

Center clusters in the Yang-Mills vacuum

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Abstract

Properties of local Polyakov loops for $SU(2)$ and $SU(3)$ lattice gauge theory at finite temperature are analyzed. We show that spatial clusters can be identified where the local Polyakov loops have values close to the same center element. For a suitable definition of these clusters the deconfinement transition can be characterized by the onset of percolation in one of the center sectors. The analysis is repeated for different resolution scales of the lattice and we argue that the center clusters have a continuum limit.

1 Introductory remarks

Understanding the mechanisms that drive the transition to the deconfined regime is one of the great open problems of QCD. In particular with the expected new results from the experiments at the RHIC, LHC and GSI facilities also the theoretical side is challenged to contribute to our understanding of confinement and the transition to the deconfined phase.

Since phase transitions are non-perturbative phenomena, the applied methods must be non-perturbative approaches. A particularly powerful technique is the lattice formulation of QCD, where numerical simulations have become reliable quantitative tools of analysis.

An interesting idea, which is partly rooted in the lattice formulation, is the Svetitsky-Jaffe conjecture [1] which links the deconfinement transition of an $SU(N)$ gauge theory in $d + 1$ dimensions to the magnetic transition of a d -dimensional spin system which is invariant under the center group \mathbb{Z}_N . The spins of the system are related [2] to the local Polyakov loops, which are static quark sources in the underlying gauge theory.

Having identified an effective spin system which describes $SU(N)$ gauge theory at the deconfinement transition, it is natural to ask whether one can turn the argument around and identify characteristic features of a spin system directly in the corresponding gauge theory. Furthermore one may analyze whether the gauge-spin relation holds only at the critical temperature T_c or also in a finite range of temperatures around T_c .

A particular property of many discrete spin systems is the percolation of suitably defined clusters of spins at the magnetic transition. Since the spin systems relevant for gauge theories have the discrete \mathbb{Z}_N invariance, one may expect to find some kind of percolation phenomenon for center degrees of freedom at the deconfinement transition of the gauge theories. Indeed, for the case of $SU(2)$ lattice gauge theory studies of percolation properties can be found in the literature [3, 4, 5], and more recently first results for $SU(3)$ [6] as well as full QCD [7] were presented.

Establishing finite clusters below T_c and percolating infinite clusters above T_c gives rise to a tempting interpretation of the deconfinement transition: The size of finite clusters in the confining phase might be related to the maximal distance one can place a quark and an anti-quark source such that they still have a non-vanishing vacuum expectation value. For larger distances the two sources always end up in different clusters and average to zero independently. Above T_c there exists an infinite cluster and with a finite probability the two sources are correlated also at arbitrary distances such that they can move freely.

However, the above sketched picture hinges crucially on the scaling proper-

ties of the center clusters – a question that so far has not been addressed in the literature. A spin system has an intrinsic scale: The lattice constant of the underlying grid. In lattice gauge theory the situation is different: There one is interested in studying the system for finer and finer lattices in order to learn about the continuum limit. For the percolation picture this implies that when measured in lattice units, the clusters have to be larger for finer lattices. Only then the size of the cluster in physical units, e.g., the diameter of the cluster multiplied with the lattice constant in fm can approach a finite value and can be assigned a physical meaning. If no such scaling behavior can be established the clusters are merely lattice artifacts.

In this article we compare for SU(3) and SU(2) lattice gauge theory the evidence for the existence of center clusters and their percolation at T_c . Particular focus is put on the analysis of the scaling behavior of the clusters. We study the flow of the cluster parameters as a function of the lattice spacing and demonstrate that a continuum limit for the cluster picture is plausible.

2 Conventions and setting of our analysis

In our analysis we explore pure SU(3) and SU(2) lattice gauge theory at temperatures below and above the deconfinement transition. The basic observable we analyze is the local Polyakov loop $L(\vec{x})$ defined as

$$L(\vec{x}) = \text{Tr} \prod_{t=1}^{N_t} U_4(\vec{x}, t) . \quad (1)$$

$L(\vec{x})$ is the ordered product of the SU(3) or SU(2) valued temporal gauge variables $U_4(\vec{x}, t)$ at a fixed spatial position \vec{x} , where N_t is the number of lattice points in time direction and Tr denotes the trace over color indices. The loop $L(\vec{x})$ thus is a gauge transporter that closes around compactified time. Often also the spatially averaged loop P is considered, which we define as

$$P = \frac{1}{V} \sum_{\vec{x}} L(\vec{x}) , \quad (2)$$

where V is the spatial volume. Due to translational invariance P and $L(\vec{x})$ have the same vacuum expectation value.

The Polyakov loop corresponds to a static quark source and its vacuum expectation value is (after a suitable renormalization) related to the free energy F_q of a single quark, $\langle L(\vec{x}) \rangle = \langle P \rangle \propto \exp(-F_q/T)$, where T is the temperature (the Boltzmann constant is set to 1 in our units). Below the critical temperature

T_c quarks are confined and F_q is infinite, implying $\langle L(\vec{x}) \rangle = \langle P \rangle = 0$. The transition from the confined to the deconfined phase is of first order for the case of SU(3), while it is second order for SU(2) gauge theory.

The deconfinement transition of pure Yang-Mills theory may also be interpreted as the spontaneous breaking of center symmetry. For SU(3) the elements z of the center group \mathbb{Z}_3 are a set of three phases, $z \in \{1, e^{i2\pi/3}, e^{-i2\pi/3}\}$, while for SU(2) we have the center group \mathbb{Z}_2 with $z \in \{1, -1\}$. In a center transformation all temporal links in a fixed time slice are multiplied with an element z of the center group. While the action and the path integral measure are invariant under a center transformation, the local and averaged Polyakov loops transform non-trivially as

$$L(\vec{x}) \longrightarrow z L(\vec{x}) \quad \text{and} \quad P \longrightarrow z P. \quad (3)$$

The non-vanishing expectation value $\langle L(\vec{x}) \rangle = \langle P \rangle \neq 0$, which we find above T_c , thus signals the spontaneous breaking of the center symmetry.

