Private Discovery of Common Social Contacts

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Abstract. The increasing use of computing devices for social interactions fuels the proliferation of online social applications, yet prompts a number of privacy concerns. One common problem occurs when two unfamiliar users, in the process of establishing social relationships, want to assess their social proximity by discovering mutual social contacts. In this paper, we introduce *Private Contact Discovery*, a novel cryptographic primitive that lets two users, on input their respective contact lists, learn their common contacts (if any), and nothing else. We present an efficient and provably secure construction, that (i) prevents arbitrary list manipulation by means of contact certification, and (ii) guarantees user authentication and revocability. Following a rigorous cryptographic treatment of the problem, we define *contact-hiding* security and prove it for our solution, under the RSA assumption in the Random Oracle Model (ROM). We also show that other related cryptographic techniques are unsuitable in this context. Experimental analysis attests to the practicality of our technique, which achieves computational and communication overhead almost linear in the number of contacts.

1 Introduction

The increasing volume of electronic interactions and digital contents shared on the Web motivates the need for efficient privacy-enhancing techniques. One interesting problem occurs when two users want to *privately discover their common contacts*: in doing so, users should learn only contacts they share.

We consider, for instance, the discovery of mutual social-network *friends*. Online social networks provide friends with services to share interests, activities, or pictures, and help them build and reflect social relations. Popular social network websites, such as Facebook, Linkedin, or MySpace, involve millions of active users, who access their services *ubiquitously* — e.g., 150 of 500 million Facebook users access it from their mobile devices. Other projects, such as Nokia's Awarenet [1], aim at letting users in physical proximity interact using their mobile phones, without relying on a central server or using an Internet connection.

One of the first steps toward establishing social-network relationships is to verify the existence of mutual friends. Consider the following scenarios: (1) a social network user wants to extend her network and is willing to establish friendships with other users with whom she has common *friends*; (2) a mobile phone user would like to interact with users in physical proximity (e.g., in a bar or on the subway), given that they have some common friends in a given social network, such as Facebook. One crucial problem, in these scenarios, is discovering mutual friends in a *privacy-preserving* manner.

A naïve solution would require users to reveal their friends to each other. Clearly, this solution does not preserve users' privacy, since their complete lists would be exposed. Another trivial solution would employ and trust a central server to find and output the mutual friends. However, a central server is not necessarily trusted and is not always be available. Also, such a server would learn not only users' friends, but also which users become friends, how, when, and where. For instance, in scenario (2) above, mobile phone users might be willing to discover their mutual friends on a social network (e.g., Facebook) but may not be connected to the Internet or they may want to operate outside the social network website.

In order to protect privacy, we are faced with a couple of fundamental issues. First, we need to prevent a malicious user from manipulating her list of friends, e.g., by populating it with her best guesses of other user's list to maximize the amount of information learned, or by "impersonating" unwarranted friendships. Then, as friend relationships may vary over time, we need an efficient mechanism allowing to *revoke* friendships.

In this paper, we introduce the concept of **Private Contact Discovery**, a novel general construct geared to preserve user privacy, not only in social network interactions, but also in any other application that uses personal contact lists. We design a cryptographic protocol involving two users, Alice and Bob, on input their contact lists, outputting only the list of mutual contacts (if any). The protocol prevents users from claiming unwarranted friendships by introducing *friendship certification.* For instance, in order to include Carol in her contact list, Alice needs to obtain a certificate from Carol attesting this friendship. Then, when Alice interacts with Bob, not only the entries in her contact list are hidden from Bob, but also the possession of corresponding certificates with respect to non-common friends (and vice-versa). Our protocol does not require any trusted server nor is bound to a specific network infrastructure, and can be used in both centralized and distributed environments.

1.1 Unsuitability of Related Concepts

The problem of Private Contact Discovery bears some resemblance with several cryptographic constructs. We review them below and discuss why they are inappropriate for the problem we consider.

