

The cosmological constant and the relaxed universe

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Abstract. We study the role of the cosmological constant (CC) as a component of dark energy (DE). It is argued that the cosmological term is in general unavoidable and it should not be ignored even when dynamical DE sources are considered. From the theoretical point of view quantum zero-point energy and phase transitions suggest a CC of large magnitude in contrast to its tiny observed value. Simply relieving this disaccord with a counterterm requires extreme fine-tuning which is referred to as the old CC problem. To avoid it, we discuss some recent approaches for neutralising a large CC dynamically without adding a fine-tuned counterterm. This can be realised by an effective DE component which relaxes the cosmic expansion by counteracting the effect of the large CC. Alternatively, a CC filter is constructed by modifying gravity to make it insensitive to vacuum energy.

1. Introduction

The old CC problem is of theoretical nature [1]. It is not forbidden to explain the current accelerated expansion of the universe by a small CC or vacuum energy density $\Lambda_{\text{obs}} \sim 10^{-47} \text{ GeV}^4$. The cosmological term can be considered as the zero-order term in almost every action functional and it is not absent unless further assumptions are made. If it were a free parameter its observationally favoured smallness would not pose any problem. And due to its minimalist nature it would be a perfect candidate to explain the late-time expansion history.

However, modern theories suggest the existence of large contributions to the CC Λ which have to sum up to the tiny value Λ_{obs} . This is a highly non-trivial requirement because all those parts are usually not related to each other and they can be of different magnitude and sign. Perhaps the worst part comes from quantum field theory in the form of vacuum or zero-point energy. Since every field mode provides a contribution related to its energy ω , the integral over all modes in the energy spectrum results into an infinite CC contribution. For a massless field ω is just the mode momentum p and one finds the quartically divergent result

$$\Lambda_0 \sim \int_0^\infty d^3p p. \quad (1)$$

Common estimations impose an ultra-violet (UV) cutoff at a high-energy scale p_{max} , which replaces the upper bound of the integral. This naive procedure yields the often presented number $|\Lambda_0/\Lambda_{\text{obs}}| \sim 10^{123}$ when the maximal momentum is assumed to be around the Planck mass $M_P \sim 10^{19} \text{ GeV}$. So far no evidence has been found for the existence of a cutoff indicating that p_{max} should be at least above $M_{\text{ew}} \sim 10^2 \text{ GeV}$, which is roughly the energy scale accessible to current accelerator experiments. Obviously, this value is still too high. The energy cutoff

compatible with Λ_{obs} would be of the order of neutrino masses $p_{\text{max}} \sim 10^{-3}$ eV, which is in clear contrast to quantum field theory tested up to the electro-weak scale M_{ew} . However, it should be noted that vacuum energy¹ cannot be measured in the lab because it couples only to gravity. Therefore, a low UV cutoff just for the zero-point energy part could be allowed by observations, but it would be awkward from the theoretical point of view. Finally, using renormalisation the divergence in Eq. (1) can be formally removed, but finite terms remain typically of the order m^4 for fields with mass m . Despite the fact that we do not know how to handle the term in (1), there is no reason to believe that it is small or zero. Instead, it might be a large quantity according to the above considerations.

Next, a more quantitative CC contribution comes from the symmetry breaking of the electro-weak sector of the standard model. As the Higgs field acquires a non-zero vacuum expectation value in the broken phase the value of the potential in the new minimum is different from its minimal value in the unbroken phase. Thus, the phase transition induces a finite vacuum energy shift $\mathcal{O}(M_{\text{ew}}^4)$ again much larger than Λ_{obs} . Apart from this classical term, quantum corrections render the situation worse as new terms occur at every order in perturbation theory [2].

Another source of vacuum energy could be primordial inflation, where a large positive value of the inflaton potential drives the rapid accelerated expansion in the early universe. At the end of the inflationary period the inflaton scalar approaches the minimum of its potential, where the effective vacuum energy is much lower than before. However, this does not imply that it is zero or of the observed CC value, it could be much larger in magnitude and even negative. In that case the well known Big Bang evolution had stopped already far in the past.

