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QCD transition temperature: full staggered result

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We conclude our investigations on the QCD cross-over transition temperatures with 2+1 staggered flavours and one-link stout improvement. We extend our previous two studies [Phys. Lett. B643 (2006) 46, JHEP 0906:088 (2009)] by choosing even finer lattices (N_t=16) and we work again with physical quark masses. These new results [for details see JHEP 1009:073,2010] support our earlier findings. We compare them with the published results of the hotQCD collaboration. All these results are confronted with the predictions of the Hadron Resonance Gas model and Chiral Perturbation Theory for temperatures below the transition region. Our results can be reproduced by using the physical spectrum. The findings of the hotQCD collaboration can be recovered only by using a distorted spectrum. This analysis provides a simple explanation for the observed discrepancy in the transition T between our and the hotQCD collaborations.

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Introduction. One of the most interesting quantities that can be extracted from lattice simulations is the transition temperature T_c at which hadronic matter passes to a deconfined phase. T_c has been vastly debated over the last few years, due to the disagreement on its value observed by different lattice collaborations, which in some cases is as high as 20% of the absolute value. Indeed, the analysis of the hotQCD collaboration (performed with two different improved staggered actions, asqtad and p4, and with physical strange quark mass and somewhat larger than physical u and d quark masses, $m_s/m_{u,d}$=10), indicates that the transition region lies in the range $T = (185-195)$ MeV. Different observables lead to the same value of T_c (for the latest published result and for references see [1]). The authors expect that $m_s/m_u = 20$ yields about 5 MeV shift (towards the smaller values) in the T dependence of the studied observables. On the other hand, the results obtained by our collaboration using the staggered stout action (with physical light and strange quark masses, thus $m_s/m_{u,d} \simeq$28) are quite different: T_c lies in the range 150-170 MeV, and it changes with the observable used to define it [2, 3]. This is not surprising, since the transition is a cross-over [4]: in this case it is possible to speak about a transition region, in which different observables may have their characteristic points at different T values, and the T dependences of the various observables play a more important role than any single T_c value. Unfortunately, the 25-30 MeV discrepancy was observed between the two groups for the T dependences of the various observables, too.

A lot of effort has been invested, to clarify the discrepancy between the results of the two collaborations. (Note, that quite recently preliminary results were presented [5] and the results of the hotQCD collaboration moved closer to our results. We include some of these data in our comparisons.) In Refs. [2, 3], we emphasized the role of the proper continuum limit with physical $m_s/m_{u,d}$, showing how the lack of them can distort the result. In [6] we pointed out that the continuum limit can be approached only if one reduces the unphysical pion splitting (the main motivation of our choice of action). An interesting application of these observations was studied in [7]. These authors have performed an analysis within the Hadron Resonance Gas model (HRG). They show that, to reproduce the lattice results for the asqtad and p4 actions of the hotQCD collaboration, it is necessary to distort the resonance spectrum away from the physical one in order to take into account the larger quark masses used in these lattice calculations, as well as finite lattice spacing effects. As we will see, no such distortion is needed to describe our data, and the discrepancy between the two collaborations has its roots in the above mentioned lattice artifacts.

From the lattice point of view, we present our most recent results for several physical quantities: our previous works [2, 3] have been extended to an even smaller lattice spacing (down to $a \lesssim 0.075$ fm at T_c), corresponding to N_t=16. We use physical light and strange quark masses: we fix them by reproducing f_K/m_π and f_K/m_K and by this procedure [3] we get $m_s/m_{u,d} = 28.15$.

First we give the details of our numerical simulations. Then we present the results of our simulations for different observables. We also present some aspects of the Hadron Resonance Gas model and the comparison between lattice and HRG model results. Finally we conclude.

Details of the lattice simulations. We use [2, 3] a tree-level Symanzik improved gauge, and a stout-improved staggered fermionic action (see Ref. [9] for details). The stout-smearing is an important part of the framework, which reduces the taste violation.

