Inflation with improved D3-brane potential and the fine tunings associated with the model

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We investigate brane-antibrane inflation in a warped deformed conifold background that includes contributions to the potential arising from imaginary anti-self-dual (IASD) fluxes including the term with irrational scaling dimension discovered recently. We find that the model can give rise to required number of e-foldings; observational constraint on COBE normalization is easily satisfied and low value of the tensor to scalar ratio of perturbations is achieved. We observe that these corrections to the effective potential help in relaxing the severe fine tunings associated with the earlier analysis.

PACS numbers:

I. INTRODUCTION

Cosmological inflation [1] is a mechanism, for the universe to undergo a brief period of accelerated expansion. This is postulated to cure some of the intrinsic problems, like the horizon and flatness problems, associated with the standard Big Bang model. This inflationary scenario not only explains the large-scale homogeneity of our universe but also provides a way, via quantum fluctuation, to generate the primordial inhomogeneities which is the seed for understanding the structure formation in the universe. Such inhomogeities have been observed as anisotropies in the temperature of the cosmoc microwave background. Thus the theoretical framework of inflation i.e. a scalar field, called as inflaton, slowly rolling over the slope of its potential are subject to observational constraints [2, 3]. However, the initial conditions for inflation and the form of the potential function are expected to come from a fundamental theory of gravity and not to be chosen arbitrarily. In this context, enormous amount of efforts are underway to derive inflationary models from string theory, a consistent quantum field theory around the Planck's scale and considered to be ultraviolet-complete theory of gravity. Discovery of D-branes, gauge/gravity duality and various nonperturbative aspects in string theory have also played crucial role in building and testing inflationary models of cosmology.

In past few years, many inflationary models have been constructed from the four dimensional string theory making use of D-branes. The examples are inflation due to tachyon condensation on a non-BPS brane, inflation due to the motion of a D3-brane towards an anti-D3-brane [4–7], inflation due to geometric tachyon arising from the motion of a probe brane in the background of a stack of either NS5-branes or the dual D5-branes [8]. However, these models do not take into account the details of compactification and the effects of moduli stabilization. Such issues could be addressed only when it was learnt [9] that the background fluxes sourced by D3 and D5-branes can stabilize the axio-dilaton and the complex structure moduli of type IIB string theory compactified on an orientifold of a Calabi-Yau threefold. Moreover, the back reaction of these D-branes yields the geometry of a throat [10] which could be glued smoothly to the compact Calabi -Yau manidold. Further important progress was achieved when it was shown in Ref. [11] that the Kähler moduli fields also can be stabilized by a combination of fluxes and nonperturbative effects via gauge dynamics of either an Euclidean D3-brane or from a stack of D7-branes, wrapping super-symmetrically a four cycle of the compact manifold, placed around the base of the throat.

The above results enabled to construct an inflationary model [12] which took into account of the compactification data (see also Refs. [13]) since the inflaton potential is obtained by performing string theoretic computations involving the details of fluxes and warping. In this scenario inflation is realized by the motion of a D3-brane, placed in the compact manifold, towards a distant static anti-D3-brane sitting at the tip of the throat. The radial separation between the two is considered to be the inflaton field. The effect of the moduli stabilization resulted in a large mass to the inflaton field which turned out to be of the order of Hubble parameter and hence spoils the inflation. As a possibility for circumventing this problem, it was proposed [14–16] to embed the D7-branes such that at least one of the four-cycles carrying the nonperturbative effects descend down a finite distance into the warped throat which implied that the probe D3-brane is constrained to move only inside the throat. This consideration lead to the inflaton potential having an inflection point. The inflation dynamics however, within a single throat model [17], revealed that when the spectral index of scalar perturbation reaches the scale invariant value, the amplitude tends to be larger than

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the COBE normalized value by about three order of magnitude, making the model unrealistic. Such problems could be solved if another non-inflating throat is added to the compactification procedure [18].