Private Set Intersection (PSI). PSI techniques [9–11, 16, 18, 19, 25, 27, 28] allow two parties to compute the intersection of their input sets, such that they learn nothing beyond the set intersection (and set sizes). One assumption implicit to PSI is that parties do not manipulate their input sets. (Note that such an assumption occurs not only in presence of honest-but-curious but also arbitrarily malicious adversaries [17]). In other words, initial inputs are assumed to be valid — an unrealistic assumption for the applications we consider.

Authorized PSI (APSI). APSI [10, 11] extends PSI by ensuring that inputs are authorized by an appropriate Certification Authority (CA). Thus, unless they hold an authorization on their inputs (typically, in the form of a digital signature), parties do not learn whether the corresponding input belongs to the set intersection. Similarly, Private Intersection of Certified Sets [6] allows a trusted CA to ensure that all protocol inputs in PSI are valid and bound to each protocol participant. Note, however, that these constructs involve one single CA, whereas, every user in the context of Contact Discovery would act as an independent CA for her contacts. Also, one could think that the social network provider can act as the CA certifying friendships: this, however, does not yield a solution to the problem we consider. In fact, in APSI, authorizations are signatures: assuming that Alice and Bob are both friends with Carol, Alice would have a signature on a message in the form of "Carol→Alice", while Bob – on "Carol→Bob". Therefore, since (Authorized) Private Set Intersection techniques only output matching elements, they do not yield a solution for privately discovering common contacts (in fact, messages representing common friendships would not match).

Affiliation-Hiding Authentication (AHA). AHA protocols, also called *Secret Handshakes* (SH), allow two parties with certificates issued by the same organization—called *Group Authority* (GA)—to *privately* authenticate each other. Specifically, one party can prove to the other that it has a valid certificate, yet this proof hides the identity of the issuing organization, unless the other party also has a valid certificate from the same organization [2, 3, 7, 21–23, 26, 32, 33, 36, 38]. Other protocols efficiently support multiple credentials, i.e, multiple Group Authorities [5, 24, 30], and are more closely related to the Contact Discovery problem. Specifically, Jarecki and Liu [24] introduce a multiple-credential Affiliation-Hiding Authentication scheme which achieves practical overhead (linear in the number of credentials), and is secure under GapDH assumption. Recently, Manulis, Pinkas, and Poettering [30] proposed another multiple-credential AHA, secure under RSA assumption. Thus, one could think that Private Contact Discovery can be instantiated with an efficient multiple-credential AHA scheme. For instance, every user could act as a Group Authority (GA) and issue credentials as a friendship certification: whenever two users want to discover whether or not they have common contacts, they execute AHA on input their credentials. However, this approach presents several shortcomings. In fact, multiple-credential AHA schemes, such as [24, 30], assume that GAs are unconditionally trusted and always follow protocol specification. While this assumption might be realistic in classic AHA scenarios (where GAs are often considered trustworthy authorities, such as courts or investigation agencies), it is not reasonable, in the context of Contact Discovery, to trust all social network users. Consider the case of the AHA scheme in [24]: in the process of obtaining credentials from GAs, users need to surrender their secret keys. These keys are not dependent on the specific group but are valid for all of them. If Eve certifies Bob to be her friend, she would obtain Bob's secret keys, thus, she may impersonate Bob and/or test Bob's friendship with other users. Although recent results in [32,33] relax some of the trust assumptions on GAs in AHA protocols, it is not clear how to efficiently extend them to the multiple credential setting.

Friend-of-Friend. Prior work has attempted to solve problems similar to the one considered in this paper. Von Arb et al. [37] present a mobile social networking platform which enables *Friend-of-Friend* (FoF) detection in physical proximity. Privacy of friend lists is provided using PSI techniques [20, 27]. As discussed earlier, this approach fails to effectively guarantee privacy, as contact lists can be artificially expanded. Freedman and Nicolosi [15] propose two solutions for the FoF problem, in the context of trust establishment in email whitelisting. One solution is based on hash functions and symmetric encryptions, the other on bilinear maps. Both solutions leverage friendship attestation but do not support user revocation — a necessary requirement in our context. Also note that, as opposed to our protocol: (1) their solution based on symmetric encryption allows users to maliciously transfer attestations to other users, and (2) their technique using bilinear maps involves a quadratic number of bilinear map operations.