In analogy with what we did in [2, 3], we set the scale at the physical point by simulating at $T = 0$ with physical quark masses [3] and reproducing the kaon and pion masses and the kaon decay constant. This gives an uncertainty of about 2% in the scale setting, which propagates in the

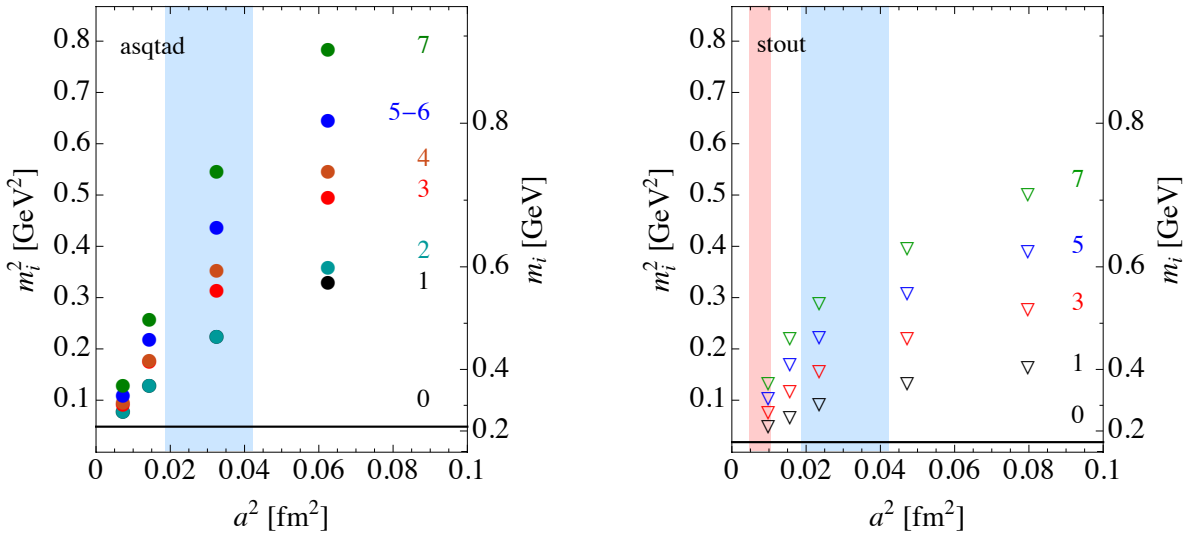

Figure 1: Pion mass splitting, as functions of a^2. Left: asqtad action [10]. Right: stout action. In both panels, the blue band indicates the relevant range of lattice spacings for a thermodynamics study at N_t=8 between T=120 and 180 MeV. The red band in the right panel corresponds to the same T range and N_t=16.

uncertainty in the determination of the T values listed.

The pion splittings of a staggered framework are proportional to $(\alpha_s a^2)$ for small a. It has to vanish in the continuum limit. Once it shows an $\alpha_s a^2$ dependence (in practice a^2 dependence with a subdominant logarithmic correction) we are in the scaling region. This is an important check for the validity of the staggered framework at a given lattice spacing. In Fig. 1 we show the leading order a^2-behavior of the masses of the pion multiplets calculated with the asqtad (left) and stout (righ) actions. It is evident that the continuum expectation is reached faster in the stout action than in the asqtad one. In addition, in the present paper we push our results to $N_t = 16$, which corresponds to even smaller lattice spacings and mass splittings than those used in [3].

Lattice results. We present our lattice results for the strange quark number susceptibility, Polyakov loop and two different definitions of the chiral condensate. After performing a continuum extrapolation, we extract the values of T_c associated to these observables. The T dependence of an observable contains much more information than the location of a peak or inflection point (which are usually hard to determine precisely for such a broad transition). We perform a HRG analysis and compare our results with those of the hotQCD Collaboration later.

Quark number susceptibilities increase during the transition, therefore they can be used to identify this region. In the left panel of Fig. 2 we show our results for the strange quark number susceptibility for $N_t = 10$, 12, 16. The gray band shows our continuum extrapolation.

The Polyakov loop indicates the transition, since it exhibits a rise in the transition region. In the right panel of Fig. 2 we plot the renormalized Polyakov loop as a function of T. We use our renormalization procedure of [2], in order to compare our results with those obtained by the hotQCD collaboration [1] we use the same renormalization constant. The various N_t data sets together with the continuum extrapolated result are presented. As it is expected from a broad cross-over the rise of the Polyakov loop is pretty slow as we increase T (c.f. [1, 2, 3]).

