

# Semi-Holographic Universe

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By assuming that a dark component (dark energy) in the universe strictly obeys the holographic principle, that is, its entropy is one fourth of the apparent horizon, we find that the existence of the other dark component (dark matter) is compulsory, as a compensation of dark energy, based on the first law of thermodynamics. By using the method of dynamical system analysis, we find that there exists a stable dark energy-dark matter scaling solution at late time, which is helpful to solve the coincidence problem. For reasonable parameters, the deceleration parameter is well consistent with current observations.

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## I. INTRODUCTION

The existence of dark energy is one of the most significant cosmological discoveries over the last century [1]. Various models of dark energy have been proposed, such as a small positive cosmological constant, quintessence, k-essence, phantom, holographic dark energy, etc., see [2] for a recent review. However, although fundamental for our understanding of the universe, its nature, especially in the theoretical aspect, remains a completely open question nowadays.

The holographic principle is a very important idea in high energy physics and gets more and more attentions in different branches of physics. t' Hooft proposed the first version of the holographic principle and named it [3]. In this version, the holographic principle declares that the true laws of inside any surface are actually a description of how its image evolves on that surface. He guesses that the horizon of a black plays something as a computer, where the entropy of a black hole is determined by its event horizon area. That is to say, we can count the number of microstates of a black hole on a surface. Generally speaking, people should apply the quantum gravity to the

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system where the density and curvature are large enough to the order of Planck scale, such as a black hole and the initial cosmic singularity. More or less wonderfully, in quantum field theory, the ultraviolet (UV) cut-off and infrared (IR) cut-off are suggested to be related to each other [4]. Therefore, we should not be surprised if the dark energy problem is finally proved to be a problem of quantum gravity, though its characteristic energy scale is very low. However, when we apply the holographic principle to the universe, the first problem we confront is which surface is the proper surface that the information of the volume “holographing to”? In the case of black hole, the event horizon is a proper holography of the black hole. In the case of cosmology the result is not so evident. If we just take the event horizon as the holography for mimicking the case of black hole, we may be embarrassed in a decelerating universe, since it has no event horizon at all. So, we should find some more fundamental analogies between black hole physics and cosmology to find the proper surface.

In the context of black hole physics we believe that the event horizon of a black hole is the proper surface since every physical quantity of the black hole, especially the entropy, shows itself properly on that surface. And they strictly obey the first law of the thermodynamics. An interesting progress is that Einstein equation can be reproduced from the proportionality of entropy and horizon area together with the first law of thermal dynamics,  $\delta Q = TdS$ , jointing to heat, entropy, and temperature, where the temperature is the Unruh temperature to an observer just behind a causal Rindler horizon [5]. This work pioneers the way how to find the proper holography besides the case of black hole. In the case of dynamical solution, a similar procedure reproduces the Friedmann equation. One needs to apply only the first law of thermodynamics to the trapped surface (apparent horizon) of an FRW universe and assume the geometric entropy given by a quarter of the apparent horizon area and the temperature given by the inverse of the apparent horizon [6]. There are several arguments that the apparent horizon should be a causal horizon and is associated with the gravitational entropy and Hawking temperature [7]. Hence it seems that the apparent horizon is the right holography of the universe [11].

Up to now, our arguments are at the level of sophistication. Now we turn our sight to the realistic universe. There are several different components including baryon matter, non-baryon dark matter, and maybe dark energy etc, in our universe. We know the properties of baryon matter well. But, sadly, its entropy obeys the “volume law” rather than the “area law”. And its entropy is far from saturating the holographic bound. Also the total entropy of all the known matters is still much lower than the holographic bound [8]. So it is interesting to see what will happen if a dark component of the universe strictly obeys the holographic principle?

This paper is organized as follows: In the next section we will study a model in which a component satisfies the holographic principle. The dynamical analysis is left in section III. We find that there exists a stable dark matter-dark energy solution at late time, which is helpful to solve the coincidence problem. We present our conclusion and some discussions in section IV.

