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Non-perturbatively improved clover action for SU(2) gauge + fundamental and adjoint representation fermions

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The research of strongly coupled beyond-the-standard-model theories has generated significant interest in non-abelian gauge field theories with different number of fermions in different representations. Motivated by the increased interest to various technicolor scenarios, we study the non-perturbative improvement of the Wilson-clover action with $SU(2)$ gauge fields and 2 flavors of fermions in the fundamental and adjoint representations. The Sheikholeslami-Wohlert coefficients are fixed using Schrödinger functional boundary conditions. The adjoint representation theory is a candidate for a "minimal technicolor" theory, already studied on the lattice using unimproved Wilson fermions.

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1. Introduction

Gauge theories with fermions in non-fundamental representations have recently been proposed as candidates for phenomenologically viable Technicolor models. A particularly simple example of these is so-called *minimal walking technicolor* (MWT), a theory with SU(2) gauge fields and two adjoint representation fermions [1, 2, 3]. Analytical studies suggest that this theory has either an infrared stable fixed point or the coupling constant "walks," i.e. evolves very slowly over some energy range. This question is inherently non-perturbative and lattice simulations are needed.

Initial lattice studies of MWT suggest that the theory has an infrared stable fixed point [4, 5, 6, 7, 8, 9, 10]. However, these investigations were made with non-improved Wilson fermion action, which is known to have large $O(a)$ cutoff effects. The cutoff effects can be especially significant when the evolution of the coupling constant is measured with the Schrödinger functional method [4, 7]; where, with the unimproved action, it was not possible to perform the continuum limit in a controlled fashion. Thus, it is important to use actions which have as small cutoff effects as possible. This becomes especially important at relatively large bare lattice couplings, which have been observed to be necessary in order to study the relevant physical domain.

Our aim is to calculate the $O(a)$ improvement coefficients for the Wilson-clover action for MWT, and, for comparison, also for SU(2) gauge theory with two fundamental fermion flavours. We do this in two stages: in this work we describe the non-perturbative evoluation of the clover (Sheikholeslami-Wohlert) coefficient in these two theories using the Schrödinger functional method [11, 12, 13, 14]. In order to apply the Schrödinger functional for the calculation of the coupling constant, we also need to calculate various "boundary improvement" coefficients. This we do using perturbation theory, and the calculation is described in these proceedings in ref. [16].

2. The Models

We study the nonperturbative order a improvement of two lattice gauge models, one where two flavors of fermions couple to the fundamental representation of an SU(2) gauge field, and one where they couple to the adjoint representation. They share the same standard Wilson gauge action for SU(2). The fermion action is

$$S_F = a^4 \sum_x \bar{\psi}(x)\,(D+m_0)\,\psi(x) \tag{2.1}$$

$$D = \frac{1}{2}\left[\gamma_\mu(\nabla^*_\mu + \nabla_\mu) - a\nabla^*_\mu\nabla_\mu\right] + c_{\rm SW}\frac{ia}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}, \tag{2.2}$$

where $\hat{F}_{\mu\nu}$ is the symmetrized field strength tensor (clover) and

$$\nabla_\mu\psi(x) = \frac{1}{a}\left[\widetilde{U}_\mu(x)\psi(x+\hat{\mu}) - \psi(x)\right], \qquad \nabla^*_\mu\psi(x) = \frac{1}{a}\left[\psi(x) - \widetilde{U}^\dagger_\mu(x-\mu)\psi(x-\hat{\mu})\right]. \tag{2.3}$$

Here $\widetilde{U}$ is the gauge link in the appropriate fermion representation, that is, it is the standard fundamental SU(2) link for fundamental representation fermions, and for adjoint fermions it is

$$\widetilde{U}^{ab}_\mu(x) = 2\mathrm{Tr}\,\left(\lambda^a U_\mu(x)\lambda^b U^\dagger_\mu(x)\right), \qquad \lambda^a = \frac{1}{2}\sigma^a,\ \ a = 1,2,3. \tag{2.4}$$

3. Schrödinger functional scheme

Our method for determining the improvement coefficient c_{SW} follows refs. [12, 13, 14], where the Schrödinger functional scheme is used to determine c_{SW} in the case of *QCD*. However, for the adjoint representation fermions (and, in general, for fermions in higher representations) the method is modified, as described below.