II. POTENTIAL FROM COMPACTIFICATION EFFECTS AND ITS IMPLICATION FOR INFLATION

The possibility of a realistic model for brane inflation, within the large volume compactification and a single throat scenario, received further attention when the authors of Ref. [19] observed that there are corrections to the inflaton potential which arise from the compactification effects in the ultra violet (UV). Thus the assumption of the D7-brane descending into the throat was relaxed. Instad, using AdS/CFT correspondence, the throat geometry (conifold) has been treated as an approximate conformal field theory with a high cut off scale M_{UV} . In this context, the position of the probe D3-brane is identified with the Coulomb branch vev of a field in the gauge theory which couples to bulk moduli fields. This coupling changes the Kähler potential and hence the inflaton potential. It was noted that the leading contribution to the inflaton potential comes from the coupling of a chiral operator with dimension $\Delta = 3/2$ in the dual gauge theory to the bulk field. Taking this contribution in to account, the full inflaton potential takes the form:

$$V(\phi) = D \left[1 + \frac{1}{3} \left(\frac{\phi}{M_{pl}} \right)^2 - C_{3/2} \left(\frac{\phi}{M_{UV}} \right)^{3/2} - \frac{3D}{16\pi^2 \phi^4} \right]$$
 (1)

where ϕ is the canonically normalized inflaton field related to the position of the D3-brane; $C_{3/2}$ is a positive constant and $D \sim 2a_0^4T_3$. Here a_0 is the minimal warp factor at the tip of the throat and T_3 is the tension of D3-brane. Note that the third term in the above equation is the contribution coming from the compactification effect in the UV and the rest of the potential is the same as in [12].

The inflationary dynamics, using the above potential, was investigated in [20] and the reheating issue was discussed in [21]. It was observed that the parameter $C_{3/2}$ in the above potential has to be severely fine tuned for the inflation model to be consistent with the WMAP five years data. This fine tuning becomes worse for the consistency with recent WMAP seven years data.

On the other hand, recently the authors of Refs. [22] have performed a detailed analysis of the potential on the Coulomb branch of the conifold gauge theory and have found many more corrections to the inflaton potential. The general structure of the corrections arising from UV deformations of the background take the form (See [22] for details):

$$V_c(\phi) = \sum_i C_i \frac{\phi^{\Delta_i}}{M_{UV}^{\Delta_i - 4}} \tag{2}$$

where M_{UV} is a UV mass scale related to the ultraviolet location at which the throat is glued into the compact bulk. While, the constant coefficients C_i are left undetermined, the scaling dimensions Δ_i are found to be $1, 3/2, 2, 5/2, 2.79, \cdots$. These contributions arise from various sources. For example, terms with scaling dimensions Δ_i with $i = 3/2, 2, \cdots$ come from homogeneous solution (an arbitrary harmonic function on the conifold) and with $i = 1, 2, 5/2, 2.79, \cdots$ come from inhomogeneous solutions sourced by fluxes. The term with scaling dimension 2.79 corresponds to a flux perturbation dual to a non-chiral operator which is generically present but is not captured via perturbations of the superpotential.

In this note, we reinvestigate the inflation dynamics by including these new corrections. We find that these corrections not only help in constructing a viable inflation model consistent with WMAP seven years data but also the fine tuning problem, mentioned above, is considerably relaxed. For this purpose, we write the full inflaton potential, including V_c as given above, as follows:

$$\mathcal{V} = \mathcal{D} \left[1 - C_1 x - C_{3/2} x^{3/2} + \left(\frac{\alpha^2}{3} - C_2 \right) x^2 - C_{5/2} x^{5/2} - C_{2.79} x^{2.79} - \frac{3\mathcal{D}}{16\pi^2 \alpha^4 x^4} \right]$$
(3)

where $x = \phi/M_{UV}$, $\mathcal{V} = V/M_{UV}^4$, $\mathcal{D} = D/M_{UV}^4$ and $\alpha = M_{UV}/M_{pl}$. In what follows, we shall consider the inflationary dynamics of the field with the modified potential given by (3).

III. SLOW ROLL INFLATION

In this section, we shall study the dynamics of inflation based upon the improved D brane potential and demonstrate that the model under consideration based upon (3) performs much better than the earlier models of D-brane inflation.