Non-Cryptographic Techniques. Besides the work focused on protecting privacy by means of cryptographic techniques, other solutions targeting the discovery of common contacts have been proposed in different and broader contexts. Some techniques address the Friend-of-Friend problem with none or unclear privacy properties [8, 29]. Other solutions analyze, to a higher extent, social relationships, without focusing on privacy. For instance, [35] uses random walks to discover *communities* in large social-network graphs, [39, Chapter 12] formalizes the problem of dynamically identifying core communities (i.e., sets of entities with frequent and consistent interactions), [40] builds a prediction model to identify certain social structures, e.g., friendship ties and family circles, while [13] attempts at identifying communications that substantiate social relationship types.

Remark 1. In this paper, we focus on the problem of Private Contact Discovery. Although prior work has analyzed many privacy concerns in the context of social networking, we highlight the need for a cryptographic treatment of them. We define formal privacy requirements and provide provably secure solutions with practical overhead, which can be used to address more complex challenges.

1.2 Contribution and Organization

This paper makes several contributions. First, we formally define Private Contact Discovery, a cryptographic primitive that lets two users discover their mutual contacts, without leaking information on any other contacts, and without relying on a central server. We approach the problem from a formal point of view, providing rigorous privacy definitions and security model. We define *contact-hiding* as the main property of Contact Discovery. Finally, we propose a very efficient solution, secure under the RSA assumption in ROM, which also supports efficient revocation. Performance analysis attests to the practicality of our technique, which incurs computational and communication complexities almost linear in the number of alleged contacts. Our solution is based on a recent construct, namely Index-Hiding Message Encoding (IHME) [30, 31].

Paper Organization. After preliminaries in Section 2, we introduce Private Contact Discovery and present our solution in Section 3. Next, we formalize the security model for Private Contact Discovery in Section 4. Section 5 concludes the paper. In Appendix, we review an IHME construction and present the proof of contact-hiding security of our solution.

2 Preliminaries

In this section, we present cryptographic assumptions and building blocks.

Definition 1 (RSA Assumption on Safe Moduli). *Let* $\mathsf{RSA\text{-}G}(\kappa')$ *be a probabilistic algorithm that outputs pairs* (N, e)*, where* $N = PQ$ *for random* κ'*-bit primes* $P \neq Q$ *such that* $P = 2P' + 1, Q = 2Q' + 1$ *for primes* P', Q'*, and* $e \in \mathbb{Z}_{\varphi(N)}$ *is coprime to* $\varphi(N)$*. The RSA-success probability of a PPT solver* $\mathcal{A}$ *is defined as*

$$\mathsf{Succ}^{\mathsf{rsa}}_{\mathcal{A}}(\kappa') = \Pr\big[(N, e) \leftarrow \mathsf{RSA\text{-}G}(\kappa');\ z \xleftarrow{\$} \mathbb{Z}_N;\ m \leftarrow \mathcal{A}(N, e, z);\ m^e = z \pmod N\big].$$

The RSA assumption on Safe Moduli *states that the maximum RSA-success probability* $\mathsf{Succ}^{\mathsf{rsa}}(\kappa')$ *(defined over all PPT solvers* $\mathcal{A}$*) is negligible in* κ'*.*

One important building block of our protocol are 'Index-Hiding Message Encoding' (IHME) schemes, recently introduced by Manulis, Pinkas, and Poettering [30], which we review below. Definition 2 recalls the concept of 'Index-Based Message Encoding' (IBME) [30], an encoding technique that combines a set of input messages, $m_1, \ldots, m_n \in \mathcal{M}$ (where $\mathcal{M}$ is the message space), into a single data structure $\mathcal{S}$. Any message can be individually recovered from $\mathcal{S}$ using its *index*, which is arbitrarily chosen from an index set $\mathcal{I}$ specified at encoding-time.

