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Strange Quark Mass and 1^{++} Nonet Singlet-Octet Mixing Angle

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Abstract

We compute the strange quark mass from the analysis of the $f_1(1420) - f_1(1285)$ mass difference QCD sum rule, where the operator-product-expansion series is up to dimension six and to $\mathcal{O}(\alpha_s^3)$ accuracy. We obtain bounds for the strange quark mass 125 MeV $\leq \overline{m}_s(1\text{ GeV}) \leq$ 230 MeV (i.e. 95 MeV $\leq \overline{m}_s(2\text{ GeV}) \leq$ 174 MeV) and for the singlet-octet mixing angle $2^\circ \leq \theta \leq 68^\circ$. Two strategies are taken into account to further determine the mixing angle θ. (i) First, in the previous study the Gell-Mann-Okubo mass formula together with the $K_1(1270) - K_1(1400)$ mixing angle $\theta_{K_1} = (-34 \pm 13)^\circ$ which was extracted from the data for $\mathcal{B}(B \to K_1(1270)\gamma), \mathcal{B}(B \to K_1(1400)\gamma), \mathcal{B}(\tau \to K_1(1270)\nu_\tau)$, and $\mathcal{B}(\tau \to K_1(1420)\nu_\tau)$, gave $\theta = (23^{+17}_{-23})^\circ$. (ii) Second, from the study of the ratio for $f_1(1285) \to \phi\gamma$ and $f_1(1285) \to \rho^0\gamma$ branching fractions, we have two-fold solution $\theta = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$. Combining these two analyses, we thus obtain $\theta = (19.4^{+4.5}_{-4.6})^\circ$.

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1. Introduction. The light quark masses are important parameters in the standard model. The quark mass term mixes left- and right-handed quarks in the QCD Lagrangian. The existence of non-zero light quark masses results in the spontaneous $SU(3)_L \times SU(3)_R$ chiral symmetry breaking of the QCD to be $SU(3)_V$. Due to the symmetry breaking we have eight massless Goldstone bosons: π, K, η, in the massless quark limit. However, actually π, K, and η are not massless mesons because the light quark masses do not vanish. The strange quark mass measures not only the chiral but also vector flavor $SU(3)$ symmetry breaking which can be observed in the hadron mass spectrum and transition amplitudes, *e.g.*, phenomenologies in B factories. Furthermore, the theoretical prediction for ϵ'/ϵ also depends on the accuracy of the strange quark mass [1]. Nevertheless, the quarks are not directly observed due to the color confinement.

For the aforementioned reasons, the strange quark mass has been evaluated in various frameworks. Together with the experimental data, the techniques of the chiral perturbation theory and current algebra can determine quark mass ratios [2, 3]. Many attempts have been made to compute $\overline{m}_s$ using QCD sum rules, finite energy sum rules [4, 5, 6, 7, 8, 9, 10] and lattice QCD [11, 12, 13]. The running strange quark mass in the $\overline{\rm MS}$ scheme at a scale $\mu \approx 2$ GeV is $\overline{m}_s = 101^{+29}_{-21}$ MeV (80-130 MeV) given in the current particle data group (PDG) average [14], where $\overline{m}_s$ is estimated from SU(3) splitting in hadron masses. This error is still large. It should be important if the error can be further reduced.