## II. THE MODEL

There are decisive evidences that our observable universe evolves adiabatically after inflation in a comoving volume, that is, there is no energy-momentum flow between different patches of the observable universe so that the universe keeps homogeneous and isotropic after inflation. That is the reason why we can use an FRW geometry to describe the evolution of the universe. In an adiabatically evolving universe, the first law of thermodynamics equals the continuity equation. In a comoving volume the first law reads,

$$dU = TdS - pdV, \quad (1)$$

where  $U = \Omega_k \rho a^3$  is the energy in this volume,  $T$  denotes temperature,  $S$  represents the entropy of this volume, and  $V$  stands for the physical volume  $V = \Omega_k a^3$ . Here,  $\Omega_k$  is a factor related to the spatial curvature, for spatially flat case  $\Omega_0 = \frac{4}{3}\pi$ , in this paper we only consider the spatially flat model,  $\rho$  is the energy density and  $a$  denotes the scale factor. For examples, in the case of radiation  $p = 1/3\rho$ , then we derive  $\rho \sim a^{-4}$ ; in the case of dust  $p = 0$ , then we obtain  $\rho \sim a^{-3}$ ; and in the case of vacuum  $p = -\rho$ , then  $\rho = \text{constant}$ .

Since we know little about the properties of dark energy, especially in the theoretical side, it is reasonable to study the possibility of a non-adiabatical dark energy. Next we consider the non-adiabatical evolutions. When the term  $TdS$  does not equal zero, the results are completely different. For instance, it is neither sufficient nor necessary that  $\rho = \text{constant}$  implies  $p = -\rho$ . We will show it is just the case if a dark component strictly obeys the holographic principle. To our knowledge, this point scarcely gets any attention in the literatures.

Based on the investigations in [6, 7], the entropy in the apparent horizon is

$$S = \frac{8\pi^2 \mu^2}{H^2}, \quad (2)$$

where  $H$  is the Hubble parameter,  $\mu$  denotes the reduced Planck mass. This equation implies that the entropy is exactly one-fourth of the area of the apparent horizon. So, in a comoving volume the entropy becomes,

$$S_c = \frac{8\pi^2 \mu^2}{H^2} \frac{a^3}{H^{-3}} = 8\pi^2 \mu^2 H a^3. \quad (3)$$

In the above equation we have used an assumption that the entropy is homogeneous in the observable universe. This is not a tough assumption for we have no good reason that the entropy density in one region is larger than other regions. Evidently, the entropy in a comoving volume is not constant. However, as we discussed before, our observable universe evolves adiabatically after inflation in any comoving volumes. Thus, if a dark component, which is called dark energy, satisfies the holographic principle, it requires the other compensative dark component, which is assumed to be dark matter, such that the total entropy in a comoving volume keeps constant. In this sense, our universe only partly obeys the holographic principle, which indicates the title: Semi-Holographic Universe.

With the above supposition and conventions the entropy of the dark energy satisfies (3),

$$S_{de} = 8\pi^2 \mu^2 H a^3. \quad (4)$$

Correspondingly, the entropy of dark matter in this comoving volume should be

$$S_{dm} = C - S_{de}, \quad (5)$$

where  $C$  is a constant, representing the total entropy of the comoving volume. The Hubble parameter  $H$  is determined by the Friedmann equation,

$$H^2 = \frac{1}{3\mu^2}(\rho_{dm} + \rho_{de} + \Lambda), \quad (6)$$

where  $\rho_{dm}$  denotes the density of non-baryon dark matter,  $\rho_{de}$  denotes the density of dark energy, and  $\Lambda$  is the cosmological constant (or vacuum energy). In this preliminary research, we omit the baryon matter since its partition is very small and does little work in the late time universe. However we introduce the cosmological constant because we want not only to consider a more general case, but also to show that the holographic dark energy and dark matter require each other even if there is a cosmological constant. And furthermore, we will see that the cosmological constant plays an important role in the final state of the universe.