It is unlikely that the sum Λ of all contributions mentioned above is of the order Λ_{obs} , instead it is expected to be of the order of the largest contribution. To obtain consistency between the theoretical value Λ and the observed one, the easiest way is adding a counterterm Λ_{ct} which matches $(-\Lambda)$ to very high precision such that $\Lambda_{\text{ct}} + \Lambda \approx \Lambda_{\text{obs}}$. This procedure is considered to be highly unnatural since it requires an enormous amount of fine-tuning in Λ_{ct} , e.g. fixing up to 122 decimal digits.

Motivated by the problems of understanding the zero-point energy in Eq. (1), it has become popular to “replace” the CC by dynamical DE. Note that in general this does not address the old CC problem. Instead it introduces more steps. First, any CC originating e.g. from the sources mentioned above has to be removed somehow. Clearly, without further explanation it means that a fine-tuned counterterm is added which cancels Λ . This step is often suppressed in the description of DE models. Second, a new dynamical source is introduced, which drives the late-time accelerated expansion [3]. Obviously, the second step requires the first one but not vice-verse. The reason is that making the CC vanish or much smaller than Λ_{obs} requires even more fine-tuning than adjusting it to match exactly the observed value. On the other hand, one could argue that a yet unknown theory of quantum gravity (or an undiscovered feature of the known candidates) makes the CC vanish and clears the path for dynamical DE models. However, this unknown but probably powerful theory could also arrange the CC to match its observed value and no additional dynamical DE component would be needed. Moreover, it seems that at the moment observations do not require a dynamical nature of DE [4]. Of course, the situation might change in the future when observational data becomes better and more precise. But even in that case the problem of getting rid of a large Λ persists and the CC as a static DE component is by no way ruled out. Accordingly, the relevant question about DE and the accelerated expansion is not whether there is either a static CC or a dynamical component, but if we need only a CC or a CC plus something dynamical. More complicated but completely analogous to dynamical DE components are modified gravity theories with built-in late-time acceleration [5] or unconventional cosmological models, where the observed acceleration is just

¹ We mean its absolute value, not finite differences related to the Casimir effect.

an apparent effect. It will be interesting to know whether nature exhibits more complexity than the simplest explanation. In the “worst” case we have to deal with a mixture of many sources for acceleration in addition to the old CC problem.

In the following we concentrate on the latter problem, i.e. we accept the existence of a presumed large value $|\Lambda| \gg \Lambda_{\text{obs}}$ as a part of the energy content of the universe. Conventional energy sources like dust and radiation dilute with the expansion of the universe, and without counter-measures the large CC term eventually starts dominating the cosmic expansion very early in the cosmic history. As a result, $\Lambda < 0$ initiates a collapse and the universe dies in a Big Crunch. For $\Lambda > 0$ an inflationary epoch with a large expansion rate H starts and continues forever because the constant Λ does not offer a graceful exit. In order to avoid these catastrophic scenarios and to permit a reasonable Big Bang-style universe, we have to neutralise the effect of Λ . Of course, we want to avoid the introduction of a fine-tuned counterterm, instead we look for a dynamical mechanism or a suitable modification of gravity, which tames the large CC term. Here, we discuss three recent approaches in this context. A relaxation model with a variable CC (including the large term Λ), a similar model implemented as modified gravity in the metric formalism, and finally a CC filter in a setup using the Palatini formalism. All these models have in common that they do not need a fine-tuned counterterm despite the existence of a large CC.

2. Relaxing a large CC

Let us begin with a simple example of the relaxation mechanism. To neutralise the effect of the large CC we introduce an additional component of opposite sign, which adjusts to the value of Λ dynamically. In our first model the total DE density is given by $\rho_\Lambda = \Lambda + \beta/f$ with a constant parameter β and a function f . According to the Friedmann equation the Hubble expansion rate H is sensitive to the sum of all energy densities

$$\rho_c = \frac{3H^2}{8\pi G_N} = \rho_m + \rho_\Lambda = \rho_m + \Lambda + \frac{\beta}{f}, \quad (2)$$