Many sum rule calculations for $\overline{m}_s$ were done from the analysis of various channels of scalar, pseudoscalar, or vector currents. In this letter, we shall obtain the mass difference QCD sum rules for the $f_1(1420)$ and $f_1(1285)$ to determine the magnitude of the strange quark mass. The mass splitting is owing to the SU(3) flavor symmetry breaking. These two mesons with quantum number $J^{PC} = 1^{++}$ are the members of the 1^3P_1 states in the quark model language, and are mixtures of the pure octet f_8 and singlet f_1, where the mixing is characterized by the mixing angle θ. We perform a comprehensive study of the constraint on $\overline{m}_s$, which is as a function of θ, by considering theoretical uncertainties from all inputs. We thus get the lower bound on the strange quark mass: $\overline{m}_s(1\text{GeV}) \geq 125$ MeV (i.e., $\overline{m}_s(2\text{GeV}) \geq 95$ MeV), which sufficiently improves the current PDG result. In section 2, we shall present detailed discussions on the determination of the mixing angle θ. Substituting the $K_1(1270) - K_1(1400)$ mixing angle, which was extracted from the $B \to K_1\gamma$ and $\tau \to K_1\nu_\tau$ data, to the Gell-Mann-Okubo mass formula, we can derive the value of θ. Alternatively, from the analysis of the decay ratio for $f_1(1285) \to \phi\gamma$ and $f_1(1285) \to \rho^0\gamma$, we have a much accurate estimation for θ. We calculate the mass difference sum rule $m_{f_1(1420)} - m_{f_1(1285)}$ in section 3. From the sum rule analysis, we obtain the constraint ranges for $\overline{m}_s$ and θ. Finally, a brief summary is given in section 4.

2. Singlet-octet mixing angle of the 1^{++} nonet. In the quark model, $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and K_{1A} are classified in 1^{++} multiplets, which, in terms of spectroscopic notation $n^{2S+1}L_J$, are 1^3P_1 p-wave mesons. Analogous to η and η', because of SU(3) breaking effects, $f_1(1285)$ and $f_1(1420)$ are the mixing states of the pure octet f_8 ad singlet f_1,

$$|f_1(1285))\rangle = |f_1\rangle \cos\theta + |f_8\rangle \sin\theta, \qquad |f_1(1420)\rangle = -|f_1\rangle \sin\theta + |f_8\rangle \cos\theta\,. \tag{1}$$

In the present letter, we adopt

$$f_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s), \tag{2}$$

$$f_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s), \tag{3}$$

where there is a relative sign difference between the $\bar{s}s$ contents of f_1 and f_8 in our convention. From the Gell-Mann-Okubo mass formula, the mixing angle θ satisfies

$$\cos^2\theta = \frac{3m^2_{f_1(1285)} - \left(4m^2_{K_{1A}} - m^2_{a_1}\right)}{3\left(m^2_{f_1(1285)} - m^2_{f_1(1420)}\right)}, \tag{4}$$

where

$$m^2_{K_{1A}} = \langle K_{1A}|\mathcal{H}|K_{1A}\rangle = m^2_{K_1(1400)}\cos^2\theta_{K_1} + m^2_{K_1(1270)}\sin^2\theta_{K_1}, \tag{5}$$

with $\mathcal{H}$ being the Hamiltonian. Here θ_{K_1} is the $K_1(1400) - K_1(1270)$ mixing angle. The sign of the mixing angle θ can be determined from the mass relation [14]

$$\tan\theta = \frac{4m^2_{K_{1A}} - m^2_{a_1} - 3m^2_{f_1(1420)}}{3m^2_{18}}, \tag{6}$$

where $m^2_{18} = \langle f_1|\mathcal{H}|f_8\rangle \simeq (m^2_{a_1} - m^2_{K_{1A}})2\sqrt{2}/3 < 0$, we find $\theta > 0$. Due to the strange and non-strange light quark mass difference, K_{1A} is not the mass eigenstate and it can mix with K_{1B}, which is one of the members in the 1^1P_1 multiplets. From the convention in [15] (see also discussions in [16, 17]), we write the two physical states $K_1(1270)$ and $K_1(1400)$ in the following relations,

$$\begin{aligned}
|K_1(1270)\rangle &= |K_{1A}\rangle\sin\theta_K + |K_{1B}\rangle\cos\theta_K, \\
|K_1(1400)\rangle &= |K_{1A}\rangle\cos\theta_K - |K_{1B}\rangle\sin\theta_K.
\end{aligned} \tag{7}$$