### III. DYNAMICAL ANALYSIS

To investigate the evolution in a more detailed way, we take a dynamical analysis of the universe. The holographic principle requires that the temperature [6]

$$T = \frac{H}{2\pi}. \quad (7)$$

By using (7), (6), and (4), the first law of thermal dynamics (1) becomes the evolution equation of dark energy,

$$\frac{2}{3}\rho'_{de} = \rho_{dm}(1 - w_{dm}) - \rho_{de}(1 + 3w_{de}) + 2\Lambda, \quad (8)$$

where a prime denotes the derivative with respect to  $\ln a$ ,  $w_{dm}$  indicates the equation of state (EOS) of dark matter, and  $w_{de}$  represents the EOS of dark energy. Similarly, we derive the evolution equation of dark matter,

$$\frac{2}{3}\rho'_{dm} = -\rho_{dm}(3 + w_{dm}) + \rho_{de}(-1 + w_{de}) - 2\Lambda. \quad (9)$$

For convenience we introduce two new dimensionless functions to represent the densities,

$$u \triangleq \frac{\rho_{dm}}{3\mu^2 H_0^2}, \quad (10)$$

$$v \triangleq \frac{\rho_{de}}{3\mu^2 H_0^2}, \quad (11)$$

and a dimensionless cosmological constant

$$\lambda \triangleq \frac{\Lambda}{3\mu^2 H_0^2}, \quad (12)$$

where  $H_0$  denotes the present Hubble parameter. Then the equation set (9), (8) becomes

$$\frac{2}{3}u' = -u(3 + w_{dm}) + v(-1 + w_{de}) - 2\lambda, \quad (13)$$

$$\frac{2}{3}v' = u(1 - w_{dm}) - v(1 + 3w_{de}) + 2\lambda, \quad (14)$$

respectively. We note that the time variable does not appear in the dynamical system (13) and (14) because time has been completely replaced by scale factor.

Before presenting the numerical examples for special parameters we study the analytical property of this system. The critical points of dynamical system (13) and (14) are given by

$$u'_c = v'_c = 0, \quad (15)$$

which yields,

$$u_c = -\lambda \frac{w_{de} + 1}{2w_{de} + w_{dm}w_{de} + 1}, \quad (16)$$

$$v_c = \lambda \frac{w_{\text{dm}} + 1}{2w_{\text{de}} + w_{\text{dm}}w_{\text{de}} + 1}. \quad (17)$$

So, finally the universe enters a de Sitter phase, and the ratio of dark matter over dark energy is

$$\frac{u_c}{v_c} = -\frac{1 + w_{\text{de}}}{1 + w_{\text{dm}}}. \quad (18)$$

We see that the final ratio is independent of the cosmological constant. There are two reasonable cases: case I,  $w_{\text{de}} < -1$  and  $w_{\text{dm}} > -1$ ; case II,  $w_{\text{de}} > -1$  and  $w_{\text{dm}} < -1$ , since we should require both of the final densities of dark matter and dark energy are positive. Physically, it is easy to understand. Since one of the dark components flows out entropy (surely with energy) to the other dark component, at the same time it keeps a constant density, its apparent EOS should be less than  $-1$ , like phantom, for similar mechanism, see [9].

Now we consider the degenerated case in which  $\lambda = 0$ . Under this condition the equation set (13) and (14) become homogeneous. A non-trivial solution implies its determinate of coefficients equals zero,

$$1 + 2w_{\text{de}} + w_{\text{dm}}w_{\text{de}} = 0. \quad (19)$$

Under this condition the ratio of dark matter over dark energy reaches the same as in the case with a  $\lambda$ ,

$$\frac{u_c}{v_c} = -\frac{1 + w_{\text{de}}}{1 + w_{\text{dm}}}. \quad (20)$$

But, this ratio of dark matter and dark energy will keep the same value in the whole history of the universe, that is, dark matter and dark energy always evolve in the same way: It is not a very interesting case.