where ρ_c is the critical energy density, G_N is Newton’s constant, and ρ_m denotes the energy density of matter and radiation. Without the β/f term the constant Λ would control the expansion when ρ_m becomes sufficiently small. However, we want to have a well-behaved late-time evolution with H of the order of the currently observed Hubble rate H_0 . For this purpose consider the dynamical term defined by $f = H^2$ and β having the opposite sign of Λ [6]. As a result of the small Hubble rate at late times only the term $(\Lambda + \beta/H^2)$ is relevant, which is obvious after dividing Eq. (2) by $|\Lambda| \gg \rho_c, \rho_m$,

$$1 + \frac{\beta}{\Lambda H^2} = \frac{\rho_c - \rho_m}{\Lambda}. \quad (3)$$

The right-hand side is much smaller than unity and can be safely neglected. Therefore, we find the consistent solution $H^2 \rightarrow H_e^2 = -\beta/\Lambda$ describing a de Sitter cosmos with a low Hubble rate H_e , which complies with current observations. Without the term β/f the expected value of H^2 would be proportional to Λ and thus very large. Here, H_e^2 is inversely proportional to the large CC indicating a low Hubble rate. Note also the absence of fine-tuning in the parameter β . Small changes in β only lead to small changes in H_e^2 , hence, it is not necessary to fix a lot of decimal digits.

The example $f = H^2$ works well at late times when H is small such that the dynamical term β/H^2 becomes large and dynamically relaxes the large CC term. At earlier times in the cosmic history, the Hubble rate was much larger and we have to enhance the function f for the relaxation mechanism to work. First, consider the radiation era, where the scale factor $a \propto t^{1/2}$ scales like the square root of the cosmic time t and the deceleration variable $q = -\ddot{a}/(aH^2) \lesssim 1$ is

very close to unity. Using this property the function $f \propto H^2(q-1)$ would be sufficiently small to allow $\beta/f \propto (q-1)^{-1}$ to counteract the large constant Λ in a similar way as at late times. Also here $\rho_c, \rho_m \ll |\Lambda|$ can be neglected in the Friedmann equation yielding $\beta/(H^2(q-1)) + \Lambda = 0$. As a result, the large value of H in the radiation era implies $q \approx 1$ as the solution of this equation, which means that the cosmological evolution will be that of a radiation universe despite the large CC term. Moreover, since H decreases with time the deceleration q will slightly change its value but it stays very close to unity. It is clear that replacing $(q-1)$ in f by $(q - \frac{1}{2})$ yields a cosmological expansion behaviour with $q \approx \frac{1}{2}$ for large Hubble rates. Consequently, the effect of the CC is neutralised also in the matter era, where $q = \frac{1}{2}$ follows from the scale factor $a \propto t^{2/3}$.

In summary, a large CC can be relaxed in all major epochs of a Big Bang universe by constructing an adequate function f in Eq. (2). In Ref. [7] the following model was proposed,

$$f = \frac{4(2-q)}{(1-q)} \left(\frac{1}{2} - q \right) H^2 + y 72(1+q^2)(1-q)H^6, \quad (4)$$

which allows the relaxation of the large CC in all cosmological epochs. The different powers of H in f ensures the correct temporal sequence. For very large H the last term $\propto H^6(1-q)$ is responsible for the radiation era, while the first term $\propto H^2(\frac{1}{2} - q)$ relaxes the CC in the matter and the final de Sitter eras. The parameter y is related to the radiation-matter transition. Note that Eq. (4) can be written in terms of the Ricci scalar $R = 6H^2(1-q)$, the squared Ricci tensor $Q = R_{ab}R^{ab} = 12H^4(q^2 - q + 1)$ and the squared Riemann tensor $T = R_{abcd}R^{abcd} = 12H^4(q^2 + 1)$, i.e. $f = (R^2 - Q + yR^2T)/R$. Moreover, the model features an implicit interaction with dark matter [8] as well as interesting tracking properties [7] and a reasonable evolution of perturbations [9].