The mixing angle was found to be $|\theta_{K_1}| \approx 33^\circ, 57^\circ$ in [15] and $\approx \pm 37^\circ, \pm 58^\circ$ in [18]. A similar range $35^\circ \lesssim |\theta_{K_1}| \lesssim 55^\circ$ was obtained in [19]. The sign ambiguity for θ_{K_1} is due to the fact that one can add arbitrary phases to $|\bar{K}_{1A}\rangle$ and $|\bar{K}_{1B}\rangle$. This sign ambiguity can be removed by fixing the signs of decay constants $f_{K_{1A}}$ and $f^\perp_{K_{1B}}$, which are defined by

$$\langle 0|\bar{\psi}\gamma_\mu\gamma_5 s|\bar{K}_{1A}(P,\lambda)\rangle = -i\, f_{K_{1A}}\, m_{K_{1A}}\, \epsilon^{(\lambda)}_\mu, \tag{8}$$

$$\langle 0|\bar{\psi}\sigma_{\mu\nu} s|\bar{K}_{1B}(P,\lambda)\rangle = i f^\perp_{K_{1B}}\, \epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_{(\lambda)} P^\beta, \tag{9}$$

where $\epsilon^{0123} = -1$ and $\psi \equiv u$ or d. Following the convention in [17], we adopt $f_{K_{1A}} > 0$, $f^\perp_{K_{1B}} > 0$, so that θ_{K_1} should be negative to account for the observable $\mathcal{B}(B \to K_1(1270)\gamma) \gg \mathcal{B}(B \to K_1(1400)\gamma)$ [20, 21]. Furthermore, from the data of $\tau \to K_1(1270)\nu_\tau$ and $K_1(1400)\nu_\tau$ decays together with the sum rule results for the K_{1A} and K_{1B} decay constants, the mixing angle $\theta_{K_1} = (-34 \pm 13)^\circ$ was obtained in [21]. Substituting this value into (4), we then obtain $\theta^{\rm quad} = (23^{+17}_{-23})^\circ$

[22], i.e., $\theta^{\rm quad} = 0^\circ - 40^\circ$ [2]. Since $K^*\overline{K}$ and $K\overline{K}\pi$ are the dominant modes of $f_1(1420)$ whereas $f_0(1285)$ decays mainly to the 4π states, this suggests that the quark content is primarily $s\bar{s}$ for $f_1(1420)$ and $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ for $f_1(1285)$. Therefore, the mixing relations can be rewritten to exhibit the $n\bar{n}$ and $s\bar{s}$ components which decouple for the ideal mixing angle $\theta_i = \tan^{-1}(1/\sqrt{2}) \simeq 35.3^\circ$. Let $\bar{\alpha} = \theta_i - \theta$, we rewrite these two states in the flavor basis [3],

$$\begin{aligned} f_1(1285) &= \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\cos\bar{\alpha} + \bar{s}s\,\sin\bar{\alpha}\,, \\ f_1(1420) &= \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\sin\bar{\alpha} - \bar{s}s\,\cos\bar{\alpha}\,. \end{aligned} \tag{10}$$

An alternative strategy for extracting θ can be achieved from the ratio of $f_1(1285) \to \phi\gamma$ and $f_1(1285) \to \rho^0\gamma$ branching fractions. Because the electromagnetic (EM) interaction Lagrangian is given by

$$\begin{aligned} \mathcal{L}_I &= -A^\mu_{em}(e_u\bar{u}\gamma_\mu u + e_d\bar{d}\gamma_\mu d + e_s\bar{s}\gamma_\mu s) \\ &= -A^\mu_{em}\left((e_u + e_d)\frac{\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d}{2} + (e_u - e_d)\frac{\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d}{2} + e_s\bar{s}\gamma_\mu s\right), \end{aligned} \tag{11}$$