If the present dark energy dominated universe can be an attractor of the dynamical evolution, it is helpful to overcome the coincidence problem. To confirm the critical point of the system is an attractor, we need the stability property of it. Imposing a perturbation to the critical points, we derive,

$$\frac{2}{3}(\delta\rho_{\text{dm}})' = -\delta\rho_{\text{dm}}(3 + w_{\text{dm}}) + \delta\rho_{\text{de}}(-1 + w_{\text{de}}), \quad (21)$$

$$\frac{2}{3}(\delta\rho_{\text{de}})' = \delta\rho_{\text{dm}}(1 - w_{\text{dm}}) - \delta\rho_{\text{de}}(1 + 3w_{\text{de}}). \quad (22)$$

Note that we have assumed both  $w_{\text{dm}}$  and  $w_{\text{de}}$  are constant from the beginning. The eigenvalues of this system read,

$$l_1 = \frac{1}{2} \left( -w_{\text{dm}} + w_{\text{de}} + \sqrt{16 + w_{\text{dm}}^2 + 32w_{\text{de}} + 14w_{\text{dm}}w_{\text{de}} + w_{\text{de}}^2} \right), \quad (23)$$

$$l_2 = \frac{1}{2} \left( -w_{\text{dm}} + w_{\text{de}} - \sqrt{16 + w_{\text{dm}}^2 + 32w_{\text{de}} + 14w_{\text{dm}}w_{\text{de}} + w_{\text{de}}^2} \right), \quad (24)$$

Stability implies all of real parts of the eigenvalues are less than zero, which requires,

$$w_{\text{de}} < \text{Min} \left\{ -1, -\frac{1}{2 + w_{\text{dm}}} \right\}, \quad (25)$$

when  $w_{\text{dm}} > -2$ . The system will be unstable for any  $w_{\text{de}}$  when  $w_{\text{dm}} \leq -2$ . Therefore, case I is stable while case II is unstable.

The most significant parameter from the viewpoint of observations is the deceleration parameter  $q$ , which carries the total effects of cosmic fluids. Using (6), (8), and (9) we obtain the deceleration parameter in this model

$$q = -\frac{\ddot{a}}{a} \frac{1}{H^2} = \frac{1}{2} \frac{\rho_{\text{dm}}(1 + 3w_{\text{dm}}) + \rho_{\text{de}}(1 + 3w_{\text{de}}) - 2\lambda}{\rho_{\text{dm}} + \rho_{\text{de}} + \lambda}. \quad (26)$$

For a numerical example, we take the terminal ratio of dark energy to dark matter 1 : 1, the present dark matter partition  $u_0 = 0.25$ , the present holographic dark energy partition  $v_0 = 0.01$ , correspondingly  $\lambda = 0.74$ .

Figure 1 illuminates the evolution of deceleration parameter. As a simple example we just set  $w_{\text{dm}} = -0.4$ ,  $w_{\text{de}} = -1.4$ . From the figure one see that current  $q \sim -0.75$  and at the high redshift region it goes to 0.5, which is well consistent with current observations [2, 10]. As a comparison, we plot the evolution of the deceleration parameter in a spatially flat  $\Lambda$ CDM, in which we set  $\Omega_{\text{dm}} = 0.25$ . One sees that the deceleration parameter more swiftly approaches 0.5 (standard dark matter model, SCDM for short) in this semi-holographic model than that of  $\Lambda$ CDM.

One may be confused why we can endow a negative EOS to the dark matter. In fact, its evolution do not depend on this apparent EOS, but the effective EOS. We define the effective EOS as the following procedure. Supposing the dark matter evolves adiabatically itself, we obtain its evolution from (1),

$$d\rho_{\text{dm}} + 3(\rho_{\text{dm}} + p_{\text{eff}}) \frac{da}{a} = 0, \quad (27)$$

where  $p_{\text{eff}}$  denotes the effective pressure of dark matter. Then we obtain

$$w_{\text{dme}} \triangleq \frac{p_{\text{eff}}}{\rho_{\text{dm}}} = \frac{1}{2} + \frac{1}{2}w_{\text{dm}} + \frac{v}{2u}(1 - w_{\text{de}}) + \frac{\lambda}{u}, \quad (28)$$

which is a variable in the evolution history of the universe. Figure 2 displays  $w_{\text{dme}}$  as a function of  $\ln a$ , in which we set the same parameters as in figure 1. This figure shows that the dark matter is effectively very stiff in the current time, but quickly gets softer and becomes ordinary dust in