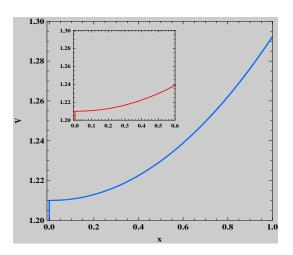


FIG. 1: Plot of the effective potentials given by (1) and (3), the insert displays the effective potential (3) for $C_1 = 10^{-7}$, $C_{3/2} = 0.006232$, $\alpha^{-1} = 2.11869$, $C_2 = 10^{-6}$, $C_{5/2} = C_{2.79} = 2 \times 10^{-5}$, $\mathcal{D} = 1.21 \times 10^{-17}$.

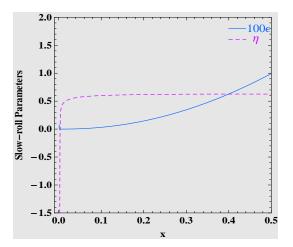


FIG. 2: Plot of the slow roll parameters $\epsilon \& \eta$ for the effective potential (3) for the same set of parameters as in Fig 1

The modified potential allows to easily generate enough inflation without involving the fine tuning of initial conditions for slowly rolling inflaton. The corrected potential also allows to decrease the fine tuning of model parameters present in earlier models.

For the sake of convenience, let us cast the evolution equation for the field , $\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$ and the Friedmann equation $H^2 = \left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)/3M_{pl}^2$ in the autonomous form

$$\frac{dx}{dN} = \frac{y}{\mathcal{H}} \tag{4}$$

$$\frac{dy}{dN} = -3y - \frac{1}{\mathcal{H}} \frac{d\mathcal{V}}{dx} \tag{5}$$

$$\mathcal{H}^2 = \frac{\alpha^2}{3} \left(\frac{1}{2} y^2 + \mathcal{V}(x) \right) \tag{6}$$

where $y = \dot{x}/M_{UV}^2$, $\mathcal{H} = H/M_{UV}$, and N designates the number of e-foldings. In the scenario under consideration, the mobile D3-brane moves towards the anti-D3 brane located at the tip of the throat corresponding to x = 0 and thus we have 0 < x < 1 since $\phi < \phi_{UV} \sim M_{UV}$.

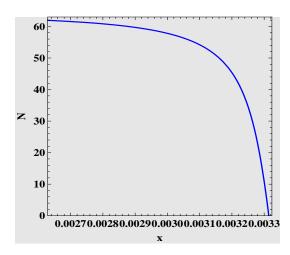


FIG. 3: Plot of the number of e-folds for the effective potential (3) versus x. The numerical values of model parameters is same as in Fig 1

The slow roll parameters for the generic field range are

$$\epsilon = \frac{1}{2\alpha^2} \left(\frac{\mathcal{V}_{,x}}{\mathcal{V}(x)} \right)^2 \simeq \frac{1}{2\alpha^2} \left[2(\frac{\alpha^2}{3} - C_2)x - C_1 - \frac{3C_{3/2}}{2}x^{1/2} - \frac{5}{2}C_{5/2}x^{3/2} - 2.79C_{2.79}x^{1.79} + \frac{3\mathcal{D}}{4\pi^2\alpha^4x^5} \right]^2 \tag{7}$$

$$\eta = \frac{1}{\alpha^2} \frac{\mathcal{V}_{,xx}}{\mathcal{V}(x)} \simeq \frac{1}{\alpha^2} \left[2(\frac{\alpha^2}{3} - C_2) - \frac{3C_{3/2}}{4x^{1/2}} - \frac{15}{4}C_{5/2}x^{1/2} - 4.99C_{2.79}x^{0.79} - \frac{15\mathcal{D}}{4\pi^2\alpha^4x^6} \right]$$
(8)

In Fig.1, we plot the effective potential for a generic choice of parameters. And we note that $|\epsilon| < |\eta|$ in the case under consideration in the field range of interest, see Fig.2. Thus it is sufficient to consider η for discussing the slow roll conditions. The field potential should be monotonously increasing function of x to ensure realistic motion of D brane, i.e., it should be allowed to move towards the origin where the \bar{D} is located. As was pointed out in Ref[20], the field potential (3) may be monotonously increasing (decreasing) or even acquiring a minimum for 0 < x < 1 depending upon the numerical values of the model parameters,. The latter imposes constraint on the parameters of the model if it is to be viable for inflation.