We shall work with lattices of size $L^3 \times T$. In the Schrödinger functional scheme the spatial gauge fields are fixed to constant values at time slices $x_0 = 0$ and $x_0 = T$, chosen so that these generate a chromoelectric background field.

For fundamental fermions we use color diagonal background fields as in ref. [12]

$$U_k(x_0 = T) \,=\, \exp(iC'), \;\; C' = -\frac{\pi}{4}\frac{a\sigma^3}{L} \tag{3.1}$$

$$U_k(x_0 = 0) \,=\, \exp(iC), \;\; C = -\frac{3\pi}{4}\frac{a\sigma^3}{L}. \tag{3.2}$$

These generate a chromoelectric background field $\propto \sigma^3$. Different boundary conditions give rise to different cutoff effects in fermion propagation when the source is at $x_0 = 0$ or at $x_0 = T$. The idea is to find the value of c_{SW} which maximizes the symmetry between the two cases, leading to automatic $O(a)$ improvement.

For adjoint representation fermions, however, complications emerge. Using Eq. (2.4) we immediately notice that the boundary matrices (3.1), (3.2) are transformed to form

$$\widetilde{U}_k = \begin{pmatrix} \ldots & \ldots & 0 \\ \ldots & \ldots & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Thus, there is a component of the adjoint fermion spinor which simply does not see the background field. This feature is independent of the color structure chosen for the boundary conditions. It turns out that regardless of how the fermion sources or the constant boundary conditions are chosen, at long distances the fermions propagate as if there is no background field. In other words, the adjoint fermions “see” the background electric field only at short distances.

This property gives the background field method significantly less leverage for determining c_{SW} for adjoint representation fermions. In order to maximize the effect of the different boundaries, we choose to maximize the difference between the two boundaries, and we use the following asymmetric "non-Abelian" boundary conditions: links at the upper $x_0 = T$ boundary are chosen to be trivial

$$U(x_0 = T, k) = I \tag{3.3}$$

and at the lower boundary $x_0 = 0$ we use

$$U(x_0 = 0, k) = \exp(aC_k), \;\; C_k = \frac{\pi}{2}\frac{\tau^k}{iL}. \tag{3.4}$$

These boundary conditions do not make the problem to vanish, but ameliorate it to a degree. We also note that these boundary conditions are useful only for determining c_{SW}, not for evaluating the coupling constant.

We define the fermion mass through the partial conservation of the axial current (PCAC) relation:

$$m_Q(x_0) = \frac{1}{2}\frac{\frac{1}{2}(\partial_0^* + \partial_0) f_A(x_0) + c_A a \partial_0^* \partial_0 f_P(x_0)}{f_P(x_0)} \equiv r(x_0) + c_A s(x_0), \tag{3.5}$$

where

$$A_\mu^a = \bar{\psi}(x)\gamma_5\gamma_\mu \frac{1}{2}\sigma^a \psi(x), \tag{3.6}$$

$$P^a = \bar{\psi}(x)\gamma_5 \frac{1}{2}\sigma^a \psi(x), \tag{3.7}$$

$$f_A(x_0) = -a^6 \sum_{\mathbf{y},\mathbf{z}} \langle A_0^a(x)\bar{\zeta}(\mathbf{y})\gamma_5 \frac{1}{2}\sigma^a \zeta(\mathbf{z})\rangle, \tag{3.8}$$

$$f_P(x_0) = -a^6 \sum_{\mathbf{y},\mathbf{z}} \langle P^a(x)\bar{\zeta}(\mathbf{y})\gamma_5 \frac{1}{2}\sigma^a \zeta(\mathbf{z})\rangle. \tag{3.9}$$

Here the sources $\zeta(\mathbf{z})$ live on the time slice $x_0 = 0$. The term proportional to c_A in Eq. (3.5) is irrelevant in the continuum limit, but it is needed to cancel $O(a)$-contributions to the axial current in the Wilson action.