In our study we analyze the behavior of the local Polyakov loops $L(\vec{x})$ using quenched SU(3) and SU(2) configurations at finite temperature. For both gauge groups we use the Lüscher-Weisz gauge action [8] on lattices with different sizes ranging from $20^3 \times 6$ to $40^3 \times 12$. For SU(3) the lattice constant a was determined in [9] using the Sommer parameter, while for SU(2) we express dimensionful quantities using suitable powers of the string tension σ at zero temperature. For SU(3) the critical temperature determined in [10] is used, for SU(2) it was determined using the Polyakov loop susceptibility [5]. All errors we show are statistical errors determined with single elimination Jackknife.

3 Distribution properties of local Polyakov loops

We begin our analysis with studying the distribution properties of the local Polyakov loops $L(\vec{x})$ defined in (1). While $L(\vec{x})$ is a real number for the gauge group SU(2) it is complex for SU(3). For the latter case we decompose the local Polyakov loops into modulus and phase,

$$L(\vec{x}) = \rho(\vec{x}) e^{i\varphi(\vec{x})}. \quad (4)$$

In [6] it was demonstrated that the distribution of the modulus $\rho(\vec{x})$ is a rather unspectacular quantity. It is very well described by the Haar measure distribution $P(\rho) = \int dU \delta(\rho - |\text{Tr}U|)$, where dU denotes the integration according to Haar measure over SU(3) group elements U . In particular the distribution of the modulus is almost entirely insensitive to temperature, lattice volume and

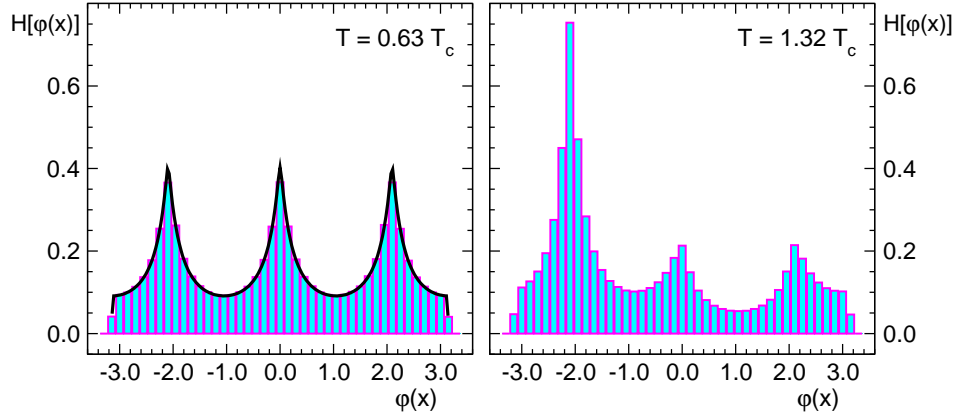


Figure 1: Histograms for the distribution of the phases of local Polyakov loops for the case of SU(3). The results are from our $40^3 \times 6$ SU(3) ensembles and we compare two temperatures, $T = 0.63 T_c$ (lhs. plot) and $T = 1.32 T_c$ (rhs.). The curve superimposed on the low temperature histograms is the Haar measure distribution $P(\varphi) = \int dU \delta(\varphi - \arg \text{Tr} U)$. The high temperature results are for the sector of gauge configurations characterized by the SU(3) center element $z = e^{-i2\pi/3}$.

resolution scale [6]. Thus we conclude that the first order transition of SU(3) lattice gauge theory into the deconfined phase is not driven by a change in the distribution of the modulus.

The situation is different for the distribution of the phase $\varphi(\vec{x})$, where indeed we observe a strong change in the distribution as one crosses into the deconfined phase. This is illustrated in Fig. 1, where we show histograms $H[\varphi(\vec{x})]$ for the distribution of the phases $\varphi(\vec{x})$ comparing the two temperatures $T = 0.63 T_c$ and $T = 1.32 T_c$. Below T_c (lhs. plot), the distribution shows three pronounced peaks located at the center phases $-2\pi/3, 0$ and $+2\pi/3$. The whole $T = 0.63 T_c$ histogram perfectly matches the Haar measure distribution $P(\varphi) = \int dU \delta(\varphi - \arg \text{Tr} U)$ which we superimpose as a full curve on the histograms in the lhs. plot of Fig. 1.

Above T_c (rhs. plot) the distribution changes: One of the peaks has grown, while the other two peaks have shrunk. We stress at this point, that the selection which peak becomes enhanced is a matter of spontaneous symmetry breaking. In the rhs. plot of Fig. 1 the system has chosen the sector which is characterized by a phase $e^{-i2\pi/3}$ of the spatially averaged Polyakov loop P (compare Eq. (2)). In case one of the other two center sectors is selected, the whole distribution in