Lets us emphasize that the model exhibits sensitivity with respect to its parameters $C_{3/2}$ and \mathcal{D} . Since the last term in Eq.(7) is always positive, the monotonicity of $\mathcal{V}(x)$ can be ensured provided we require smaller and smaller values of $C_{3/2}$. This is because we need to avoid the occurrence of a minimum of the potential as we move towards the origin before the last term in Eq.(7) could become dominant. For generic numerical values of $\mathcal{D}^{1/4} \sim 10^{-4}$ required for observational constraints to be satisfied, we find numerical values of $C_{3/2}$ much smaller than one. In case of the effective potential (3), it is possible to make the potential flat near the origin, see Fig.1. In this case, the field range viable for inflation is narrow and the potential should be made sufficiently flat to derive required number of e-foldings. In particular, it means that the sow roll parameter ϵ is very small leading to low value of tensor to scalar ratio of perturbations. This feature, however, becomes problematic for scalar perturbations. Indeed, since $\delta_H^2 \propto \mathcal{V}/\epsilon$ and $\mathcal{V} \sim \mathcal{D}$, smaller values of ϵ lead to larger values of density perturbations. One of the difficulties associated with the model described by (1) as well as by (3) is the unusual feature of \mathcal{D} appearing not only as an over all scale in the expression of the effective potential but also its appearance in the expressions for slow roll parameters. This fact brings in difficulties for setting the COBE normalization right. The aforementioned features of the model makes the search difficult for viable parameters and gives rise to fine tuning of model parameters specially in case of the effective potential (1). Worse is the fine tuning associated with the initial conditions for the inflaton. Since there is very narrow interval for the field viable for inflation and we can not make the potential arbitrarily flat in view of the COBE normalization, the model based on potential (1) becomes extremely fine tuned. For generic values of parameters, the initial value of the field x_i should be tuned typically to the level of one part in 10^{-7} for obtaining e-folds around sixty and to satisfy the other observational constraints. Any deviation of numerical values of x_i ($x_i = 0.0033050$)[20] beyond the said accuracy makes the inflaton hit the singularity before it could make the required number of e-folds. As a result, the inflationary scenario based upon the effective potential (1) becomes heavily constrained and that too with severe fine tuning of the parameters.

On the contrary, for the case of the improved effective potential given by (3), the fine tuning problem is alleviated. We might think that the latter is achieved at the cost of four new terms in the potential with corresponding free

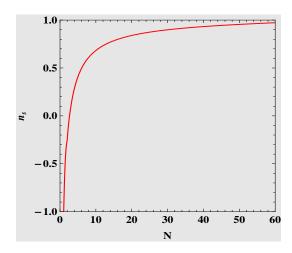


FIG. 4: Plot of the spectral index n_S versus the field number of e-folds starting from the end of inflation for the effective potential (3). For the set of parameters as in Fig 1, n_S reaches the observed value for $N \simeq 60$.

C_1	10^{-7}	1.9×10^{-7}	10^{-7}	10^{-7}	10^{-7}	10^{-7}	10^{-7}	10^{-7}
$C_{3/2}$	0.006232	0.006232	0.006233	0.006232	0.006232	0.006232	0.006232	0.006232
α^{-1}	2.11869	2.11869	2.11869	2.11890	2.11869	2.11869	2.11869	2.11869
C_2	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-10}	10^{-6}	10^{-6}	10^{-6}
$C_{5/2}$	2×10^{-5}	2×10^{-5}	2×10^{-5}	2×10^{-5}	2×10^{-5}	10^{-4}	2×10^{-5}	2×10^{-5}
$C_{2.79}$	2×10^{-5}	2×10^{-5}	2×10^{-5}	2×10^{-5}	2×10^{-5}	2×10^{-5}	10^{-10}	2×10^{-5}
\mathcal{D}	1.21×10^{-17}	1.21×10^{-17}	1.21×10^{-17}	1.21×10^{-17}	1.21×10^{-17}	1.210×10^{-17}	1.21×10^{-17}	1.20×10^{-17}

TABLE I: A possible variation of model parameters consistent with observational bounds: $\delta_H^2 = (2.349 - 2.529) \times 10^{-9} \& n_s = 0.949 - 0.977 (WMAP + BAO + H_0)$ (Tensor to scalar ratio of perturbations is low in the model and does not impose constraint on the model parameters).