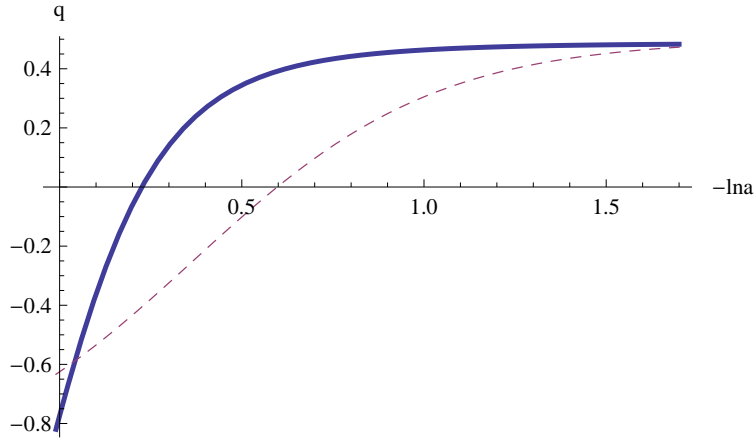


FIG. 1: The evolutions of  $q$  in semi-holographic universe (solid curve) and in  $\Lambda$ CDM (dashed curve), respectively.

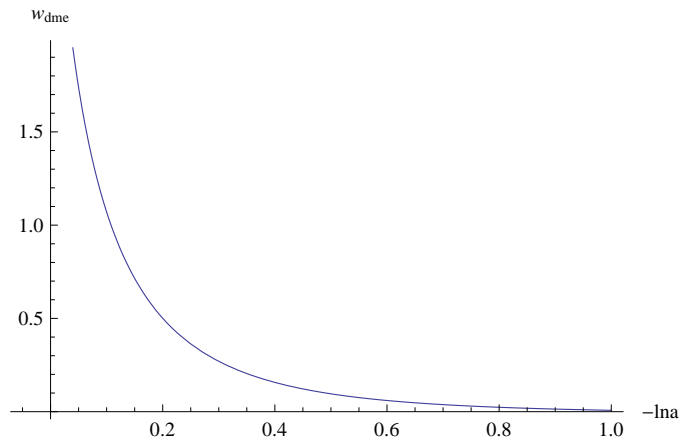


FIG. 2: The effective EOS of dark matter  $w_{dme}$  as a function of  $-\ln a$ .

a higher redshift region. The corresponding effective EOS of dark energy  $w_{dee}$  is illuminated in Figure 3. From this figure we know that the dark energy currently evolves as phantom and becomes a cosmological constant in some high redshift region.

Associating figure 1 with figure 2 and 3, we conclude that the universe is dominated by dark matter, whose effective EOS  $w_{eff} = 0$ , in some high redshift region, such as  $z > 1.5$  ( $-\ln a > 0.92$ ) and hence essentially SCDM recovers.

The other point deserves to note is that we only introduce a little bit of holographic dark energy (in the above example 1%), the final state changes heavily. It evolves into a dark energy-dark matter scaling solution, which shed light on the coincidence problem. The dark matter will be diluted rapidly if we do not introduce it, which yields coincidence.