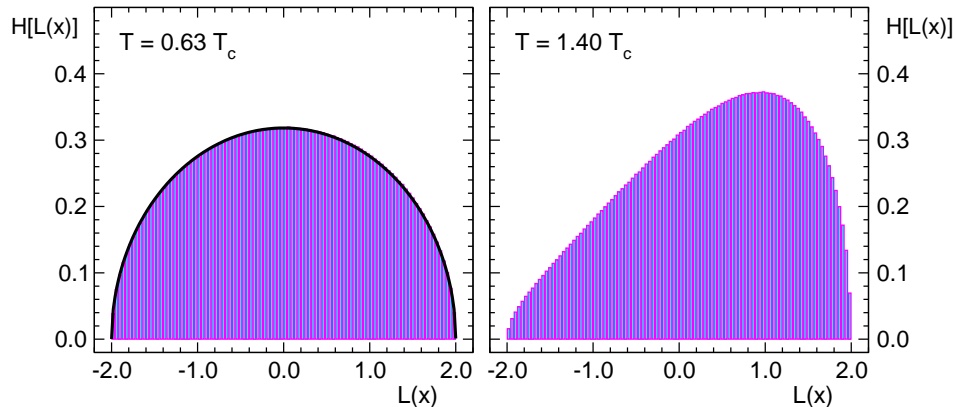


Figure 2: Histograms for the distribution of the local Polyakov loops for the case of $SU(2)$. The results are from our $40^3 \times 6$ $SU(2)$ ensembles and we compare two temperatures, $T = 0.63 T_c$ (lhs. plot) and $T = 1.40 T_c$ (rhs.). The curve superimposed on the low temperature histograms is the Haar measure distribution $P(L) = \int dU \delta(L - \text{Tr } U)$. The high temperature results are for the sector of gauge configurations characterized by the $SU(2)$ center element $z = +1$.

the rhs. plot is shifted periodically by $\pm 2\pi/3$. It is interesting to note, that also above T_c we still see the subdominant peaks which correspond to the center sectors that were not selected in the act of spontaneous symmetry breaking. This already hints at the possibility that spatial bubbles with phases $\varphi(\vec{x})$ near the subdominant center phases might exist also above T_c .

Let us now come to the case of $SU(2)$. There the local Polyakov loops $L(\vec{x})$ are real and we can directly look at their distribution. In Fig. 2 we show histograms $H[L(\vec{x})]$ for the distribution of $L(\vec{x})$, again comparing two temperatures, $T = 0.63 T_c$ (lhs. plot) and $T = 1.4 T_c$ (rhs.). Similar to the $SU(3)$ case we find that below T_c the distribution very closely follows the Haar measure distribution $P(L) = \int dU \delta(L - \text{Tr } U)$, which we superimpose as a full curve in the corresponding plot. Above T_c we observe a deformation of the distribution favoring positive values. However, again we stress that this is a manifestation of spontaneous symmetry breaking, since here we use high temperature configurations which are characterized by a positive value of the spatially averaged Polyakov loop P . In case the system spontaneously selects the sector of the other center element $z = -1$, which is characterized by negative values of P above T_c , the distribution in the rhs. plot of Fig. 2 would display its peak at

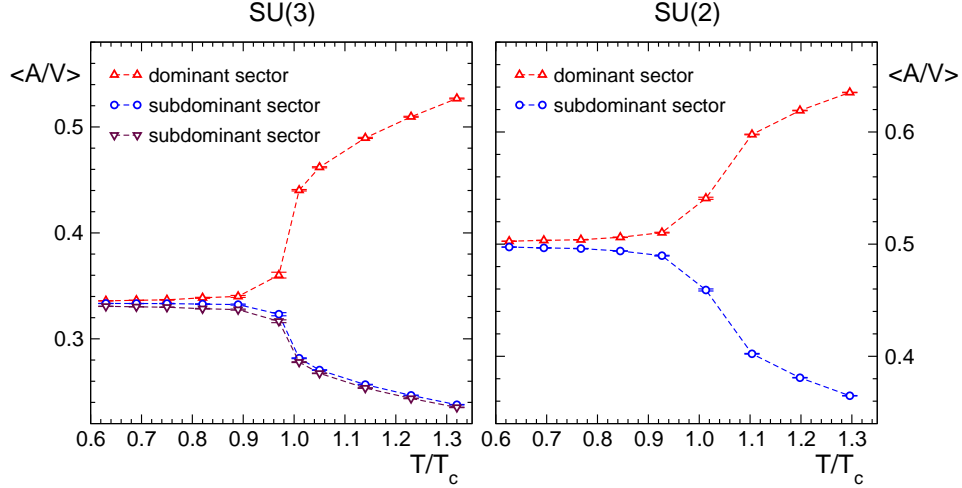


Figure 3: Abundance A of sites in each sector normalized by the spatial volume V as a function of temperature. The results are for our $40^3 \times 6$ lattices and we compare the case of SU(3) (lhs. plot) with SU(2) gauge theory (rhs.).

negative values of $L(\vec{x})$.

The spontaneous symmetry breaking, which leads to a non-vanishing expectation value of the Polyakov loop above T_c , can now be directly related to the distributions of the local Polyakov loops $L(\vec{x})$. In particular the relative distributions in the three (two) center sectors for gauge group SU(3) (gauge group SU(2)) drive the change of the expectation value of the Polyakov loop. The three sectors of SU(3) are here defined by the three intervals $[-\pi, -\pi/3]$, $[-\pi/3, +\pi/3]$ and $[\pi/3, \pi]$, the phases $\varphi(\vec{x})$ can fall into. The two sectors for the case of SU(2) are distinguished by the sign of $L(\vec{x})$.

Below T_c all three (two) sectors are populated with an equal number of sites \vec{x} . When all center sectors are equally populated, the Polyakov loop expectation value vanishes as the center elements add up to zero, i.e., $1 + e^{i2\pi/3} + e^{-i2\pi/3} = 0$ for SU(3), and $1 + (-1) = 0$ for SU(2). Above T_c one of the center sectors is more populated and thus the contributions do no longer add up to zero. In Fig. 3 we show the abundance A of sites in each sector normalized by the spatial volume V as a function of temperature and compare SU(3) (lhs. plot) and SU(2) (rhs.). For low temperature each of the sectors is for the case of SU(3) populated with roughly a third of the lattice sites (half of the lattice sites for SU(2)). Near T_c one of the sectors starts to increase the abundance of

sites, while the other sectors become depleted. It is interesting to note that for the case of SU(3), where the transition is first order, the curves show a rather sudden change near T_c , while for SU(2), where the transition is of second order, the behavior is smoother (as expected).