we therefore obtain

$$\begin{aligned} \frac{\mathcal{B}(f_1(1285) \to \phi\gamma)}{\mathcal{B}(f_1(1285) \to \rho^0\gamma)} &= \left(\frac{\langle\phi|e_s\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(e_u - e_d)(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2|f_1(1285)\rangle}\right)^2 \underbrace{\left(\frac{m^2_{f_1} - m^2_\phi}{m^2_{f_1} - m^2_\rho}\right)^3}_{\text{phase factor}} \\ &= \underbrace{\left(\frac{-e/3}{2e/3 + e/3}\right)^2}_{\text{EM factor}} \left(\frac{\langle\phi|\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2|f_1(1285)\rangle}\right)^2 \underbrace{\left(\frac{m^2_{f_1} - m^2_\phi}{m^2_{f_1} - m^2_\rho}\right)^3}_{\text{phase factor}} \\ &\approx \frac{4}{9}\left(\frac{m_\phi f_\phi}{m_\rho f_\rho}\right)^2 \tan^2\bar{\alpha}\left(\frac{m^2_{f_1} - m^2_\phi}{m^2_{f_1} - m^2_\rho}\right)^3, \end{aligned} \tag{12}$$

where $f_1 \equiv f_1(1285)$, and we have taken the single-pole approximation [4]:

$$\begin{aligned} \frac{\langle\phi|\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2|f_1(1285)\rangle} &\approx \frac{m_\phi f_\phi g_{f_1\phi\phi}}{m_\rho f_\rho g_{f_1\rho\rho}/\sqrt{2}}\frac{\sin\bar{\alpha}}{\cos\bar{\alpha}/\sqrt{2}} \\ &\approx \frac{m_\phi f_\phi}{m_\rho f_\rho} \times 2\tan\bar{\alpha}\,. \end{aligned} \tag{13}$$

[2] Replacing the meson mass squared m^2 by m throughout (4), we obtain $\theta^{\rm lin} = (23^{+17}_{-23})^\circ$. The difference is negligible. Our result can be compared with that using $\theta_{K_1} = -57^\circ$ into (4), one has $\theta^{\rm quad} = 52^\circ$.

[3] In PDG [14], the mixing angle is defined as $\alpha = \theta - \theta_i + \pi/2$. Comparing it with our definition, we have $\alpha = \pi/2 - \bar{\alpha}$.

[4] The following approximation was used in [23]:

$$\frac{\langle\phi|\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2|f_1(1285)\rangle} \approx 2\tan\bar{\alpha}\,.$$

Using $f_\rho = 209 \pm 1$ MeV, $f_\phi = 221 \pm 3$ MeV [24], and the current data $\mathcal{B}(f_1(1285) \to \phi\gamma) = (7.4 \pm 2.6) \times 10^{-4}$ and $\mathcal{B}(f_1(1285) \to \rho^0\gamma) = (5.5 \pm 1.3)\%$ [14] as inputs, we obtain $\bar{\alpha} = \pm(15.8^{+4.5}_{-4.6})^\circ$, i.e., two fold solution $\theta = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$. Combining with these two analyses, we thus find that $\theta = (19.4^{+4.5}_{-4.6})^\circ$ is much preferred and in good agreement with experimental observables.

3. Mass of the strange quark. We proceed to evaluate the strange quark mass from the mass difference sum rules of the $f_1(1285)$ and $f_1(1420)$ mesons. We consider the following two-point correlation functions,

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle 0|\mathrm{T}(j_\mu(x) j_\nu^\dagger(0))|0\rangle = -\Pi_1(q^2) g_{\mu\nu} + \Pi_2(q^2) q_\mu q_\nu \,, \tag{14}$$

$$\Pi'_{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle 0|\mathrm{T}(j'_\mu(x) j'^\dagger_\nu(0))|0\rangle = -\Pi'_1(q^2) g_{\mu\nu} + \Pi'_2(q^2) q_\mu q_\nu \,. \tag{15}$$

