in Eq. (24) and $w_{\rm eff} \equiv p_{tot}/\rho_{tot} = (2q-1)/3$ as functions of redshift z with the best-fit model parameters. It is easy to find the transition redshift $z_t = 0.7398$ where the universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). Since the best-fit β is positive, dark energy decays into dark matter ($Q > 0$) when $z > z_t$, dark matter decays into dark energy ($Q < 0$) when $z < z_t$. The interaction Q crosses the non-interacting line ($Q = 0$) at z_t.

IV. CONCLUDING REMARKS

Recently, Cai and Su [15] argued that the sign of interaction Q in the dark sector changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$, by using a model-independent method to deal with the observational data. In fact, this result raises a remarkable problem, since most of the familiar interactions cannot change their signs in the whole cosmic history. Motivated by the work of Cai and Su, we have proposed a new type of interaction in a previous work [16]. The key ingredient is the deceleration parameter q in the interaction Q. Therefore, the interaction Q can change its sign when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). In the present work, we considered the cosmological constraints on this new type of sign-changeable interactions, by using the latest observational data. We found that the constraints on the model parameters are fairly tight. In particular, the key parameter β has been constrained to a very narrow range.

Some remarks are in order. Firstly, we briefly consider the comparison of these models. For convenience, we also consider the well-known ΛCDM model in addition. In fact, it corresponds to the decaying Λ model with $Q = 0$. Fitting ΛCDM model to the observational data considered in the present work, it is easy to find the corresponding best-fit parameter $\Omega_{m0} = 0.2701$, whereas $\chi^2_{min} = 542.919$. Of course, we would

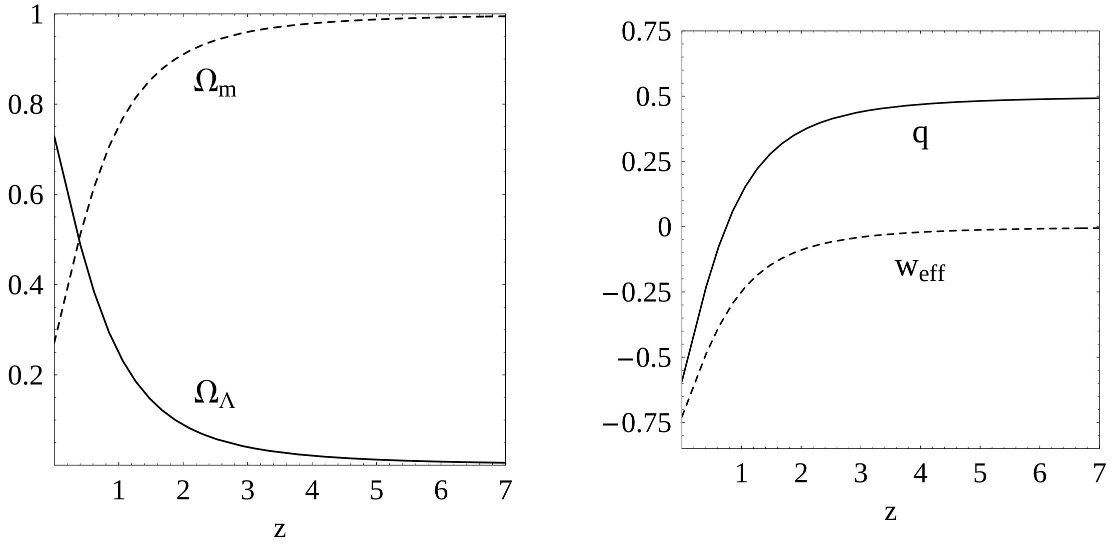

FIG. 8: The same as in Fig. 2, but for the case of $Q = 3\beta q H\rho_\Lambda$.

like to also consider the decaying Λ model with a traditional interaction $Q = 3\beta H\rho_m$ which cannot change its sign in the whole cosmic history. The corresponding $E(z)$ can be found in e.g. [12], namely

$$E(z) = \left[\frac{\Omega_{m0}}{1-\beta}(1+z)^{3(1-\beta)} + \left(1 - \frac{\Omega_{m0}}{1-\beta}\right)\right]^{1/2}. \tag{54}$$