Analogously with Eqs. (3.8,3.9), we define another set of correlation functions, $f_A'(T-x)$, $f_P'(T-x)$ and $r'(T-x)$, $s'(T-x)$, where the source is now at time slice $x_0 = T$. In order to obtain an expression which is independent of c_A we consider the combination [13]

$$M(x_0,y_0) = r(x_0) - s(x_0)\frac{r(y_0) - r'(y_0)}{s(y_0) - s'(y_0)}, \tag{3.10}$$

which coincides with m_Q up to $\mathscr{O}(a^2)$ corrections. In our calculations here we define the fermion mass with $m = M(T/2, T/4)$.

Further defining M' with obvious replacements to (3.10) gives us two correlation functions which, in the absence of cutoff effects, are equal. Thus, the quantity

$$\Delta M \equiv M(\frac{3}{4}T, \frac{1}{4}T) - M'(\frac{3}{4}T, \frac{1}{4}T) \tag{3.11}$$

vanishes up to corrections of $\mathscr{O}(a^2)$ if $c_{\rm SW}$ has its proper value.[1]

In order to achieve full $O(a)$ improvement in the Schrödinger functional schema we need to cancel also the $O(a)$ errors caused by the fixed boundaries. We treat these by using 1-loop improved boundary conditions, discussed in ref. [16]. These are not necessary, however, for the calculation here.

4. The simulations and results

In order to evaluate $c_{\rm SW}$ we used the following routine: we choose lattice volume $L^3 \times T = 8^3 \times 16$ for both fundamental and adjoint representation fermions, and a set of values of the lattice coupling β.

[1] To be more precise, at tree level ($g = 0$, $c_{\rm SW} = 1$) both ΔM and $m = M(T/2, T/4)$ have small values, which depends on the boundary conditions and the lattice size. In order to obtain the correct weak coupling limit We actually match ΔM and m to these tree-level values, not to zero (see ref. [13]).

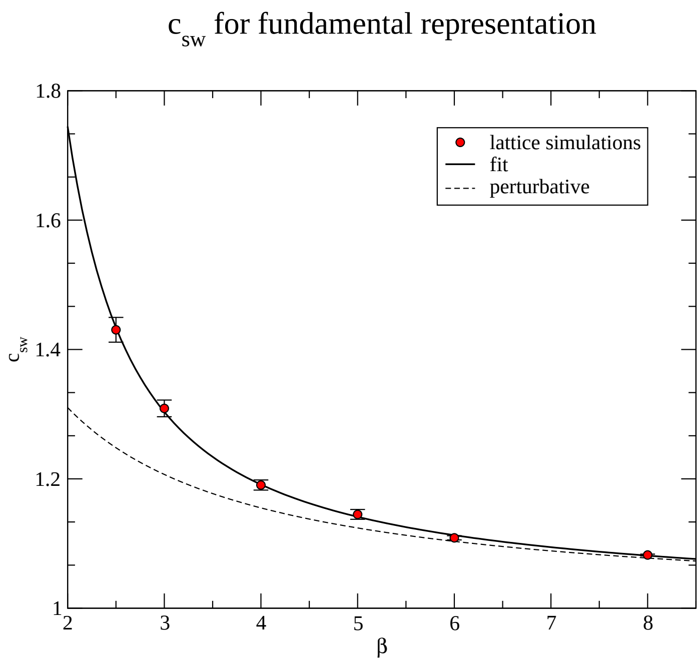

Figure 1: c_{SW} for two flavors of fundamental representation fermions. The solid line is the interpolating fit, Eq. (4.1), and the dashed line is the 1-loop perturbative value