The interpolating currents satisfying the relations:

$$\langle 0|j_\mu^{(\prime)}(0)|f_1^{(\prime)}(P,\lambda)\rangle = -i f_{f_1^{(\prime)}} m_{f_1^{(\prime)}} \epsilon_\mu^{(\lambda)}, \tag{16}$$

are

$$j_\mu = \cos\theta j_\mu^{(1)} + \sin\theta j_\mu^{(8)} \,, \tag{17}$$

$$j'_\mu = -\sin\theta j_\mu^{(1)} + \cos\theta j_\mu^{(8)} \,, \tag{18}$$

where

$$j_\mu^{(1)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s) \,, \tag{19}$$

$$j_\mu^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s) \,, \tag{20}$$

and we have used the short-hand notations for $f_1 \equiv f_1(1285)$ and $f'_1 \equiv f_1(1420)$. In the massless quark limit, we have $\Pi_1 = q^2\Pi_2$ and $\Pi'_1 = q^2\Pi'_2$ due to the current conservation of j_μ and j'_μ. Here we focus on $\Pi_1^{(\prime)}$ since it receives contributions only from axial-vector ($^3P_1$) mesons, whereas $\Pi_2^{(\prime)}$ contains effects from pseudoscalar mesons. The lowest-lying $f_1^{(\prime)}$ meson contribution can be approximated via the dispersion relation as [25]

$$\frac{m^2_{f_1^{(\prime)}} f^2_{f_1^{(\prime)}}}{m^2_{f_1^{(\prime)}} - q^2} = \frac{1}{\pi}\int_0^{s_0^{f_1^{(\prime)}}} ds \frac{\mathrm{Im}\Pi_1^{(\prime)\mathrm{OPE}}(s)}{s - q^2} \,, \tag{21}$$

where $\Pi_1^{(\prime)\mathrm{OPE}}$ is the QCD operator-product-expansion (OPE) result of $\Pi_1^{(\prime)}$ at the quark-gluon level [17], and $s_0^{f_1^{(\prime)}}$ is the threshold of the higher resonant states. The four-quark condensates are expressed as

$$\langle 0|\bar{q}\Gamma_i\lambda^a q\bar{q}\Gamma_i\lambda^a q|0\rangle = -a_2\frac{1}{16N_c^2}\mathrm{Tr}(\Gamma_i\Gamma_i)\mathrm{Tr}(\lambda^a\lambda^a)\langle\bar{q}q\rangle^2 \,, \tag{22}$$

where $a_2 = 1$ corresponds to the vacuum saturation approximation. In the present work, we have $\Gamma = \gamma_\mu$ and $\gamma_\mu\gamma_5$, for which $a_2 = 1 \sim 1.2$ was estimated in [26]. For $\Pi_1^{(\prime)\mathrm{OPE}}$, we take into account the terms with dimension ≤ 6, where the term with dimension=0 is up to $\mathcal{O}(\alpha_s^3)$, and the terms with dimension=4 are up to $\mathcal{O}(\alpha_s^2)$. We do not include the radiative correction to the dimension=6 terms since all the uncertainties can be lumped into a_2. Note that such radiative corrections for terms with dimensions=0 and 4 are the same as the vector meson case and can read from [6].

Further applying the Borel (inverse-Laplace) transformation [25],

$$\mathbf{B}[f(q^2)] = \lim_{\substack{n\to\infty \\ -q^2\to\infty \\ -q^2/n^2=M^2\text{fixed}}} (-q^2)^{n+1}\left[\frac{d}{dq^2}\right]^n f(q^2), \tag{23}$$

to both sides of (21) to improve the convergence of the OPE series and further suppress the contributions from higher resonances, the sum rules thus read