Definition 2 (Index-Based Message Encoding). *An* index-based message encoding *scheme* $(\mathsf{iEncode}, \mathsf{iDecode})$ *over an index space* $\mathcal{I}$ *and a message space* $\mathcal{M}$ *consists of two efficient algorithms:*

$\mathsf{iEncode}(\mathcal{P})$**:** *On input a tuple of index/message pairs* $\mathcal{P} = \{(i_1, m_1), \ldots, (i_n, m_n)\} \subseteq \mathcal{I} \times \mathcal{M}$*, with distinct indices* $i_1, \ldots, i_n$*, this algorithm outputs an encoding* $\mathcal{S}$*.*

$\mathsf{iDecode}(\mathcal{S}, i)$**:** *On input of an encoding* $\mathcal{S}$ *and an index* $i \in \mathcal{I}$ *this algorithm outputs a message* $m \in \mathcal{M}$*.*

An index-based message encoding scheme is correct *if* $\mathsf{iDecode}(\mathsf{iEncode}(\mathcal{P}), i_j) = m_j$ *for all* $j \in \{1, \ldots, n\}$ *and all tuples* $\mathcal{P} = \{(i_1, m_1), \ldots, (i_n, m_n)\} \subseteq \mathcal{I} \times \mathcal{M}$ *with distinct indices* i_j*.*

Further, [30] defines IBME schemes that provide *index-hiding* security – *'Index-Hiding Message Encoding'* (IHME) schemes. Informally, an IHME scheme guarantees that no adversary, by inspecting an IBME structure $\mathcal{S}$, can learn any useful information about the deployed indices, even if she knows some of the indices and/or messages. We refer to Appendix A for the formal definition of the index-hiding property and for the polynomial-based construction of IHME from [30]. This is defined such that $\mathcal{I} = \mathcal{M} = \mathbb{F}$ for an arbitrary finite fields $\mathbb{F}$ (e.g., $\mathbb{F} = GF(p)$ in [30]), and provides the index-hiding property in an information-theoretic sense. Our protocol uses the IHME construction from [30] in a black-box way, thus proposing an interesting application of this recent primitive. During our experimental analysis, we will also consider several optimizations recently proposed in [31].

3 Private Contact Discovery

3.1 Contact Discovery: Syntax and Correctness

A Contact Discovery Scheme CDS is defined as a tuple of four algorithms:

$\mathsf{Init}(1^\kappa)$: It is an algorithm executed once by each user U. On input of a security parameter 1^κ, it initializes internal parameters $U.\mathsf{params}$ and clears U's contact revocation list, i.e., $U.\mathsf{crl} = \emptyset$. $U.\mathsf{crl}$ is distributed and authenticated to other users (i.e., all users have always access to up-to-date $U.\mathsf{crl}$). In contrast, $U.\mathsf{params}$ is private to U.

$\mathsf{AddContact}(U, V)$: It is an algorithm executed by user U. On input user V, it adds V to U's contact list. In addition, a corresponding contact certificate $\mathsf{cc}_{U \to V}$ is output and given to V.

$\mathsf{RevokeContact}(U, V)$: It is an algorithm executed by user U. On input user V, the contact revocation list of U is updated as $U.\mathsf{crl} \leftarrow U.\mathsf{crl} \cup \{V\}$.