4 Center clusters

In the previous section we have studied the distribution of the local Polyakov loops $L(\vec{x})$. When analyzing the abundance of sites assigned to the different center sectors we found that near T_c one of the sectors starts to become more populated, thus giving rise to the non-vanishing expectation value of the Polyakov loop. In this section we now analyze the connectedness properties of sites \vec{x}, \vec{y} where the values of the local loops $L(\vec{x}), L(\vec{y})$ fall in the same center sector. For that purpose we assign sector numbers $n(\vec{x})$ to the sites \vec{x} . For the case of SU(3) the sector numbers can have three values $n(\vec{x}) \in \{-1, 0, +1\}$ assigned according to

$$n(\vec{x}) = \begin{cases} -1 & \text{for } \varphi(\vec{x}) \in [-\pi + \delta, -\pi/3 - \delta], \\ 0 & \text{for } \varphi(\vec{x}) \in [-\pi/3 + \delta, \pi/3 - \delta], \\ +1 & \text{for } \varphi(\vec{x}) \in [\pi/3 + \delta, \pi - \delta], \end{cases} \quad (5)$$

while for SU(2) we have two possibilities, $n(\vec{x}) \in \{-1, +1\}$, with

$$n(\vec{x}) = \begin{cases} -1 & \text{for } L(\vec{x}) \leq -\delta, \\ +1 & \text{for } L(\vec{x}) \geq +\delta. \end{cases} \quad (6)$$

If the phase $\varphi(\vec{x})$ (the local loop $L(\vec{x})$ for SU(2)) is not contained in one of the intervals, no sector number is assigned to the site \vec{x} , which then is no longer taken into account in the subsequent analysis. We can now define center clusters by assigning two neighboring sites \vec{x} and \vec{y} to the same cluster if $n(\vec{x}) = n(\vec{y})$.

Let us comment on the role which the parameter δ plays in our construction of the center clusters. The parameter δ allows one to cut those lattice sites where the corresponding local Polyakov loop does not clearly lean towards one of the center elements. For the case of SU(3) sites \vec{x} with phases near the minima of the distributions shown in Fig. 1 are cut, while for SU(2) the cut leads to a removal of sites where $L(\vec{x})$ is close to zero. In order to have a more accessible definition of how much we cut, instead of quoting a value of the parameter δ , from now on we express the cut in percent of sites that are not assigned a sector number $n(\vec{x})$, i.e., the percentage of sites that are removed from the analysis. To allow for a comparison of the values δ and the cut in %,

δ	sites cut, $0.69 T_c$	sites cut, $1.01 T_c$	sites cut, $1.22 T_c$
0.0	0.00 %	0.00 %	0.00 %
$0.1 \pi/3$	5.76 %	5.67 %	5.57 %
$0.2 \pi/3$	11.67 %	11.51 %	11.28 %
$0.3 \pi/3$	17.85 %	17.61 %	17.25 %
$0.4 \pi/3$	24.48 %	24.21 %	23.72 %
$0.5 \pi/3$	31.83 %	31.46 %	30.86 %
$0.6 \pi/3$	40.18 %	39.76 %	39.06 %

Table 1: Comparison of the cut parameter values δ and the fraction of cut sites in %. We show the SU(3) results for our $30^3 \times 6$ lattices and compare different temperatures.

we list the corresponding numbers for $30^3 \times 6$ at three temperatures in Table 1. The percentage of points that are cut for a given δ shows a mild temperature dependence, and when referring to these numbers in the subsequent text we quote an average value. The volume dependence is negligible.

Introducing such a cut for the analysis of cluster properties of course immediately raises the question whether such a cut does not destroy the physics one wants to study. In order to address this question we show in the lhs. plot of Fig. 4 the results for the Polyakov loop as a function of temperature for different amounts of lattice sites removed when turning on δ (we show results for SU(3) on $40^3 \times 6$). It is obvious that even a cut of almost 40 % of lattice points leads to only a small reduction of the expectation value of the Polyakov loop, indicating that the cut indeed removes only the rather unimportant fluctuations between the center values. One can even go one step further and replace the local loop $L(\vec{x})$ by the nearest center element. The result is shown in the rhs. plot of Fig. 4, again comparing different values for the cut. It is obvious that the center elements alone are sufficient to reproduce most of the Polyakov loop expectation value. Repeating the same analysis for SU(2) leads to equivalent results [5].

Having convinced ourselves that the cut does not destroy the physics we want to analyze, let us add a few more comments on the role of the cluster parameter δ . It is obvious that a small value of δ will produce denser clusters as more points are available for forming the clusters. As one increases δ the clusters become thinner and smaller. One of the motivations of this analysis is to study a possible characterization of the deconfinement transition as a percolation phenomenon. This is an idea that has been widely explored in the context of spin systems, where for many models the magnetic transition may be