3. The relaxation mechanism in modified gravity

In the previous section the large term Λ was enhanced directly by a dynamical term β/f , which provides a reasonable cosmological evolution. Now, we show that this relaxation mechanism can be implemented in an action functional for modified gravity. Consider the action

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[\frac{R}{16\pi G_N} + \mathcal{L}_{\text{mat}} - \Lambda - \beta F(R, G) \right], \quad (5)$$

which contains the Einstein-Hilbert term, the matter Lagrangian \mathcal{L}_{mat} , the large CC term Λ and the modified gravity functional $F(R, G)$ in terms of the Ricci scalar and the Gauß-Bonnet invariant $G = R^2 - 4Q + T$. As before β is a constant parameter. In the metric formalism the variation of \mathcal{S} with respect to the metric g_{ab} yields the Einstein equations

$$G_{ab} = -8\pi G_N [T_{ab} + g_{ab}\Lambda + 2\beta E_{ab}], \quad (6)$$

where T_{ab} is the energy-stress tensor of matter, and E_{ab} emerges from the new term $F(R, G)$. On a flat Friedmann-Lemaître-Robertson-Walker (FLRW) background E_{ab} is completely defined by the effective energy density ρ_F and pressure p_F . One finds

$$\rho_F = 2\beta \left[\frac{1}{2}F - 3(\dot{H} + H^2)F^R + 3H\dot{F}^R - 12H^2(\dot{H} + H^2)F^G + 12H^3\dot{F}^G \right], \quad (7)$$

where $F^{R,G}$ correspond to partial derivatives of F with respect to R and G , respectively. Next, we consider the model from Ref. [10], where $F(R, G) = 1/B$ with $B := \frac{2}{3}R^2 + \frac{1}{2}G + (yR)^3$ is very similar to f in Eq. (4) if written in terms of H and q ,

$$B = 24H^4 \left(q - \frac{1}{2} \right) (q - 2) + H^6 [6y(1 - q)]^3. \quad (8)$$

Since every derivative of F in Eq. (7) yields another factor B in the denominator of ρ_F we obtain $\rho_F = N/B^d$ with $d \geq 1$. N denotes terms involving H and its time derivatives. As a result, the structure of the energy density ρ_F is very similar to β/f from the previous section and it behaves in a similar way. ρ_F becomes sufficiently large in the radiation, matter and respectively the late-time DE eras because the terms $(q-1)$, $(q-\frac{1}{2})$ and H^4 in the denominator B are small in the corresponding epochs. Therefore, the Friedmann equation reduces to $\rho_F + \Lambda = 0$ with corrections much smaller than $|\Lambda|$. As before, this yields in a dynamical way a relaxed cosmological expansion behaviour despite the large term Λ . Note that the numerator N in ρ_F does not harm the working principle of the CC relaxation mechanism, but it provides new solutions for the late-time behaviour, e.g. accelerating power-law expansion and future singularities. We refer the reader to Ref. [2] for a detailed study of the above model and its generalisations.

4. Filtering out the CC in the Palatini formalism

In the previous section we discussed the CC relaxation mechanism via modified gravity in the metric formalism, where the connection Γ_{bc}^a is the Levi-Civita connection of the metric g_{ab} . Now, we modify gravity in the Palatini formalism in which g_{ab} and the connection Γ are treated independently. Recently, we have shown that this property allows the construction of a filter for the CC [11], which is based on the action

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[\frac{1}{2} f(R, Q) + \mathcal{L}_{\text{mat}}[g_{ab}] - \Lambda \right], \quad (9)$$

where \mathcal{L}_{mat} does not depend on Γ . The Ricci scalar $R = g^{ab}R_{ab}$ and $Q = R^{ab}R_{ab}$ are quantities which depend on g_{ab} and Γ , while the Ricci tensor $R_{ab} = \Gamma_{ab,e}^e - \Gamma_{eb,a}^e + \Gamma_{ab}^e \Gamma_{fe}^f - \Gamma_{af}^e \Gamma_{eb}^f$ is defined only by Γ . Here, we assume that Γ and R_{ab} are symmetric, which is not the most general case [12]. The variation of \mathcal{S} with respect to g_{ab} yields the Einstein equations

$$f^R R_{mn} + 2f^Q R_m^a R_{an} - \frac{1}{2} g_{mn} f = T_{mn}, \quad (10)$$

where we include the large CC Λ in the stress tensor T_{mn} in addition to ordinary matter with the energy density ρ and the pressure p . In the Palatini formalism the variation $\delta\mathcal{S}/\delta\Gamma_{bc}^a = 0$ provides the equation for the connection

$$\nabla_a[\Gamma] \left[\sqrt{|g|} (f^R g^{mn} + 2f^Q R^{mn}) \right] = 0, \quad (11)$$