$$\begin{aligned}
f_{f_1}^2 m_{f_1}^2 e^{-m_{f_1}^2/M^2} =& \int_0^{s_0^{f_1}} \frac{s\,ds\,e^{-s/M^2}}{4\pi^2}\left[1+\frac{\alpha_s}{\pi}+1.6398\frac{\alpha_s^2}{\pi^2}-\left(10.2839-\frac{b_0^2\pi^4}{3}\right)\frac{\alpha_s^3}{\pi^3}\right] \\
&-\frac{1}{12}\left(1-\frac{11}{18}\frac{\alpha_s}{\pi}\right)\langle\frac{\alpha_s}{\pi}G^2\rangle \\
&+\left[\frac{4}{27}\frac{\alpha_s}{\pi}+\left(-\frac{257}{486}+\frac{4}{3}\zeta(3)\right)\frac{\alpha_s^2}{\pi^2}\right]\sum_{q_i\equiv u,d,s}\langle\overline{m}_i\bar{q}_i q_i\rangle \\
&+\frac{\cos^2\theta}{3}\left[2a_1\left(2\overline{m}_q\langle\bar{q}q\rangle+\overline{m}_s\langle\bar{s}s\rangle\right)-\frac{352\pi\alpha_s}{81M^2}a_2\left(2\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right)\right] \\
&+\frac{\sin^2\theta}{3}\left[2a_1\left(\overline{m}_q\langle\bar{q}q\rangle+2\overline{m}_s\langle\bar{s}s\rangle\right)-\frac{352\pi\alpha_s}{81M^2}a_2\left(\langle\bar{q}q\rangle^2+2\langle\bar{s}s\rangle^2\right)\right] \\
&+\frac{2\sqrt{2}}{3}\cos\theta\sin\theta\left[2a_1\left(\overline{m}_q\langle\bar{q}q\rangle-\overline{m}_s\langle\bar{s}s\rangle\right)-\frac{352\pi\alpha_s}{81M^2}a_2\left(\langle\bar{q}q\rangle^2-\langle\bar{s}s\rangle^2\right)\right],
\end{aligned} \tag{24}$$

$$\begin{aligned}
f_{f_1'}^2 m_{f_1'}^2 e^{-m_{f_1'}^2/M^2} =& \int_0^{s_0^{f_1'}} \frac{s\,ds\,e^{-s/M^2}}{4\pi^2}\left[1+\frac{\alpha_s}{\pi}+1.6398\frac{\alpha_s^2}{\pi^2}-\left(10.2839-\frac{b_0^2\pi^4}{3}\right)\frac{\alpha_s^3}{\pi^3}\right] \\
&-\frac{1}{12}\left(1-\frac{11}{18}\frac{\alpha_s}{\pi}\right)\langle\frac{\alpha_s}{\pi}G^2\rangle \\
&+\left[\frac{4}{27}\frac{\alpha_s}{\pi}+\left(-\frac{257}{486}+\frac{4}{3}\zeta(3)\right)\frac{\alpha_s^2}{\pi^2}\right]\sum_{q_i\equiv u,d,s}\langle\overline{m}_i\bar{q}_i q_i\rangle \\
&+\frac{\sin^2\theta}{3}\left[2a_1\left(2\overline{m}_q\langle\bar{q}q\rangle+\overline{m}_s\langle\bar{s}s\rangle\right)-\frac{352\pi\alpha_s}{81M^2}a_2\left(2\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right)\right] \\
&+\frac{\cos^2\theta}{3}\left[2a_1\left(\overline{m}_q\langle\bar{q}q\rangle+2\overline{m}_s\langle\bar{s}s\rangle\right)-\frac{352\pi\alpha_s}{81M^2}a_2\left(\langle\bar{q}q\rangle^2+2\langle\bar{s}s\rangle^2\right)\right]
\end{aligned}$$

$$-\frac{2\sqrt{2}}{3}\cos\theta\sin\theta\left[2a_1\left(\overline{m}_q\langle\bar{q}q\rangle-\overline{m}_s\langle\bar{s}s\rangle\right)-\frac{352\pi\alpha_s}{81M^2}a_2\left(\langle\bar{q}q\rangle^2-\langle\bar{s}s\rangle^2\right)\right], \quad (25)$$

where

$$\overline{m}_q\langle\bar{q}q\rangle \equiv \frac{1}{2}\left(\overline{m}_u\langle\bar{u}u\rangle+\overline{m}_d\langle\bar{d}d\rangle\right), \quad \langle\bar{q}q\rangle^2 \equiv \frac{1}{2}\left(\langle\bar{u}u\rangle^2+\langle\bar{d}d\rangle^2\right),$$

$$a_1 = 1+\frac{1}{3}\frac{\alpha_s}{\pi}+\frac{11}{2}\frac{\alpha_s^2}{\pi^2}, \quad b_0=\frac{33-2n_f}{12\pi}, \quad (26)$$

with n_f the number of flavors. The mass sum rules for $f_1(1285)$ and $f_1(1420)$ can be obtained by applying the differential operator $M^4\partial\ln/\partial M^2$ to both sides of (24) and (25), respectively.