$\mathsf{Discover}(V \leftrightarrow V')$: It is an interactive algorithm (protocol), executed between users V and V', to discover common contacts. V's private input is $(\mathsf{role}, \mathsf{CL}, \mathsf{partner})$, where $\mathsf{role} \in \{\mathsf{init}, \mathsf{resp}\}$ specifies the role of the session as initializer or responder, contact list CL is a set of pairs of the form $(U, \mathsf{cc}_{U \to V})$, for some users U, and $\mathsf{partner}$ is the name/id of the supposed protocol partner. All values $\mathsf{cc}_{U \to V}$ are contact certificates previously obtained as output of $\mathsf{AddContact}(U, V)$. V''s private input is $(\mathsf{role}', \mathsf{CL}', \mathsf{partner}')$, defined analogously.

Users keep track of the state of created $\mathsf{Discover}(\mathsf{role}, \mathsf{CL}, \mathsf{partner})$ protocol sessions π through session variables that are initialized as follows: $(\pi.\mathsf{role}, \pi.\mathsf{CL}, \pi.\mathsf{partner}) \leftarrow (\mathsf{role}, \mathsf{CL}, \mathsf{partner})$, $\pi.\mathsf{state} \leftarrow \mathsf{running}$, $\pi.\mathsf{SCL} \leftarrow \emptyset$, and $\pi.\mathsf{id}$ is set to the own identity. After the protocol completes, $\pi.\mathsf{state}$ is updated to either $\mathsf{rejected}$ or $\mathsf{accepted}$. In the latter case, *shared contact list* $\pi.\mathsf{SCL}$ holds a non-empty set of user identifiers.

Definition 3 (Correctness of CDS). *Assume that users V and V' interact in a* $\mathsf{Discover}$ *protocol on input* $(\mathsf{role}, \mathsf{CL}, \mathsf{partner})$ *and* $(\mathsf{role}', \mathsf{CL}', \mathsf{partner}')$*, respectively. Let π and π' denote the corresponding sessions. Let* $\mathsf{CL}_\cap$ *denote the set of users (contacts) U that appear in both* CL *and* CL' *with the restriction that neither* $\mathsf{partner}$ *nor* $\mathsf{partner}'$ *are contained in the respective contact revocation lists. A* CDS *scheme is* correct *if: (1) π and π' complete in the same state, which is* $\mathsf{accepted}$ *iff* $(\mathsf{role} \neq \mathsf{role}' \wedge \mathsf{CL}_\cap \neq \emptyset \wedge \mathsf{partner} = \pi'.\mathsf{id} \wedge \mathsf{partner}' = \pi.\mathsf{id})$*, and (2) if the sessions accept then* $\pi.\mathsf{SCL} = \pi'.\mathsf{SCL} = \mathsf{CL}_\cap$.

3.2 Protocol Specification

We now present our CDS construction and describe the instantiation of Init, $\mathsf{AddContact}$, $\mathsf{RevokeContact}$, and $\mathsf{Discover}$ algorithms.

We assume that $\mathsf{AddContact}$ and $\mathsf{RevokeContact}$ sessions among (honest) users of CDS are protected by secure channels, whereas, during $\mathsf{Discover}$, the channel does not need to be confidential. Note that many applications that would use CDS, such as social networks or further group applications, already provide an authentication infrastructure for their users, e.g., they deploy a PKI or use some password-based techniques. Such an authentication infrastructure can then be used for various types of communication, including the execution of CDS protocols. With this assumption in mind, we can now focus on the core functionality of the CDS scheme, namely the private discovery of shared contacts, for which potential attacks may be mounted by other application users, i.e., from the inside.

Let $\kappa, \kappa' \in \mathbb{N}$ denote security parameters, where κ' is polynomially dependent on κ. As a building block, our construction utilizes the $\mathsf{IHME} = (\mathsf{iEncode}, \mathsf{iDecode})$ scheme from [30] (see also Section 2 and Appendix A), defined over finite field $\mathbb{F} = GF(p) \cong \mathbb{Z}_p$, where p is the smallest prime number satisfying $p > 2^{2\kappa' + \kappa}$. In addition, the protocol makes use of two hash functions

$$H : \{0,1\}^* \to [0, p-1] \qquad \text{and} \qquad H^* : \{0,1\}^* \to [0, p-1],$$

modeled as random oracles in our security analysis. For convenience, for each $N \in \mathbb{N}$ of length $2\kappa'$, we define:

$$H_N : \{0,1\}^* \to \mathbb{Z}_N;\ x \mapsto H^*(N \,\|\, x) \bmod N.$$

The four algorithms and protocols of CDS are instantiated as follows:

$\mathsf{Init}(1^\kappa)$. The setup routine run by each user mainly consists of the generation of safe RSA parameters. Given security parameter κ', two κ'-bit safe primes $P = 2P' + 1$ and $Q = 2Q' + 1$ are picked randomly. The RSA modulus is set to $N = PQ$, and a pair $e, d \in \mathbb{Z}_{\varphi(N)}$ is chosen s.t. $ed = 1 \pmod{\varphi(N)}$. Observe that $\varphi(N) = (P-1)(Q-1) = 4P'Q'$.
The largest element order in $\mathbb{Z}_N^\times$ is $\lambda(N) = \mathrm{lcm}(P-1, Q-1) = 2P'Q' = \varphi(N)/2$. [22] and [32] show that for half of the elements $g \in \mathbb{Z}_N^\times$ it holds that $\mathrm{ord}(g) = \lambda(N)$ and $-1 \notin \langle g \rangle$, i.e., $\mathbb{Z}_N^\times \cong \langle -1 \rangle \times \langle g \rangle$. Let the Init algorithm find such $g \in \mathbb{Z}_N^\times$ (e.g., by random sampling and testing) and assign $U.\mathsf{params} \leftarrow (N, e, d, g)$.
Finally, the algorithm clears U's contact revocation list by setting $U.\mathsf{crl} \leftarrow \emptyset$.

$\mathsf{AddContact}(U, V)$. This routine is executed by user U, on input $U.\mathsf{params} = (N, e, d, g)$ and identifier id of a user V. Contact certificate $\mathsf{cc}_{U \to V} = (N, e, g, \sigma_V)$ with $\sigma_V = (H_N(\mathsf{id}))^d \bmod N$, i.e., the Full-Domain-Hash RSA signature [4] on id, is confidentially handed out to V.

$\mathsf{RevokeContact}(U, V)$. User U revokes given user V by inserting V into its contact revocation list: $U.\mathsf{crl} \leftarrow U.\mathsf{crl} \cup \{V\}$. It is assumed that an up-to-date version of this list is distributed authentically to all users.

$\mathsf{Discover}(V \leftrightarrow V')$. The contact discovery protocol is executed between two users V and V' with inputs $(\mathsf{role}, \mathsf{CL}, \mathsf{partner})$ and $(\mathsf{role}', \mathsf{CL}', \mathsf{partner}')$, respectively (see Section 3.1 for a description of parameters). The protocol is specified in detail in Figure 1. Each user obtains its contact list and, for each entry in it, she parses the friendship certificate (lines 2–3). Next, our protocol combines Okamoto's technique [34] for RSA-based identity-based key agreement (lines 4–5 and 18) with a special padding scheme (introduced in [12]) to hide the size of deployed RSA-moduli (lines 6–7 and 17). Note that several instances of Okamoto's protocol can be run in parallel — one for each contact in contact list CL — and all transferred messages are IHME-encoded into a single structure before transmission (lines 8 and 10). Upon receiving the IMHE-encoded structure, each user, for every unrevoked certificate, decodes the messages (line 16) and removes the probabilistic padding applied in lines 6–7 (line 17). As we demonstrate in Section 3.3, values r calculated in line 18 are equal for both protocol participants, when computed for the same common contact. Confirmation messages (c_0, c_1) are derived from this value (lines 19–20), IHME-encoded in lines 23–24, and verified after the last communication round (line 28). Each common contact is then added in the SCL list (line 29). Note that contact revocation is handled in lines 15 and 22. Finally, the protocol terminates with "accept", unless SCL is empty, in which case "reject" is returned (lines 31-34).