where ∇_a is the covariant derivative in terms of the yet unknown Γ . For solving this set of equations we used the method described in Ref. [13]. In the following we summarise the results for the CC filter model defined by the action functional

$$f(R, Q) = \kappa R + z \quad \text{with} \quad z := \beta \left(\frac{R^2}{B} \right)^m, \quad (12)$$

where κ, β are parameters and $m > 0$ is dimensionless. Here, the function $B := R^2 - Q$ is similar to f in Sec. 2, however, note that R, Q and B are in general different from their metric versions. From Eq. (10) we obtain two algebraic equations for the two unknowns R and B . First, its trace $(-2 - \frac{4}{3}m)z = \kappa R - 4\Lambda + 3p - \rho \approx -4\Lambda$ tells us that z is approximately constant and of the order of the large CC. The second equation reads $\kappa R + r = \frac{2}{9}mz \frac{B}{R^2} + \epsilon$, where we introduced $r := \rho + p$. ϵ denotes suppressed corrections which are neglected in the following. Then, we combine both equations with z from Eq. (12) and find

$$(\kappa R + r)^3 (\kappa R)^4 = \rho_e^7 := \kappa^4 \left(\frac{2}{9}mz \left(\frac{\beta}{z} \right)^{\frac{1}{m}} \right)^3, \quad (13)$$

indicating that the Ricci scalar R can be expressed only by the energy density ρ and the pressure p of matter in r and by the constant ρ_e . Moreover, we observe that vacuum energy terms with equation of state $p = -\rho$ do not contribute to R . Obviously, the large CC Λ is filtered out, it appears only in the constant ρ_e , which can be adjusted by the parameter β .

In the following we restrict the cosmological discussion to negative values of ρ_e . In the early universe, when $-\rho_e \ll r \ll |\Lambda|$ we find $\kappa R = -r + \mathcal{O}(\epsilon)$ from Eq. (13). This implies that the parameter κ is negative since $r > 0$ for ordinary matter. At late times, when $r \ll -\rho_e$, the Ricci scalar becomes constant, $\kappa R = \rho_e - \frac{3}{7}r + \mathcal{O}(\epsilon)$. Next, let us express the matter energy density by a power-law in the cosmic scale factor, $\rho \propto a^{-3(\omega+1)}$, with the matter equation of state ω . Thus, R and B can be expressed in terms of $a(t)$, which allows calculating the Palatini connection Γ via Eq. (11) as a function of $a(t)$ and its derivatives. From Γ the Ricci tensor $R_{ab}[\Gamma]$ can be determined and the corresponding Ricci scalar $R = g^{ab}R_{ab}[\Gamma]$ must be equal to $R(r)$, which we found from Eq. (13). With the ansatz $a(t) \propto t^s$ for the scale factor we obtain the following equations for the Hubble rate H . In the era where r is dominated by dust matter with $\omega = 0$ we find $(-\kappa)H^2 = \frac{4}{27}\rho_{\text{dust}}$, whereas in the radiation dominated epoch with $\omega = \frac{1}{3}$ we obtain $(-\kappa)H^2 = \frac{4}{25}\rho_{\text{radiation}}$. Obviously, these results are very close to general relativity when the parameter $\kappa \sim 1/G_N$ is suitably chosen. Moreover, at late times we find the de Sitter solution $\kappa R = \kappa(3H^2) = \rho_e$, where the late-time Hubble rate can be adjusted by the parameter β in (13). As in the previous sections it is not necessary to fine-tune its value. Note in the solutions above the cosmic expansion is controlled by matter only, but not by the large CC term, which has been filtered out. It can be shown that the CC filter is active in black hole-like environments, too [11]. As a closing remark, our setup in the Palatini formalism leads to second order equations of motion just as general relativity.

5. Conclusions

We have discussed several arguments for the existence of a presumed large CC, which has to be neutralised to permit a reasonable evolution of the universe. We have also argued that naively replacing the cosmological term by other sources for late-time acceleration corresponds to the introduction of a fine-tuned counterterm. As an alternative to this method we have reviewed three recent approaches to relax the cosmic expansion in the presence of a large CC.

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