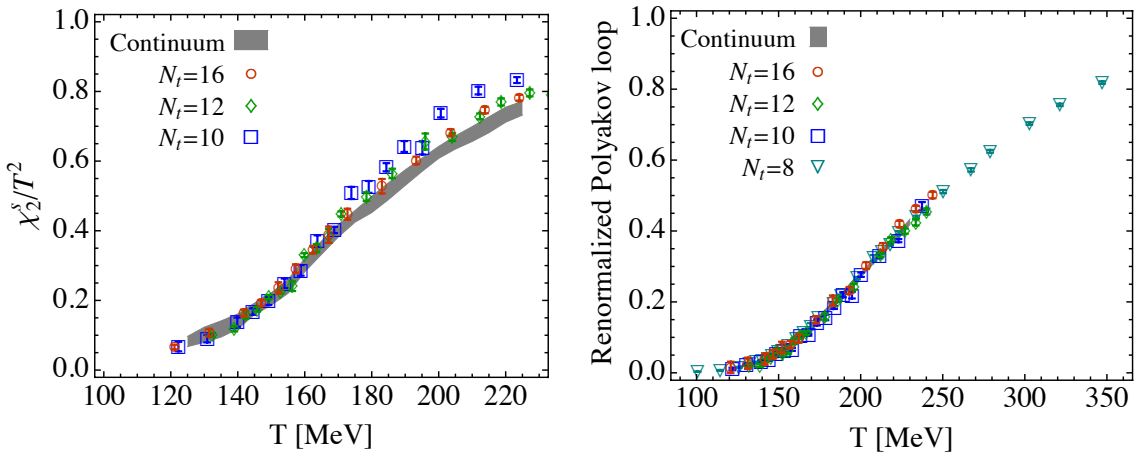

Figure 2: Strange quark number susceptibility (left) and Polyakov loop (right) as functions of T.

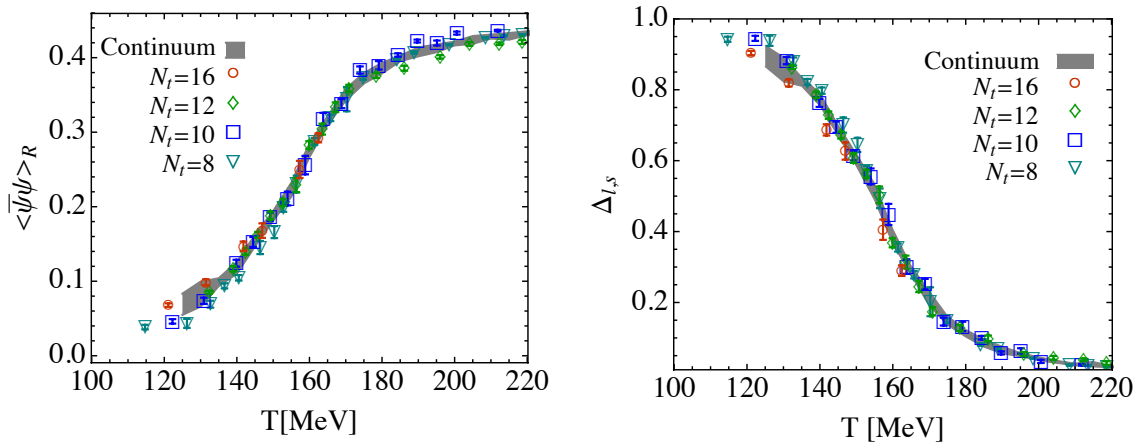

Figure 3: Left: renormalized chiral condensate $\langle\bar{\psi}\psi\rangle_R$. Right: subtracted chiral condensate $\Delta_{l,s}$.

The chiral condensate is defined as $\langle\bar{\psi}\psi\rangle_q = T\partial \ln Z/(\partial m_q V)$ for q=u,d,s. It can be taken as an indicator for the remnant of the chiral transition, since it rapidly changes around T_c. We multiply the above expression by m_q/m_π^4 to define a dimensionless renormalized chiral condensate. The individual results and the continuum extrapolation are shown in Figure 3. In order to compare our results to those of the hotQCD collaboration, we also calculate the quantity $\Delta_{l,s}$, which is defined as $[\langle\bar{\psi}\psi\rangle_{l,T} - m_l/m_s\langle\bar{\psi}\psi\rangle_{s,T}]/[\langle\bar{\psi}\psi\rangle_{l,0} - m_l/m_s\langle\bar{\psi}\psi\rangle_{s,0}]$ for l=u,d. Since the results at different lattice spacings are essentially on top of each other, we connect them to lead the eye and use this band in later comparisons (c.f. Fig. 3).

Hadron Resonance Gas model The HRG model has been widely used to study the low T phase of QCD in comparison with lattice data. In Ref. [7] an important ingredient was included, the m_π- and a-dependence of the hadron masses [11]. Here we combine these ingredients with Chiral Perturbation Theory (χPT) [12]. This opens the possibility to study chiral quantities, too.