In the numerical analysis, we shall use $\Lambda_{\rm QCD}^{(3)\rm NLO}=0.360$ GeV, corresponding to $\alpha_s(1{\rm GeV})=0.495$, $\Lambda_{\rm QCD}^{(4)\rm NLO}=0.313$ GeV, and the following values at the scale $\mu=1$ GeV [25, 27]:

$$\begin{aligned}
&\langle\alpha_s G^a_{\mu\nu}G^{a\mu\nu}\rangle = (0.474\pm0.120)\ {\rm GeV}^4/(4\pi)\,,\\
&\langle\bar{u}u\rangle \cong \langle\bar{d}d\rangle = -(0.245\pm0.010)^3\ {\rm GeV}^3\,,\\
&\langle\bar{s}s\rangle = (0.8\pm0.1)\langle\bar{u}u\rangle\,,\\
&\overline{m}_u+\overline{m}_d = (11\pm2)\ {\rm MeV}\,,\\
&a_2 \simeq 1\sim1.2\,.
\end{aligned} \quad (27)$$

We do not consider the isospin breaking effect between $\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$ since $\langle\bar{d}d\rangle/\langle\bar{u}u\rangle-1\approx-0.007$ [3] is negligible in the present analysis. The threshold is allowed by $s_0^{f_1}=2.85\pm0.15$ GeV^2 and determined by the maximum stability of the mass sum rule. For an estimate on the threshold difference, we parametrize in the form $(\sqrt{s_0^{f_1'}}-\sqrt{s_0^{f_1}})/\sqrt{s_0^{f_1}}=\delta\times(m_{f_1'}-m_{f_1})/m_{f_1}$, with $\delta=1.0\pm0.3$. In other words, we assign a 30% uncertainty to the default value. We search for the allowed solutions for strange quark mass and the singlet-octet mixing angle θ under the following constraints: (i) Comparing with the observables, the errors for the mass sum rule results of the $f_1(1285)$ and $f_1(1420)$ in the Borel window 0.9 GeV$^2\le M^2\le1.3$ GeV^2 are constrained to be less than 4% in average. In this Borel window, the contribution originating from higher resonances (and the continuum), modeled by

$$\frac{1}{\pi}\int_{s_0^{f^{(\prime)}}}^{\infty} ds\, e^{-s/M^2}\,{\rm Im}\Pi_1^{(\prime)\rm OPE}(s)\,, \quad (28)$$

is less than 35% and the highest OPE term (with dimension six) at the quark level is no more than 1%. (ii) The deviation between the $f_1(1420)-f_1(1285)$ mass difference sum rule result and the central value of the data [14] is within 1σ error: $|(m_{f_1'}-m_{f_1})_{\rm sum\ rule}-144.6\ {\rm MeV}|\le1.5$ MeV. (iii) We use the following correlations for the light quark masses and condensates, $\langle\bar{q}q\rangle=-f_{\pi^+}^2m_{\pi^+}^2/[2(\overline{m}_u+\overline{m}_d)]$ and $\langle\bar{u}u+\bar{s}s\rangle=-(0.7\sim0.8)f_{K^+}^2m_{K^+}^2/\overline{m}_s$, where $f_{\pi^+}=(130.41\pm20)$ MeV and $f_{K^+}=(156.1\pm0.8)$ MeV [14]. Fig. 1 shows the allowed region in the $(\overline{m}_s,\theta)$ plane, where $\overline{m}_s$ corresponds to the scale $\mu=1$ GeV. The variations of the relevant input parameters are given by (27) and δ, s_0. The constraints of the fit are the above-mentioned three points. We obtain the bounds for the strange quark mass and singlet-octet mixing angle: 125 MeV $\le\overline{m}_s(1\ {\rm GeV})\le$ 230 MeV (i.e. 95 MeV $\le\overline{m}_s(2\ {\rm GeV})\le$ 174 MeV) and $2^\circ\le\theta\le68^\circ$. It is interesting to note that the allowed parameter spaces are quite small for the ranges: $\overline{m}_s(1\ {\rm GeV})\le130$ MeV and $\overline{m}_s(1\ {\rm GeV})\ge220$ MeV.