Fitting to the same observational data, we find the best-fit parameters $\Omega_{m0} = 0.2731$ and $\beta = -0.0021$, whereas $\chi^2_{min} = 542.735$. A conventional criterion for model comparison in the literature is χ^2_{min}/dof, in which the degree of freedom $dof = N - k$, whereas N and k are the number of data points and the number of free model parameters, respectively. We present the χ^2_{min}/dof for all the 6 models in Table I. On the other hand, there are other criterions for model comparison in the literature, such as Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). The BIC is defined by [29, 31]

$$\text{BIC} = -2\ln\mathcal{L}_{max} + k\ln N\,, \tag{55}$$

where $\mathcal{L}_{max}$ is the maximum likelihood. In the Gaussian cases, $\chi^2_{min} = -2\ln\mathcal{L}_{max}$. So, the difference in BIC between two models is given by $\Delta\text{BIC} = \Delta\chi^2_{min} + \Delta k\ln N$. The AIC is defined by [30, 31]

$$\text{AIC} = -2\ln\mathcal{L}_{max} + 2k\,. \tag{56}$$

The difference in AIC between two models is given by $\Delta\text{AIC} = \Delta\chi^2_{min} + 2\Delta k$. In Table I, we also present the ΔBIC and ΔAIC of all the 6 models considered in this work. Notice that ΛCDM has been chosen to be the fiducial model when we calculate ΔBIC and ΔAIC. From Table I, it is easy to see that the rank of models is coincident in all the 3 criterions (χ^2_{min}/dof, BIC and AIC). The ΛCDM model is still the best one. However, it is well known that ΛCDM model is plagued with the cosmological constant problem and the coincidence problem (see e.g. [1]). On the other hand, there are some observational evidences for the interaction between dark energy and dark matter [13, 14], and the coincidence problem can be alleviated in the interacting dark energy models. Therefore, it is still worthwhile to study the interacting dark energy models. Although the model with traditional interaction which cannot change its sign is very close to the other models with sign-changeable interactions, the latter are phenomenally richer (see e.g. [16]). Therefore, we consider that the models with sign-changeable interactions deserve further investigations.

Model	ΛCDM $(Q=0)$	$Q=3\beta H\rho_m$	$Q=3\beta qH\rho_m$	$Q=3\beta qH\rho_{tot}$ with $\beta\geq 0$	$Q=3\beta qH\rho_{tot}$ without $\beta\geq 0$	$Q=3\beta qH\rho_\Lambda$
Best fit	$\Omega_{m0}=0.2701$	$\Omega_{m0}=0.2731$ $\beta=-0.0021$	$\Omega_{m0}=0.2738$ $\beta=-0.010$	$\Omega_{m0}=0.2701$ $\beta=0.0$	$\Omega_{m0}=0.2764$ $\beta=-0.0247$	$\Omega_{m0}=0.2717$ $\beta=0.0136$
χ^2_{min}	542.919	542.735	542.725	542.919	542.711	542.778
k	1	2	2	2	2	2
χ^2_{min}/dof	0.9730	0.9744	0.9744	0.9747	0.9743	0.9745
ΔBIC	0	6.142	6.132	6.326	6.118	6.185
ΔAIC	0	1.816	1.806	2.0	1.792	1.859
Rank	1	4	3	6	2	5

TABLE I: Summarizing all the 6 models considered in this work.

Secondly, we note that the case of $Q=3\beta qH\rho_\Lambda$ is fairly different from the cases of $Q=3\beta qH\rho_m$ and $Q=3\beta qH\rho_{tot}$. Comparing Fig. 7 with Figs. 1, 3 and 5, it is easy to see that the direction of contours in the $\Omega_{m0}-\beta$ plane is rightward for the case of $Q=3\beta qH\rho_\Lambda$, whereas the ones are leftward for the cases of $Q=3\beta qH\rho_m$ and $Q=3\beta qH\rho_{tot}$. From Table I, we find that the best-fit β is positive for the case of $Q=3\beta qH\rho_\Lambda$, whereas the ones are negative (or zero) for the cases of $Q=3\beta qH\rho_m$ and $Q=3\beta qH\rho_{tot}$. This means that in the case of $Q=3\beta qH\rho_\Lambda$ the interaction Q crosses the non-interacting line ($Q=0$) from above to below, whereas in the cases of $Q=3\beta qH\rho_m$ and $Q=3\beta qH\rho_{tot}$ the interaction Q crosses the non-interacting line ($Q=0$) from below to above. This is physically interesting, because $Q>0$ means that the energy transfers from dark energy to dark matter, whereas $Q<0$ means that the energy transfers from dark matter to dark energy.

Finally, in this work the role of dark energy is only played by the decaying Λ (vacuum energy), whereas the parameter α in the sign-changeable interactions are chosen to be zero. So, the constraints obtained in this work cannot be directly used to the models different from the ones considered here. In fact, the interacting dark energy models with sign-changeable interactions can be generalized. For instance, one can choose the dark energy to be the one with a constant or variable EoS (including parameterized EoS, or even the ones of quintessence, phantom, k-essence, Chaplygin gas, quintom, hessence, holographic or agegraphic dark energy, and so on). Of course, if the computer is enough powerful, one can also let the parameter α be free and then constrain the models numerically.

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