The HRG model is based on the theorem of Ref. [13], which allows to calculate the micro-canonical partition function of an interacting system, for $V \to \infty$, to a good approximation, assuming that it is a gas of non-interacting free hadrons/resonances [14]. The pressure of the model can

be written as the sum of independent contributions coming from non-interacting resonances. We include all known baryons and mesons up to 2.5 GeV, as listed in the latest edition of the PDG.

We will compare the results obtained with the physical hadron masses to those obtained with the distorted one which takes into account a-effects. Each π/K in the staggered formulation is split into 16 mesons with different masses, which are all included. Similarly to Ref. [7], we will also take into account the m_π- and a-dependence of all other hadrons/resonances.

In order to calculate the chiral condensate in the HRG model, we need to know the behavior of all baryon and meson masses as functions of m_l and m_s . For the ground state hadrons we use [15]. The same study is not available for all the resonances that we include. Therefore, similarly to Ref. [7], we work under the assumption that all resonance masses behave as their fundamental states as functions of m_q. In addition, we determine the contribution of pions to the chiral condensate obtained in three-loop χPT [16]. All details of this calculation are given in [17].

In our analysis we compare two sets of lattice data:

• The first set is based on the Wuppertal-Budapest results.

• The second set is obtained by the Bielefeld-Brookhaven-Columbia-Riken Collaboration, which later merged with a part of the the MILC collaboration and formed the hotQCD collaboration.

Furthermore, we use two types of theoretical descriptions (based on hadron resonance gas model and chiral perturbation theory, for short: HRG+χPT):

• One of the theoretical descriptions is based on the physical spectrum from the PDG (we call this description “physical").

• The other theoretical approach is based on a non-physical spectrum (this spectrum is obtained by $T = 0$ simulations of the action one studies; the reason for this distortion will be explained later); we call this description “distorted".

As it is known, the Wuppertal-Budapest and the hotQCD results disagree. All characteristic T-s are higher for the hotQCD Collaboration. Note, that this discrepancy is not related to the difficulty of determining e.g. inflection points of slowly varying functions (typical for a broad cross-over). The discrepancy appears for all variables for a large T interval. As we claimed earlier [3] we observed “approximately 20–35 MeV difference in the transition regime between our results and those of the hotQCD Collaboration".

As we will see, the Wuppertal-Budapest results are in complete agreement with the “physical" HRG model and with the “physical" chiral perturbation theory, whereas the hotQCD results cannot be described this way. The hotQCD results can only be described by the “distorted" HRG+χPT.

In Fig. 4, we show results for the chiral condensate as a function of T. The left panel shows $\langle\bar{\psi}\psi\rangle_R$, while the right panel shows $\Delta_{l,s}$. From all quantities that we have calculated, a consistent picture arises: our stout results agree with the “physical” HRG+χPT predictions; whereas the observed shift in T_c between the results of the stout and the asqtad and p4 actions can be easily explained within the Hadron Resonance Gas+χPT model with “distorted” masses. Once the discretization effects, the taste violation and the heavier quark masses used in [1, 5] are taken into account, all the HRG+χPT curves for the different physical observables are shifted to higher T-s and fall on the corresponding lattice results.

As we mentioned there are proceedings contributions written by two members of the hotQCD Collaboration, in which the HISQ action is applied and preliminary results are presented. The approximately 35 MeV discrepancy for the chiral condensate curves is reduced to about 10 MeV