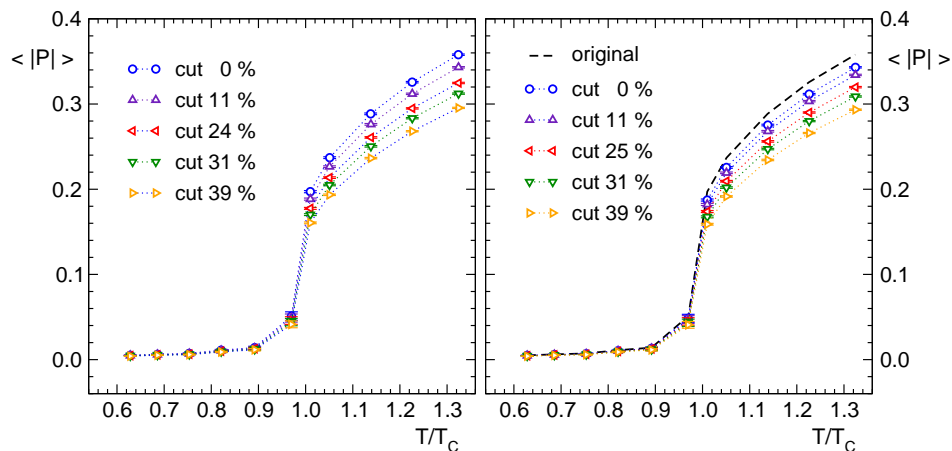


Figure 4: Lhs. plot: Expectation value of the modulus of the Polyakov loop when only those sites are taken into account that survive the cut defined in (5). Rhs.: Expectation value of the Polyakov loop when using the nearest center elements instead of the local loop $L(\vec{x})$. Both figures are for SU(3) on $40^3 \times 6$ lattices and we compare different values of the cut.

characterized by the percolation of suitably defined clusters. For these systems it is well known that a naive cluster definition, where neighboring sites with equal spin values are assigned to the same cluster, gives rise to clusters that are too dense, such that the percolation- and the magnetic transitions do not coincide. Only if the clusters are "thinned out" the two critical temperatures will agree. In particular one may use the Fortuin-Kasteleyn cluster construction [11], which is now understood to give the correct cluster description of the magnetic transition in Potts models [12]. The parameter δ which we introduce in our cluster construction plays exactly the same role as the more educated cluster definitions in spin systems, such as the Fortuin-Kasteleyn construction. As a matter of fact the introduction of a free parameter for controlling the cluster density, similar to our prescription, has been discussed in the literature [13].

There is, however, an important difference between the analysis of percolation in spin systems and in lattice gauge theory. While in the former case there is a fixed scale, the lattice spacing of the ferromagnet one studies, in the analysis of lattice gauge theories one is interested in performing the continuum limit, i.e., the limit of vanishing lattice constant $a \rightarrow 0$. The cluster picture only has a chance for a reasonable continuum limit, if the cluster diameter in lattice units diverges as one approaches $a = 0$. Only in that case the cluster

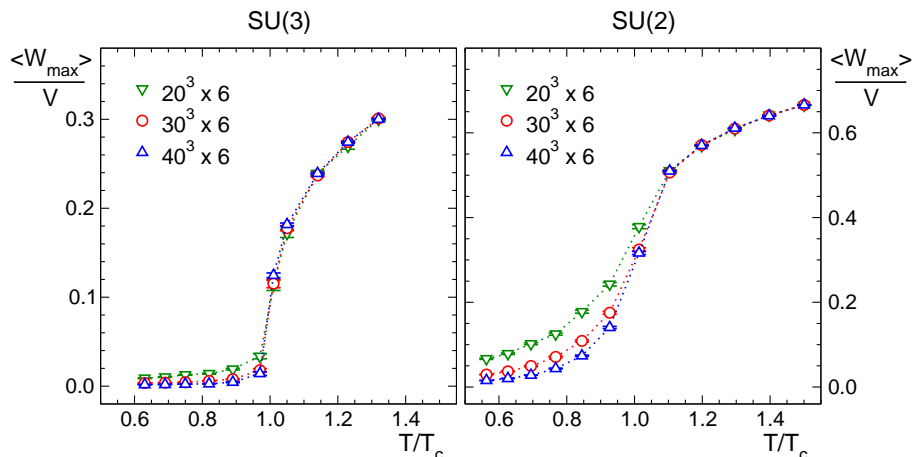


Figure 5: Weight W_{max} of the largest cluster normalized by the volume as a function of temperature. We compare the case of SU(3) (lhs. plot) with SU(2) gauge theory (rhs.) and display data for three different volumes. The cuts we use are 39 % of sites for SU(3) and 48 % for SU(2).

diameter in physical units, which is obtained as the product of the diameter in lattice units with the lattice constant a , can have a finite limit. As we will demonstrate in the next section, our cluster definition with the parameter δ is suitable for such an analysis of a possible continuum limit.

Having discussed our cluster construction and the role of the parameter δ , we now have a first look at a possible percolation behavior near T_c . For that purpose we analyze the weight W_{max} of the largest cluster, i.e., the number of sites in the largest cluster, as a function of temperature. In Fig. 5 we display the expectation value of W_{max} normalized by the spatial volume V as a function of T . We show the results for SU(3) in the lhs. plot (for a cut of 39 %) and the results for SU(2) on the rhs. (for a cut of 48 %). In both cases we compare three different volumes.

Below T_c we find that $\langle W_{max} \rangle / V$ depends on the spatial volume V . The observation that for a fixed lattice constant $\langle W_{max} \rangle / V$ decreases with increasing volume V suggests that below T_c the clusters have a finite maximal size $\langle W_{max} \rangle \sim const.$ Above T_c the volume dependence is gone and the largest cluster fills a certain fraction of the total volume with a finite density that keeps growing as one further increases the temperature. This absence of a volume dependence indicates that the clusters are percolating above T_c . For a direct analysis of the percolation probability as function of temperature see [5, 6, 7].

5 Continuum limit of the center clusters and the percolation picture

In the previous section we have demonstrated that for suitably defined center clusters percolation sets in at T_c . So far the free parameter δ which we introduced in our cluster construction was chosen arbitrarily. Now we change our approach and use a physical scale to set the cluster parameter δ . The scale we use is the diameter of the clusters in physical units. With such a prescription we can compare properties of center clusters on lattices with different resolution a .