1. For a given β, we choose initial c_{SW} (typically extrapolating from results obtained with previous values of β).
2. We choose a couple of values for $\kappa = \frac{1}{8+2am_0}$, and determine by interpolation the critical value $\kappa_c(\beta, c_{\mathrm{SW}})$ where the fermion mass $M(T/2, T/4)$ vanishes.
3. Once we have an estimate of the critical κ, we choose a new value for c_{SW} and repeat the search of κ_c.
4. At the same time, we measure $\Delta M(c_{\mathrm{SW}})$. Now we can linearly interpolate/extrapolate in c_{SW} so that ΔM vanishes, obtaining the desired value of $c_{\mathrm{SW}}(\beta)$. Using simulations at this final c_{SW} we can relocate the critical κ, if desired, and verify the results of the interpolation.

In Figs.1 and 2 we show our results for the clover coefficient c_{SW} for both fundamental and adjoint representations. The values of β used are $\beta = 2.5, 3, 4, 5, 6, 8$, and also $\beta = 10$ for the adjoint representation.

Finally, the measured values for c_{SW} can be fitted with a rational interpolating expression, which can used in simulations for this range of β-values. For fundamental representation fermions we use the perturbative 1-loop result $c_{\mathrm{SW}} = 1 - 0.1551(1)g^2 + O(g^4)$ [17] to constrain the fit:

$$c_{\mathrm{SW,fund}} = \frac{1 - 0.090254g^2 - 0.038846g^4 + 0.028054g^6}{1 - (0.1551 + 0.090254)g^2}. \tag{4.1}$$

For the adjoint representation the perturbative result is not known, and we obtain the fit result

$$c_{\mathrm{SW,adj}} = \frac{1 + 0.032653g^2 - 0.002844g^4}{1 - 0.314153g^2}. \tag{4.2}$$

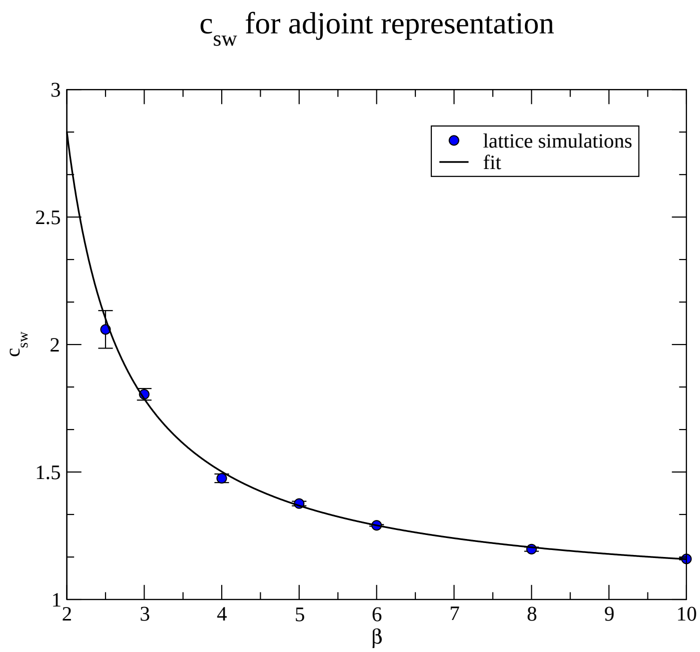

Figure 2: $c_{\rm SW}$ for two flavors of adjoint representation fermions, with the interpolating fit, Eq. (4.2).

In both cases the interpolating fits are valid for $\beta \gtrsim 2.5$. For the adjoint fermions it is difficult to reach smaller β-values because $c_{\rm SW}$ grows rapidly; and while we were able to reach $\beta = 2.3$ the errors were too large to constrain the fit (4.2) further.

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