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Towards the continuum limit of the lattice Landau gauge gluon propagator

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Abstract. The infrared behaviour of the lattice Landau gauge gluon propagator is discussed, combining results from simulations with different volumes and lattice spacings. In particular, the Cucchieri-Mendes bounds are computed and their implications for D(0) discussed.

Keywords: confinement, Landau gauge, lattice QCD, gluon propagator
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INTRODUCTION AND MOTIVATION

The link between the deep infrared behaviour of the gluon and ghost propagators and confinement, has motivated a great effort on computing these quantities on the lattice. Besides checking gluon confinement criteria, another important goal is to compare recent solutions of the Dyson-Schwinger equations with lattice results. In particular, the scaling solution [1] predicts a vanishing gluon propagator and a divergent ghost propagator at zero momentum. This solution complies with Gribov-Zwanziger [2] and Kugo-Ojima [3] confinement criteria. On the other hand, the decoupling solution [4] claims that a finite and non-vanishing zero momentum gluon propagator and a tree level like ghost propagator. The value of the zero momentum gluon propagator is connected with a dynamical generated gluon mass.

In this paper we report on our current results for the Cucchieri-Mendes bounds in SU(3) lattice gauge theory.

CUCCHIERI-MENDES BOUNDS

The Cucchieri-Mendes bounds [5] provide upper and lower bounds for the zero momentum gluon propagator of lattice Yang-Mills theories in terms of the average value of the gluon field. In particular, they relate the gluon propagator at zero momentum $D(0)$ with

$$M(0) \;=\; \frac{1}{d\,(N_c^2-1)} \sum_{\mu,a} \left| A^a_\mu(0) \right| \,, \tag{1}$$

where d is the number of space-time dimensions, and N_c the number of colors. In the above equation, $A^a_\mu(0)$ is the a color component of the gluon field at zero momentum, defined by

$$A^a_\mu(0) \;=\; \frac{1}{V} \sum_x A^a_\mu(x) \tag{2}$$

where $A^a_\mu(x)$ is the a color component of the gluon field in the real space. $D(0)$ is related with $M(0)$ by

$$\langle M(0) \rangle^2 \;\leq\; \frac{D(0)}{V} \;\leq N_d \left(N_c^2 - 1 \right) \langle M(0)^2 \rangle \,. \tag{3}$$

In the last equation $\langle\ \rangle$ means Monte Carlo average over gauge configurations. For convenience we will use the definition $N_{cd} = N_d(N_c^2-1)$. The bounds in equation (3) are a direct result of the Monte Carlo approach. The interest on these bounds comes from allowing a scaling analysis which can help understanding the finite volume behaviour of $D(0)$: assuming that each of the terms in inequality (3) scales with the volume according to A/V^α, the simplest possibility and the one considered in [5], an $\alpha > 1$ for $\langle M(0)^2 \rangle$ clearly indicates that $D(0) \to 0$ as the infinite volume is approached. In this sense, this scaling analysis allows to investigate the behaviour of $D(0)$ in the infinite volume limit.

For the SU(2) Yang-Mills theory [5], the results show a $D(0) = 0$ for the two dimensional theory, but a $D(0) \neq 0$ for three and four dimensional formulations.

RESULTS FOR SU(3) GAUGE THEORY

We have studied the Cucchieri-Mendes bounds within SU(3) lattice gauge theory for three values of the gauge coupling: $\beta = 6.0$ [6, 7], $\beta = 5.7$ [7], and $\beta = 6.2$.

Scaling analysis for $\beta = 6.0$

In table 1 we present the lattice setup for $\beta = 6.0$, pointing out the differences to [6, 7].

Figure 1 shows the results for the bounds, together with the fits to ω/V^α. Assuming this simple scaling

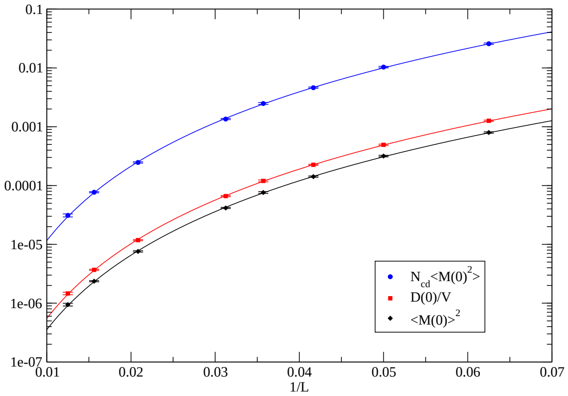

FIGURE 1. Cucchieri-Mendes bounds for $\beta = 6.0$.