In order to define the diameter of the clusters we consider the correlation functions of points within the same cluster, belonging to the center sector characterized by the sector number n ,

$$C_n(r) = \frac{1}{3N_n} \sum_{\vec{x}} \sum_{\mu=1}^3 \Delta_n(\vec{x}, \vec{x} + r\hat{\mu}). \quad (7)$$

The sector number n can have the three values $n \in \{-1, 0, +1\}$ for SU(3), while for SU(2) we have $n \in \{-1, +1\}$. By N_n we denote the total number of sites \vec{x} with section number $n(\vec{x}) = n$. The first sum runs over all lattice sites, the second one over all three spatial directions μ and by $\hat{\mu}$ we denote the corresponding unit vector. The parameter r assumes values $r = 0, 1, 2, \dots$, i.e., in (7) we consider the correlation along the coordinate axes. The function $\Delta_n(\vec{x}, \vec{y})$ is defined through

$$\Delta_n(\vec{x}, \vec{y}) = \begin{cases} 1 & \text{if } \vec{x} \text{ and } \vec{y} \text{ are in the same cluster, and this cluster is type } n, \\ 0 & \text{else.} \end{cases} \quad (8)$$

Up to some correction at short distances, these correlation functions decay exponentially in r . Similar to euclidean correlators in lattice spectroscopy, for sufficiently large r we fit them with the function

$$C_n(r) \sim A \cosh((r - L/2)/\rho), \quad (9)$$

where A and ρ are two real fit parameters and L is the number of lattice points in the spatial direction. As always, for correlation functions in a finite volume the hyperbolic cosine appears due to the periodic spatial boundary conditions which we use. An example of the correlation function and the corresponding fit is shown in Fig. 6 for the case of SU(3).

The value of $d_{lat} = 2\rho$ is now used as the diameter in lattice units. It is converted to the diameter in physical units d_{phys} by multiplication with the

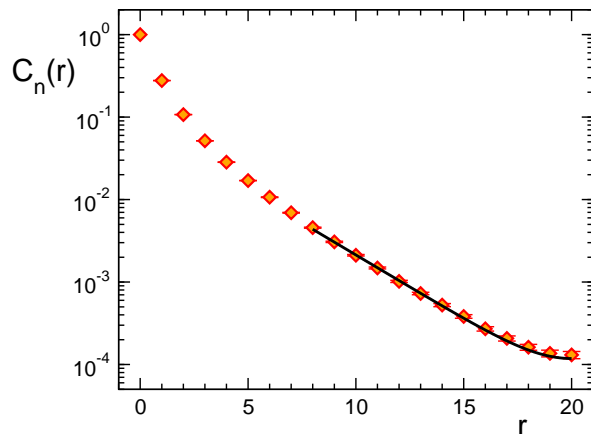


Figure 6: Example for the cluster correlation function $C_n(r)$ as a function of r (symbols), and the corresponding fit with the functional form of (9) (full curve). The data are for SU(3), $40^3 \times 12$ at $T = 0.64 T_c$ with a cut of 18 %.

lattice constant a , such that we obtain the diameter $d_{phys} = a d_{lat}$ in fm (or in units of the string tension for SU(2)). We now use the physical scale d_{phys} to set the value for the cluster parameter δ . For example we may decide to analyze clusters that have a fixed diameter of $d_{phys} = 0.5$ fm defined at some fixed temperature, e.g., $T = 0.63 T_c$. Then, working on the $T = 0.63 T_c$ ensembles, we adjust the parameter δ such, that our diameter d_{phys} has the desired value of $d_{phys} = 0.5$ fm. This procedure can now be repeated on lattices with different lattice constant a and we thus can study scaling effects and the continuum limit.

Based on that physical definition of the center clusters we now address the question how the average diameter of the clusters in physical units behaves as a function of temperature. For that purpose we proceed as described in the last paragraph and for a given lattice constant a we adjust the cluster parameter δ such that the clusters have an average diameter of $d_{phys} = 0.5$ fm (which is a reasonable mesonic scale for very heavy quarks). We then keep this value of the cluster parameter δ fixed and change the temperature, to study how the physical cluster diameter changes with T . Keeping δ fixed simply means that we work with the same cluster construction for all temperatures, i.e., the percentage of points we cut is almost the same for all T (compare Table 1). The result of this analysis is presented in Fig. 7 where we show the cluster diameter d_{phys} in fm for the case of SU(3) (lhs. plot), and for SU(2) in units of the string tension (rhs.), as a function of temperature. For both cases we find that below T_c the cluster

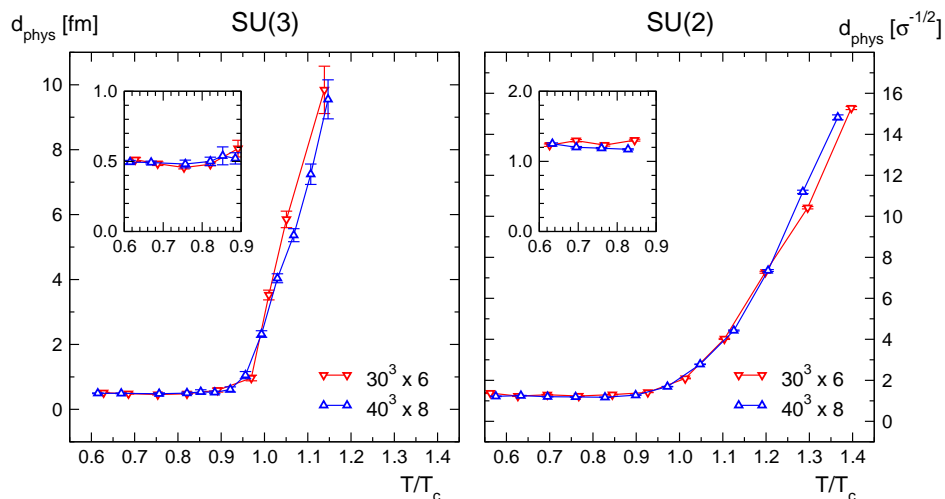


Figure 7: Diameter d_{phys} of the clusters in physical units as a function of temperature for the case of SU(3) (lhs. plot) and SU(2) (rhs.). The inserts are a zoom into the low temperature data. In both cases we compare the results for two different values of the lattice constant a .