parameters. This is indeed true. However, it should be kept in mind that these new correction terms are not inspired by phenomenological considerations but are rigorously derived using the formalism of flux compactification and Gauge/Gravity correspondence. Our investigation shows that the numerical values of the new constants are robust and do not suffer from fine tuning. The effective potential which incorporated additional corrections substantially improves the behavior of the potential for relatively large values of the field. The insert in Fig(1) displays the effective potential (3) which shows that the region of flatness substantially increases in this case, specially around the quoted value of x_i . The latter completely relaxes the fine tuning of initial conditions for slow roll inflation allowing to easily draw the required number of e-flods 60 or more, see Fig.3. In table I, we display the results of numerical simulation based upon the effective potential (3). In the table, we have presented the variation of model parameters consistent with the observationally allowed range of values of density perurbations δ_H^2 , n_S and tensor to scalar ratio of perturbations. We find that $C_{3/2}$ varies in the interval, $C_{3/2}=0.006232-0.006233$, keeping the other parameters fixed with generic numerical values given in the table. Actually, the constraint on $C_{3/2}$ is imposed from the requirement of monotonocity of the potential. Since the modification of the potential becomes more and more significant for larger values of x away from the origin, the fine tuning related to $C_{3/2}$ is not substantial: it improves by one order of magnitude only as compared to the earlier analysis based upon (1). In view of the above discussion, one might expect enormous improvement with regard to COBE normalization a la \mathcal{D} . Actually, the improvement in this case is not beyond one order of magnitude which is related to the fact that \mathcal{D} appears in the effective potential in a non-trivial way. As for α , the new corrections do not involve this parameter and it remains as fined as in the earlier models. In this case, the hunt for viable range of parameters becomes easier. In Fig.4, we have shown the spectral index versus the number of e-folds for a typical choice of parameters in the model. Though the spectral index is shown to reaches the observed value at N = 60, the larger number of e-folds can also be generated in the scenario without any difficulty.

IV. CONCLUSIONS

In this paper, we have investigated an inflationary model based upon an effective D-brane potential (3) that includes corrections to the potential arising from imaginary anti-self-dual fluxes encoded by terms containing coefficients $C_1, C_2, C_{5/2}$, and $C_{2.79}$. In absence of these terms, the monotonocity of the potential imposes tough constraints on $C_{3/2}$; it is fine tuned to the level of one part in 10^{-7} for observational constraints to be satisfied. Secondly, it is possible to make the potential flat near the origin in a very narrow field range which conflicts in general with COBE normalization. The constant \mathcal{D} requires heavy fine tuning to satisfy the COBE constraint and the observational data on the spectral index n_S . What is worse, the initial condition for the inflaton requires fine tuning at the level of one part in 10^{-6} otherwise the inflaton rolls to the region of instability before it completes 60 e-folds. The latter is related to very narrow field range of flatness of the potential (1). The corrections due to imaginary anti-self-dual fluxes included in the potential (3) allow to increase the range of flatness of the potential around x=0 thereby completely relaxing the fine tuning of initial conditions of the inflationary dynamics. The modified D-brane potential can easily give rise to a large number of e-folfings without invoking any fine tuning of initial conditions of the inflaton which is one of the major advantages of new correction terms in the effective D-brane potential. As for the new constants, they do not involve much fine tuning for observational constraints to be satisfied. The fine tuning of model parameters $C_{3/2}$ and \mathcal{D} also improves by one order of magnitude in the scenario based upon the corrected potential. Thus the inflationary scenario based upon the corrected potential performs much better than the earlier models of D-brane inflation.

Last but not least, a viable inflation should be followed by a successful reheating. Reheating in the scenario under consideration, could occur at the time of collision of D-brane with the \bar{D} -brane located at the tip of the throat which is beyond the regime of perturbative string theoretic frame work used to obtain the effective potential (3). From phenomenological considerations, it looks quite plausible to implement here the instant reheating mechanism suitable to the class of models of non-oscillatory type. Since the new corrections to the D-brane potential are insignificant in the region where inflation ends, the preheating temperature is of the same order as obtained in case of the effective potential (1)[21].