TABLE 1. Lattice setup for $\beta = 6.0$. The lattice spacing is $a = 0.1016(25)$fm.

L^4	16^4	20^4	24^4	28^4	32^4	48^4	64^4	80^4
L(fm)	1.63	2.03	2.44	2.84	3.25	4.88	6.50	8.13
# conf.	52	72	60^*	56	126	104	120	$50^\dagger$

* new ensemble
† new statistics

behaviour, our results for the exponent α support $D(0) = 0$ – see table 2. However, when one assumes a scaling behaviour like $C/V + \omega V^{-\alpha}$, the results support $D(0) \neq 0$ – see table 3. In this sense, a finite and non-vanishing value for $D(0)$ in the infinite volume is not excluded.

Concerning the fits to ω/V^{α}, the reasons for the differences in the values of α reported here and in [5] – and therefore on the behaviour of $D(0)$ in the infinite volume limit – are not clear. The simulations use different gauge groups. Although there it is generally believed that the SU(2) and SU(3) propagators are equivalent for momenta above 1 GeV [8, 9], a recent direct comparison for smaller momenta has shown a measurable difference in the infrared region [10].

Moreover, the physical volumes used in [5] are much larger – up to $(27\text{fm})^4$ – than the ones used here – up to $(8\text{fm})^4$. However, the reader should be aware that in the SU(2) case the lattice spacing used is about twice the lattice spacing considered here.

TABLE 2. Fits to ω/V^{α} using lattice data at $\beta = 6.0$.

	ω	α	χ^2_ν
$\langle M(0) \rangle$	9.53(36)	0.5255(26)	0.80
$D(0)/V$	149 ± 10	1.0542(49)	0.63
$N_{cd} \langle M(0)^2 \rangle$	2927 ± 221	1.0504(54)	0.83

TABLE 3. Fits to $C/V + \omega V^{-\alpha}$ using lattice data at $\beta = 6.0$.

	$\omega/1000$	α	$C/100$	χ^2_ν
$\langle M(0) \rangle^2$	0.23(24)	1.22(11)	0.337(50)	0.47
$D(0)/V$	0.27(23)	1.19(10)	0.49(11)	0.42
$N_{cd} \langle M(0)^2 \rangle$	7.1 ± 7.3	1.22(11)	11.0 ± 1.7	0.55

LATTICE SPACING EFFECTS IN THE GLUON PROPAGATOR

In order to disentangle possible lattice effects due to the use of a different lattice spacing, we carried out simulations at $\beta = 5.7$ and $\beta = 6.2$. The lattice setup is shown in tables 4 and 5 respectively.

TABLE 4. Lattice setup for $\beta = 5.7$. The lattice spacing is $a = 0.1838(11)$fm.

L^4	8^4	10^4	14^4	18^4	26^4	36^4	44^4
L(fm)	1.47	1.84	2.57	3.31	4.78	6.62	8.09
# conf.	56	149	149	149	132	100	55^*

TABLE 5. Lattice setup for $\beta = 6.2$. The lattice spacing is $a = 0.07261(85)$fm.

L^4	24^4	32^4	48^4	64^4	80^4
L(fm)	1.74	2.32	3.49	4.65	5.81
# conf.	51	56	87	99	15

Some differences have been seen between the gluon propagator computed at different lattice spacings for similar physical volumes. An example can be seen in figures 2 and 3, where the infrared $\beta = 6.2$ data does not agree with data from $\beta = 5.7$ and 6.0 simulations. These differences deserve further investigations to clarify any possible effects due to finite lattice spacing.

Scaling analysis for $\beta = 5.7$ and $\beta = 6.2$

In what concerns the fits to ω/V^{α}, the analysis of the data coming from both sets still supports a vanishing $D(0)$ in the infinite volume limit – see tables 6 and 7.

Similarly to the case studied before, the lattice data is also well described by the functional form $C/V + \omega V^{-\alpha}$ – see tables 8 and 9. Although the $\beta = 5.7$ case supports $D(0) \neq 0$, for $\beta = 6.2$ the statistical errors do not allow to take any conclusion. In fact, although $C = 0$ within statistical errors, we also get $\alpha = 1$. For this case, it is worth an increase of statistics.