diameter in physical units remains essentially constant at the value of d_{phys} that we dialed in at $T = 0.63 T_c$. At the critical temperature the cluster diameter starts to rise quickly indicating that the clusters start to percolate. We stress at this point that on an infinite lattice the curve for the diameter would jump to infinite slope at T_c . On a finite lattice with L lattice points in the spatial directions, the cluster diameter we can determine using the correlator $C_n(r)$ is always bounded by the finite spatial box size, giving rise to a finite slope in Fig. 7 above T_c . Comparing different values of the box size aL in physical units, we found that the results for d_{phys} at a fixed value of $T (> T_c)$ increase with L . We stress at this point that while it is obvious that at fixed T the values for d_{phys} increase with L , it is not a priori clear that for fixed L the value of d_{phys} is an increasing function of T , because we drive the temperature by changing the lattice constant a , such that also the physical volume shrinks. The fact that above T_c d_{phys} keeps growing with T is due to an increase of the density of the percolating cluster which can, e.g., be seen from Fig. 5, which shows that the number of sites in the largest cluster keeps growing above T_c .

The crucial check for our analysis of the cluster diameter in physical units is the comparison of the results obtained on ensembles with different lattice spacing a . For that purpose in Fig. 7 we compare the curves for the cluster

diameters on $30^3 \times 6$ and $40^3 \times 8$ lattices. In both cases we followed the above described procedure, adjusted the diameter to a fixed physical value d_{phys} at $T = 0.63 T_c$ and kept the corresponding cluster parameter δ for all temperatures. If the cluster picture and the percolation properties we found have a meaning in physical units, the two curves should fall on top of each other. The plots show that this is the case for both SU(3) and SU(2).

Having analyzed the percolation picture in physical units and finding universal behavior when comparing different lattice spacings, let us now turn to another conundrum related to the continuum limit of the center cluster picture. Fig. 7 suggests that the center clusters have a fixed physical diameter below T_c (at least in the range of temperatures we studied), and that this behavior can be seen on lattices with different resolution a . This poses the questions how below T_c the physical diameter

$$d_{phys} = a d_{lat} , \quad (10)$$

can remain finite in the limit $a \rightarrow 0$. Equation (10) implies that for the construction of the continuum limit the cluster diameter in lattice units d_{lat} has to diverge, in other words the clusters must become infinite in that limit.

We address this question by comparing clusters of the same physical size on lattices with different resolution a . This comparison is done at the fixed temperature $T = 0.63 T_c$. For each value of a we set the cluster parameter δ such that the physical cluster diameter d_{phys} has the desired size, e.g., $d_{phys} = 0.5$ fm. For different values of the lattice spacing a different values of δ are necessary to obtain the desired d_{phys} : For finer lattices we need larger clusters and thus can cut only fewer points than for coarse lattices. By N_{cut} we denote the number of lattice sites that are removed when constructing the clusters according to Eqs. (5) and (6). We measure the influence of the parameter δ by defining the following fraction

$$f = \frac{V - N_{cut}}{V N_c} . \quad (11)$$

This fraction measures how many sites are available (= the numerator $V - N_{cut}$), per volume (factor $1/V$), per center sector (factor $1/N_c$, where N_c is the number of center elements). In other words f measures the fraction of sites available for clusters in a given center sector. In Fig. 8 we show the flow of f with $1/N_t$. The continuum limit is reached for $1/N_t \rightarrow 0$. We compare the results for different values of d_{phys} , which for the case of SU(3) we measure in fm (lhs. plot), while for SU(2) (rhs.) we express d_{phys} in units of the string tension.

It is obvious from the plots that for a given d_{phys} the values for the available fraction f at different $1/N_t$ fall on almost perfect straight lines. Moreover,

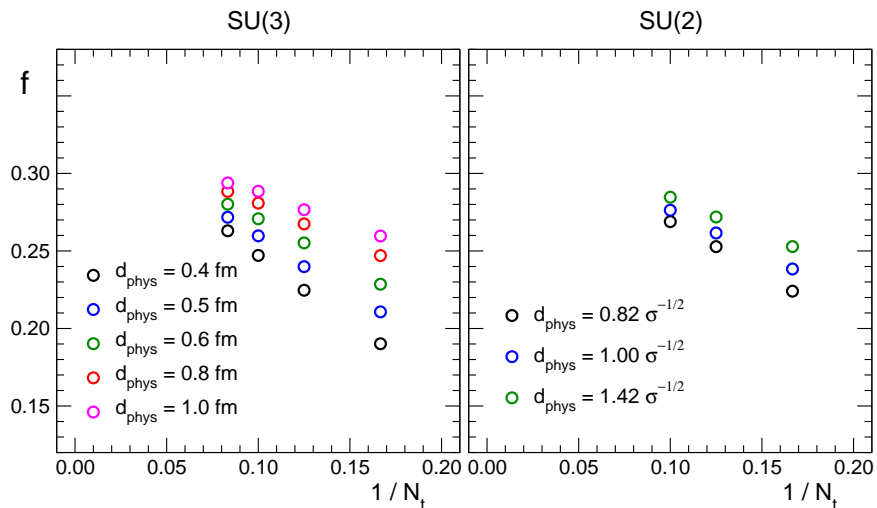


Figure 8: The fraction f of points available for clusters per center sector for the case of SU(3) (lhs. plot) and SU(2) (rhs.). We compare different physical cluster diameters, which for the case of SU(3) we give in fm, while for SU(2) the cluster diameter is given in units of the string tension.

all the lines extrapolate to roughly the same value in the continuum limit, namely $f_{cont} \sim 0.33$. We use shaded bands to mark the straight lines and their extrapolation to the continuum limit in order to indicate our estimate in the uncertainty of the analysis. We stress at this point, that the number $f_{cont} \sim 0.33$ has nothing to do with the number of colors in SU(3) – the same value is obtained also for the case of SU(2).