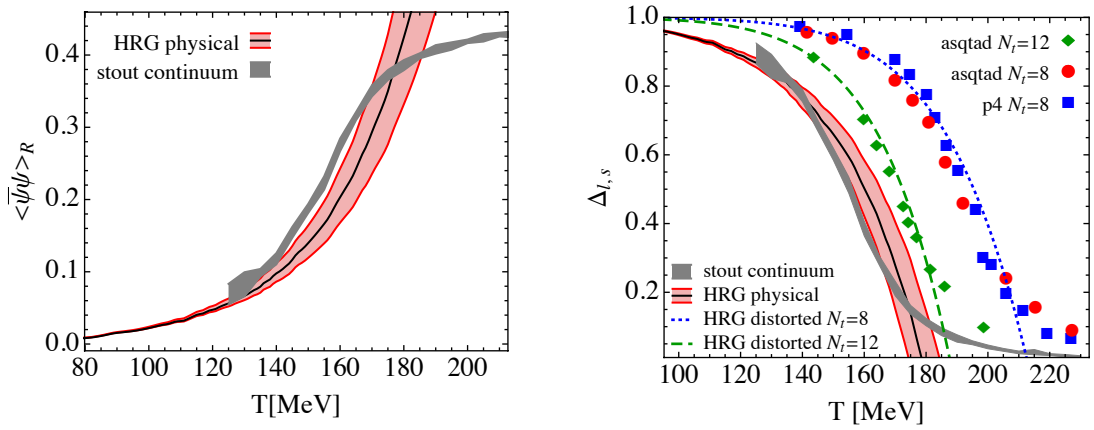

Figure 4: Left: Renormalized chiral condensate. Right: $\Delta_{l,s}$. Both as a function of T. Gray bands are our continuum results, obtained with the stout action. Full symbols are obtained with the asqtad and p4 actions [1, 5]. In both panels, the solid line is the HRG model result with physical masses. The error band corresponds to the uncertainty in the quark mass-dependence of hadron masses. The dashed lines are the HRG+χPT model result with distorted masses of the hotQCD Collaboration [1, 5] for $N_t = 8$ and $N_t = 12$.

(see Fig. 5). Note, that the continuum limit within the HISQ framework is still missing. This last important step (which needs quite some computational resources and also care) will hopefully eliminate the remaining minor discrepancy, too. The same two members of the hotQCD Collaboration presented preliminary results using the asqtad action on N_t=12 lattices [5], too. At this lattice spacing the pion splitting is smaller than on N_t=8 lattices, and the curves move closer to ours. Following these authors (Figure 5. of Ref. [5]) we zoom in into the transition region of $\Delta_{l,s}$ and on Figure 5. The stout results from a broad range of "a" (N_t=8, 10, 12 and 16) are shown with open symbols. They are all in the vicinity of our continuum estimate, indicated by the thin gray band. The hotQCD results were obtained by three different actions (p4, asqtad and HISQ) and with two different pion masses (220 and 160 MeV). They cover a broad range. The smaller the pion mass and/or splitting in the hotQCD results, the closer it is to ours.

These confirm the expectations [2, 3] that the source of the discrepancy was the lack of the proper continuum extrapolation [2] in the hotQCD result: a dominant discretization artefact within the asqtad and p4 actions is the large π splitting [6], which resulted in the distorted spectrum.

Conclusions We have presented our latest results for the QCD transition temperature. The quantities that we have studied are the strange quark number susceptibility, the Polyakov loop, the chiral condensate and the trace anomaly. We have given the complete T dependence of these quantities, which provide more information than the characteristic T values alone. Our previous results for the strange quark susceptibility, the Polyakov loop and the chiral condensate have been pushed to an even finer lattice (N_t=16). The new data corresponding to N_t=16 confirm our previous results. In order to find the origin of the discrepancy between the results of our collaboration and the hotQCD ones, we calculated these observables (except the Polyakov loop) in the Hadron Resonance Gas model. Besides using the physical hadron masses, we also performed the calculation with modified masses which take into account the heavier pions and larger lattice spacings used in [1]. We find an agreement between our data and the HRG ones with “physical” masses, while the hotQCD collaboration results are in agreement with the HRG model only if the spectrum is

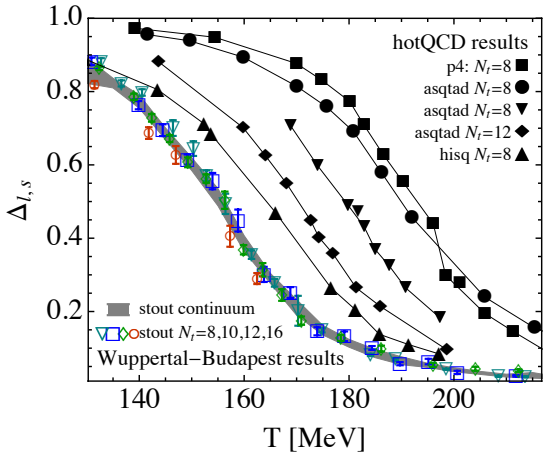

Figure 5: $\Delta_{l,s}$ as a function of T. We show a comparison between stout, asqtad, p4 and HISQ [1, 5] results. Our stout results were all obtained by the physical pion mass of 135 MeV. The full hotQCD dots and squares correspond to $m_\pi = 220$ MeV, the full triangles and diamonds correspond to $m_\pi = 160$ MeV.

"distorted" as it was directly measured on the lattice [10]. This analysis therefore provides an easy and convincing explanation of the observed shift in T_c between the two collaborations and emphasizes the role of the proper continuum limit. All the details can be found in Ref. [17].

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