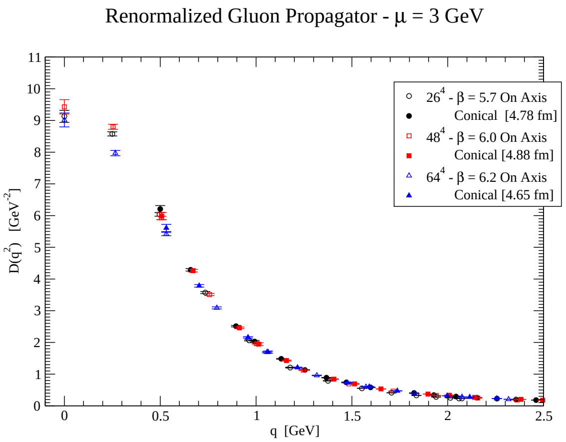

FIGURE 2. Comparing the gluon propagator computed using different lattice spacings at the same physical volume $V \sim (4.8\,fm)^4$.

TABLE 6. Fits to $\omega V^{-\alpha}$ using lattice data at $\beta = 5.7$. In order to keep $\chi^2_\nu < 2$, the 26^4 lattice data has been excluded.

	ω	α	χ^2_ν
$\langle M(0) \rangle$	4.63(12)	0.5244(23)	1.92
$D(0)/V$	32.8 ± 1.6	1.0466(42)	1.14
$N_{cd} \langle M(0)^2 \rangle$	696 ± 37	1.0488(47)	1.72

CONCLUSIONS

We have studied the scaling behaviour of Cucchieri-Mendes bounds using ensembles generated at several lattice spacings. Fits of the data to a pure power law in the volume strongly support $D(0) = 0$, but the use of other ansatze do not allow to take definitive conclusions.

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TABLE 7. Fits to $\omega V^{-\alpha}$ using lattice data at $\beta = 6.2$. Data for $M(0)$ does not include 48^4.

	$\omega/100$	α	χ^2_ν
$\langle M(0) \rangle$	0.163(11)	0.5374(47)	0.08
$D(0)/V$	3.66(46)	1.0659(84)	0.47
$N_{cd} \langle M(0)^2 \rangle$	8.4 ± 1.2	1.0725(94)	0.13

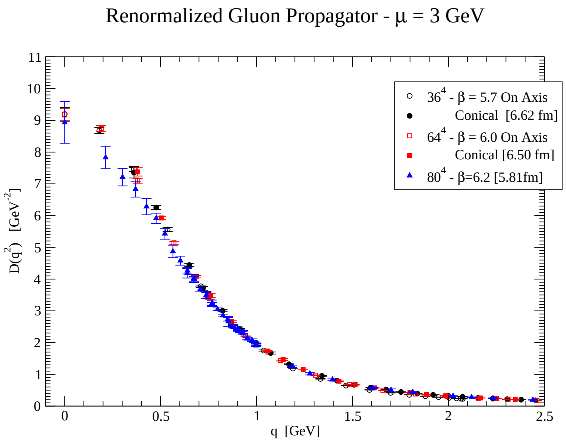

FIGURE 3. Comparing the gluon propagator computed using different lattice spacings at the same physical volume $V \sim (6.5\,fm)^4$.

TABLE 8. Fits to $C/V + \omega V^{-\alpha}$ using lattice data at $\beta = 5.7$. In order to keep $\chi^2_\nu < 2$, the 26^4 lattice data has been excluded.

	$\omega/100$	α	$C/100$	χ^2_ν
$\langle M(0) \rangle^2$	0.27(15)	1.186(90)	0.088(15)	1.80
$D(0)/V$	0.301(93)	1.122(90)	0.116(53)	1.28
$N_{cd} \langle M(0)^2 \rangle$	8.2 ± 4.2	1.172(91)	2.78(58)	1.69

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TABLE 9. Fits to $C/V + \omega V^{-\alpha}$ using lattice data at $\beta = 6.2$. Data for $M(0)$ does not include 48^4.

	$\omega/1000$	α	$C/100$	χ^2_ν
$\langle M(0) \rangle^2$	0.34(66)	1.13(29)	0.4 ± 1.2	0.13
$D(0)/V$	0.366(47)	1.07(29)	0.04 ± 5.6	0.95
$N_{cd} \langle M(0)^2 \rangle$	8.6 ± 6.7	1.08(28)	4 ± 85	0.25