ACKNOWLEDGMENTS

AD and SP thank the Centre for theoretical physics, Jamia Millia Islamia, New Delhi for hospitality. This work is partially supported by grant No. 2009/37/24 BRNS. MS thanks ICTP for hospitality where part of the work was done.

A. A. Starobinsky, Phys. Lett. B 91 (1980) 99; D. Kazanas, Astrophys. J. 241 L59 (1980); K. Sato, Mon. Not. R. Astron. Soc. 195, 467 (1981); A. H. Guth, Phys. Rev. D 23, 347 (1981).

D. N. Spergel et al., AstroAphys. J. Suppl. 148, 175(2003).

^[3] D. N. Spergel *et al.*, arXiv:astro-ph/0603452.

^[4] A. Sen, Remarks on Tachyon Driven Cosmology; arXiv:hep-th/0312153 and references therein.

^[5] A. D. Linde, Inflationary Cosmology, arXiv:0705.0164 [hep-th].

^[6] R. Kallosh, Inflation in string theory, arXiv:hep-th/0702059.

^[7] G. R. Dvali and S. H. H. Tye, Phys. Lett. B 450, 72 (1999); G. R. Dvali, Q. Shafi and S. Solganik, arXiv:hep-th/0105203;
C. P. Burgess et al., JHEP 0107, 047 (2001); G. Shiu and S. H. Tye, Phy. Lett. B 516, 421(2001); S. Alexander, Phys. Rev.D 65, 023507(2002); J. Garcia-Bellido, R. Rabadan and F. Zamora, JHEP 0201, 036 (2002); M. Gomez-Reino and I. Zavala, JHEP 0209, 020 (2002); C. P. Burgess, et al., JHEP 0203, 052(2002); N. T. Jones, H. Stoica and S. H. H. Tye, JHEP 0207, 051 (2002); D. Choudhury, D. Ghoshal, D. P. Jatkar and S. Panda, JCAP 0307, 009 (2003).

^[8] S. Thomas and J. Ward, Phys. Rev. D 72, 083519(2005); S. Panda, M. Sami, S. Tsujikawa, Phys. Rev. D 73, 023515(2006); S. Panda, M. Sami, S. Tsujikawa and J. Ward, Phys. Rev. D 73, 083512 (2006); B. Gumjudpai, T. Naskar and J. Ward, JCAP 0611, 006 (2006); E. Papantonopoulos, I. Pappa and V. Zamarias, JHEP 0605, 038 (2006); A. Sen, arXiv:hep-th/0703157.

^[9] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D 66, 106006 (2002).

^[10] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000).

^[11] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003).

^[12] S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. McAllister and S. P. Trivedi, JCAP 0310, 013 (2003).

J. P. Hsu, R. Kallosh and S. Prokushkin, JCAP 0312, 009 (2003); H. Firouzjahi and S. H. H. Tye, Phys. Lett. B 584, 147 (2004); C. P. Burgess, J. M. Cline, H. Stoica and F. Quevedo, JHEP 0409, 033 (2004); J. J. Blanco-Pillado et al., JHEP 0411, 063 (2004); JHEP 0609, 002 (2006); F. Koyama, Y. Tachikawa and T. Watari, Phys. Rev. D 69, 106001 (2004);

- J. M. Cline and H. Stoica, Phys. Rev. D **72**, 126004 (2005); R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. **97**, 191304 (2006); R. Allahverdi *et al.*, JCAP **0706**, 019 (2007).
- [14] D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt, arXiv:0705.3837 [hep-th].
- [15] D. Baumann, A. Dymarsky, I. R. Klebanov and L. McAllister, arXiv:0706.0360 [hep-th].
- [16] A. Krause and E. Pajer, arXiv:0705.4682 [hep-th].
- [17] S. Panda, M. Sami and S. Tsujikawa, Phys. Rev. D 76 (2007) 103512.
- [18] L. Hoi and J. Cline, Phys. Rev. D79 (2009) 083537.
- [19] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, arXiv:hep-th/0808.2811.
- [20] A. Ali, R. Chingangbam, S. Panda and M. Sami, Phys. Lett. B 674 (2009) 131.
- [21] S. Panda, M. Sami and I. Thongkool, Phys. Rev. D 81 (2010) 103506.
- [22] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, arXiv:hep-th/0912.4268, arXiv:hep-th/1001.5028.