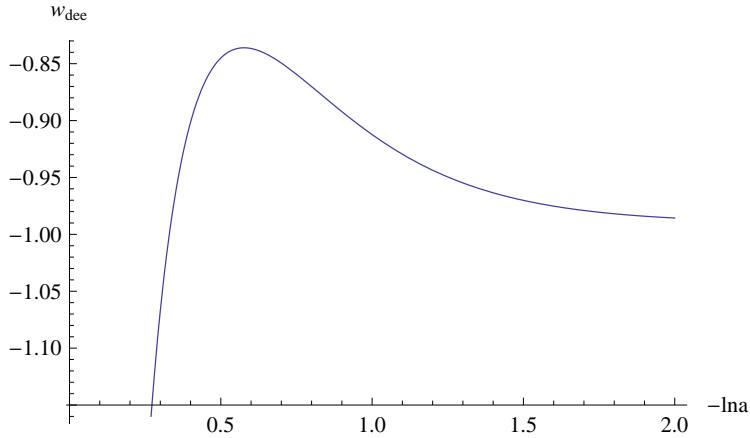


FIG. 3: The effective EOS of dark energy  $w_{dee}$  as a function of  $-\ln a$ .

#### IV. CONCLUSION AND DISCUSSION

We present a cosmological model inspired by holographic principle, especially the previous studies of the relation between thermal dynamics and general relativity, in which the entropy of the dark energy is one-fourth of the area of the apparent horizon. We find that under this condition the dark energy must evolve non-adiabatically. But the total matter in a comoving volume should evolve adiabatically. Hence a compensating component should exist, which we called dark matter.

We find a future attractor solution, which is a stable scaling solution for the dark matter-dark energy system in some proper region of the parameters  $w_{dm}$ ,  $w_{de}$ . The final ratio of dark matter to dark energy only depends  $w_{dm}$ ,  $w_{de}$ , which is independent on the initial values of the densities of dark matter and dark energy. This result is helpful to solve the coincidence problem.

In a numerical example, we find that the deceleration parameter can be well consistent with observations.

In this paper only a spatially flat universe is discussed. The model with a non-vanishing spatial curvature need to be investigated further. Also, as a phenomenological model, the parameters should be constrained by observation data further.

The derivation of Fridmann equation from the assumption that the entropy is a quarter of its apparent horizon is strict and does not depend on the energy scale. Hence, we expect that the holographic component may have remarkable effects in the early universe.

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- [1] A. G. Riess et al., *Astron. J.* 116, 1009 (1998), astro-ph/9805201; S. Perlmutter et al., *Astrophys. J.* 517, 565 (1999), astro-ph/9812133.
- [2] E. J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006) [arXiv:hep-th/0603057].
- [3] G. 't Hooft, arXiv:gr-qc/9310026; L. Susskind, *J. Math. Phys.* **36**, 6377 (1995) [arXiv:hep-th/9409089].
- [4] A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Phys. Rev. Lett.* **82**, 4971 (1999) [arXiv:hep-th/9803132].
- [5] T. Jacobson, *Phys. Rev. Lett.* 75, 1260 (1995), [gr-qc/9504004].
- [6] R. G. Cai and S. P. Kim, *JHEP* **0502**, 050 (2005) [,hep-th/0501055].
- [7] D. Bak and S. J. Rey, *Class. Quant. Grav.* 17, L83 (2000) [arXiv:hep-th/9902173]; S. A. Hayward, S. Mukohyama and M. C. Ashworth, *Phys. Lett. A* 256, 347 (1999) [arXiv:gr-qc/9810006]; S. A. Hayward, *Class. Quant. Grav.* 15, 3147 (1998) [arXiv:gr-qc/9710089].
- [8] P. Frampton, S. D. H. Hsu, D. Reeb and T. W. Kephart, arXiv:0801.1847 [hep-th].
- [9] H. S. Zhang and Z. H. Zhu, *Phys. Rev. D* **73**, 043518 (2006); Z.K. Guo and Y.Z. Zhang, *Phys.Rev.* **D71** (2005) 023501.
- [10] E. Komatsu *et al.* [WMAP Collaboration], arXiv:0803.0547 [astro-ph].
- [11] Note that in the case of a static black hole, the apparent horizon and the event horizon coincide each other. Thus we also can say the right holography of a black hole is the apparent horizon in that case.