Let us now try to understand the significance of the value $f_{cont} \sim 0.33$. For site percolation in three dimensions the critical value for the occupation probability is $P_c = 0.316$. The fraction f of points available for clusters in each of the sectors extrapolates in the continuum limit to a value of $f_{cont} \sim 0.33$ which is just above the percolation threshold $P_c = 0.316$ ¹. This suggests that the clusters indeed grow to infinity in lattice units as we approach $a = 0$, and a continuum limit of the center clusters may be possible. We stress again that our calculation should be viewed only as a first indication that a continuum limit

¹As a matter of fact it could be that f tries to extrapolate to exactly the percolation threshold 0.316, which would be an extremely beautiful result. However, with the resources currently available to us we cannot perform a calculation which is accurate enough for a determination of f_{cont} with sufficient precision to test this hypothesis.

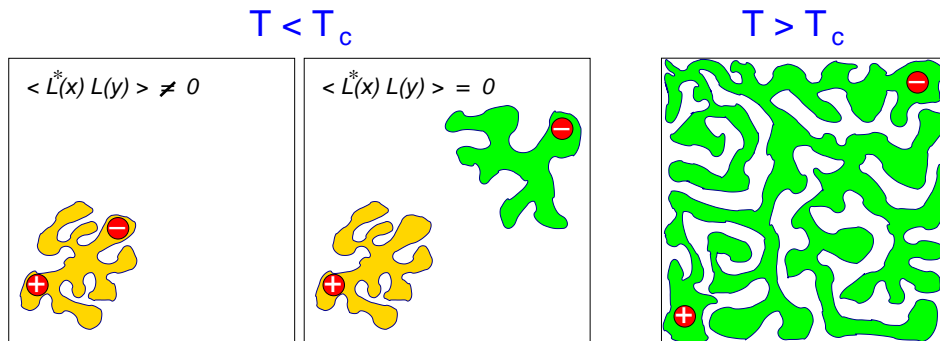


Figure 9: Illustration of the confinement mechanism and the deconfinement transition in terms of center clusters (see the text for explanations).

might be possible and certainly needs to be substantiated with a (unfortunately very expensive) high precision analysis.

6 Summary and conclusions

In this paper we analyze distribution properties of the local Polyakov loop in SU(3) and SU(2) lattice gauge theory at finite temperature. At each spatial lattice site we identify the nearest center element and study cluster properties of neighboring sites in the same center sector. While all center sectors are populated equally below T_c , above T_c one of the sectors is selected spontaneously and becomes more populated at the expense of the other, subdominant sectors. After introducing the cluster parameter δ one can construct suitable clusters which start percolating at the deconfinement transition.

We address the question whether a continuum limit for the clusters is possible by using the cluster diameter in physical units as a physical scale. Working on lattices with different lattice constant a we adjust the cluster parameter δ such that the cluster diameter has a fixed value in physical units. When approaching the continuum limit we find that the fraction of sites which are available for clusters extrapolates to a value just above the percolation threshold. This implies that in lattice units the clusters become infinite in the continuum limit, while in physical units they remain constant (below T_c). This finding indicates that the cluster picture could indeed have a well defined continuum limit. When analyzing the cluster diameter in physical units as a function of temperature, we find it is essentially constant below T_c and becomes infinite above.

The center clusters and their percolation at the deconfinement temperature

give rise to a simple picture for confinement and the deconfinement transition, which we illustrate in Fig. 9. Below T_c (lhs. and center panels in Fig. 9) the center clusters have a characteristic finite size. If one places two static sources $L(\vec{x})$ and $L(\vec{y})^*$ at a distance $|\vec{x} - \vec{y}|$ which is small enough to fit into one of the clusters (lhs. panel), the center phase information is the same at both positions \vec{x} and \vec{y} and cancels in the correlator $\langle L(\vec{x})L(\vec{y})^* \rangle$ which thus can have a non-vanishing expectation value. If the distance $|\vec{x} - \vec{y}|$ is too large to fit into a single cluster (center panel), then the two sources will always end up in different clusters. Consequently $L(\vec{x})$ and $L(\vec{y})^*$ will be subject to independent fluctuations of the center phase such that $\langle L(\vec{x})L(\vec{y})^* \rangle$ averages to zero. This averaging does not imply that the correlator vanishes abruptly above a fixed value of $|\vec{x} - \vec{y}|$, because the sizes of the clusters fluctuate giving rise to the well known exponential decay of $\langle L(\vec{x})L(\vec{y})^* \rangle$. Above T_c (rhs. panel in Fig. 9), the center clusters percolate and thus provide a coherent center information for arbitrary large distances. As a consequence, above T_c there are non-vanishing contributions to $\langle L(\vec{x})L(\vec{y})^* \rangle$ at arbitrary large distances $|\vec{x} - \vec{y}|$, and the sources are deconfined.

For a further analysis of the center cluster picture several directions need to be explored. As already discussed, the numerical evidence presented here can be improved considerably by a large scale precision study of the behavior of the center clusters using ensembles in a wide range of the lattice constant a . In particular the behavior of the fraction f of available sites for clusters per center sector should be explored closer to the continuum limit in order to precisely determine the value it extrapolates to. A second highly important question is how the center clusters change when full QCD with dynamical fermions is considered. In this case the deconfinement transition turns into a crossover and this different behavior should be reflected in the cluster properties. Preliminary results for that case can be found in [7] and a more detailed account on the dynamical case is in preparation.

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