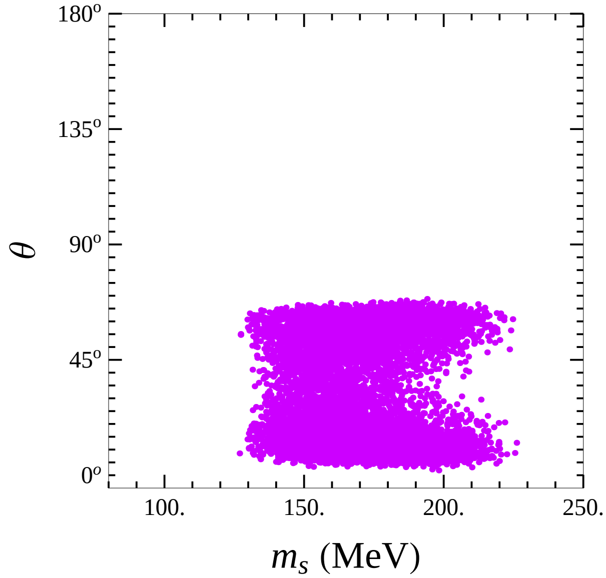

Figure 1: The allowed region in the $(\overline{m}_s, \theta)$ plane derived from the $f_1(1420) - f_1(1285)$ mass difference sum rule, where we generate 4×10^6 random points and 8055 points are satisfied with the constraints within $\overline{m}_s \in [115 \text{ MeV}, 250 \text{ MeV}]$, $\theta \in [0^\circ, 180^\circ]$ and the allowed parameter spaces (see the text for details). The $\overline{m}_s$ is at the scale 1 GeV.

4. Summary. We have estimated the strange quark mass from the analysis of the $f_1(1420) - f_1(1285)$ mass difference QCD sum rule. We have expanded the OPE series up to dimension six, where the term with dimension zero is up to $\mathcal{O}(\alpha_s^3)$, and the dimension 4 terms are up to $\mathcal{O}(\alpha_s^2)$. The bounds for the strange quark mass and mixing angle are obtained to be 125 MeV $\leq \overline{m}_s(1 \text{ GeV}) \leq$ 230 MeV (i.e. 95 MeV $\leq \overline{m}_s(2 \text{ GeV}) \leq$ 174 MeV) and $2^\circ \leq \theta \leq 68^\circ$. The lower bound for the strange quark mass sufficiently improves the current PDG result.

In addition, we have adopted two different strategies for determining the mixing angle θ: (i) Using the Gell-Mann-Okubo mass formula and the $K_1(1270) - K_1(1400)$ mixing angle $\theta_{K_1} = (-34 \pm 13)^\circ$ which was extracted from the data for $\mathcal{B}(B \to K_1(1270)\gamma), \mathcal{B}(B \to K_1(1400)\gamma), \mathcal{B}(\tau \to K_1(1270)\nu_\tau)$, and $\mathcal{B}(\tau \to K_1(1420)\nu_\tau)$, the result is $\theta = (23^{+17}_{-23})^\circ$. (ii) On the hand, from the analysis of the ratio of $\mathcal{B}(f_1(1285) \to \phi\gamma)$ and $\mathcal{B}(f_1(1285) \to \rho^0\gamma)$, we have $\bar{\alpha} = \theta_i - \theta = \pm(15.8^{+4.5}_{-4.6})^\circ$, i.e., $\theta = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$. Combining these two analyses, we deduce the mixing angle $\theta = (19.4^{+4.5}_{-4.6})^\circ$.

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