3.3 Protocol Correctness

Suppose that users V, V' have valid contact certificates $\mathsf{cc}_{U \to V}, \mathsf{cc}_{U \to V'}$, respectively, for a shared contact U. Then $\mathsf{cc}_{U \to V} = (N, e, g, \sigma_V)$ and $\mathsf{cc}_{U \to V'} = (N, e, g, \sigma_{V'})$ for common parameter set N, e, g. In a $\mathsf{Discover}(V \leftrightarrow V')$ protocol session, V would receive $\vartheta = (-1)^{b'} g^{t'} \sigma_{V'}$ from V' (lines 5 and 17) and compute $r = (\vartheta^e / H_n(\mathsf{partner}))^{2t}$ (line 18). From $\sigma_{V'} = H_n(\mathsf{partner})^d \bmod N$ (see $\mathsf{AddContact}$ algorithm) it follows that $r = g^{2ett'}$. User V' obtains the same value r by executing analogous computations (with $\mathsf{partner}'$ and t'). The protocol's correctness is now implied by IHME's correctness, and verifiable by inspection of Figure 1. The security analysis of the protocol is postponed to Section 4.3, after the specification of the security model.

3.4 Protocol Efficiency and Performance Analysis

We now discuss the efficiency of our CDS scheme. We focus on the protocol $\mathsf{Discover}$ since the algorithm Init is executed only once per user, while the al-

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    $V$ ON INPUT $(\mathsf{init}, \mathsf{CL}, \mathsf{partner})$:                                          $V'$ ON INPUT $(\mathsf{resp}, \mathsf{CL}', \mathsf{partner}')$:

1   $\mathcal{P} \leftarrow \emptyset,\ \mathcal{T} \leftarrow \emptyset$                                                   $\mathcal{P} \leftarrow \emptyset,\ \mathcal{T} \leftarrow \emptyset$
2   FOR ALL $(U, \mathsf{cc}_{U \to V}) \in \mathsf{CL}$:                                              FOR ALL $(U, \mathsf{cc}_{U \to V'}) \in \mathsf{CL}'$:
3     PARSE $\mathsf{cc}_{U \to V}$ AS $(N, e, g, \sigma_V)$                                          PARSE $\mathsf{cc}_{U \to V'}$ AS $(N, e, g, \sigma_{V'})$
4     $(b, t) \leftarrow_R \mathbb{Z}_2 \times \mathbb{Z}_{N/2}$                                            $(b, t) \leftarrow_R \mathbb{Z}_2 \times \mathbb{Z}_{N/2}$
5     $\vartheta \leftarrow (-1)^b g^t \sigma_V \bmod N$                                            $\vartheta \leftarrow (-1)^b g^t \sigma_{V'} \bmod N$
6     $k \leftarrow_R [0, \lfloor p/N \rfloor - 1]$                                                  $k \leftarrow_R [0, \lfloor p/N \rfloor - 1]$
7     $\theta \leftarrow \vartheta + kN$                                                            $\theta \leftarrow \vartheta + kN$
8     $\mathcal{P} \leftarrow \mathcal{P} \cup \{(N, \theta)\}$                                             $\mathcal{P} \leftarrow \mathcal{P} \cup \{(N, \theta)\}$
9     $\mathcal{T} \leftarrow \mathcal{T} \cup \{(U, N, e, t)\}$                                          $\mathcal{T} \leftarrow \mathcal{T} \cup \{(U, N, e, t)\}$
10  $\mathcal{M}_V \leftarrow \mathsf{iEncode}(\mathcal{P})$                       $\xrightarrow{\mathcal{M}_V}$        $\mathcal{M}_{V'} \leftarrow \mathsf{iEncode}(\mathcal{P})$
11                                                     $\xleftarrow{\mathcal{M}_{V'}}$
12  $\mathsf{sid} \leftarrow \mathcal{M}_V \,\|\, \mathcal{M}_{V'}$                                             $\mathsf{sid} \leftarrow \mathcal{M}_V \,\|\, \mathcal{M}_{V'}$
13  $\mathcal{P}' \leftarrow \emptyset,\ \mathcal{T}' \leftarrow \emptyset$                                                 $\mathcal{P}' \leftarrow \emptyset,\ \mathcal{T}' \leftarrow \emptyset$
14  FOR ALL $(U, N, e, t) \in \mathcal{T}$:                                            FOR ALL $(U, N, e, t) \in \mathcal{T}$:
15    IF $\mathsf{partner} \notin U.\mathsf{crl}$:                                                 IF $\mathsf{partner}' \notin U.\mathsf{crl}$:
16      $\theta \leftarrow \mathsf{iDecode}(\mathcal{M}_{V'}, N)$                                         $\theta \leftarrow \mathsf{iDecode}(\mathcal{M}_V, N)$
17      $\vartheta \leftarrow \theta \bmod N$                                                      $\vartheta \leftarrow \theta \bmod N$
18      $r \leftarrow (\vartheta^e / H_N(\mathsf{partner}))^{2t} \bmod N$                            $r \leftarrow (\vartheta^e / H_N(\mathsf{partner}'))^{2t} \bmod N$
19      $c_0 \leftarrow H(\mathsf{sid} \,\|\, r \,\|\, 0)$                                                  $c_0 \leftarrow H(\mathsf{sid} \,\|\, r \,\|\, 0)$
20      $c_1 \leftarrow H(\mathsf{sid} \,\|\, r \,\|\, 1)$                                                  $c_1 \leftarrow H(\mathsf{sid} \,\|\, r \,\|\, 1)$
21      $\mathcal{T}' \leftarrow \mathcal{T}' \cup \{(U, N, c_1)\}$                                         $\mathcal{T}' \leftarrow \mathcal{T}' \cup \{(U, N, c_0)\}$
22    ELSE: $c_0 \leftarrow_R [0, p-1]$                                                 ELSE: $c_1 \leftarrow_R [0, p-1]$
23    $\mathcal{P}' \leftarrow \mathcal{P}' \cup \{(N, c_0)\}$                                             $\mathcal{P}' \leftarrow \mathcal{P}' \cup \{(N, c_1)\}$
24  $\mathcal{M}'_V \leftarrow \mathsf{iEncode}(\mathcal{P}')$                      $\xrightarrow{\mathcal{M}'_V}$        $\mathcal{M}'_{V'} \leftarrow \mathsf{iEncode}(\mathcal{P}')$
25                                                     $\xleftarrow{\mathcal{M}'_{V'}}$
26  $\mathsf{SCL} \leftarrow \emptyset$                                                           $\mathsf{SCL} \leftarrow \emptyset$
27  FOR ALL $(U, N, c_1) \in \mathcal{T}'$:                                            FOR ALL $(U, N, c_0) \in \mathcal{T}'$:
28    IF $c_1 = \mathsf{iDecode}(\mathcal{M}'_{V'}, N)$:                                       IF $c_0 = \mathsf{iDecode}(\mathcal{M}'_V, N)$:
29      $\mathsf{SCL} \leftarrow \mathsf{SCL} \cup \{U\}$                                             $\mathsf{SCL} \leftarrow \mathsf{SCL} \cup \{U\}$
30
31  IF $\mathsf{SCL} \neq \emptyset$ THEN                                                  IF $\mathsf{SCL} \neq \emptyset$ THEN
32    TERMINATE WITH “ACCEPT”                                          TERMINATE WITH “ACCEPT”
33  ELSE                                                               ELSE
34    TERMINATE WITH “REJECT”                                          TERMINATE WITH “REJECT”
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Fig. 1. Specification of $\mathsf{Discover}(V \leftrightarrow V')$.

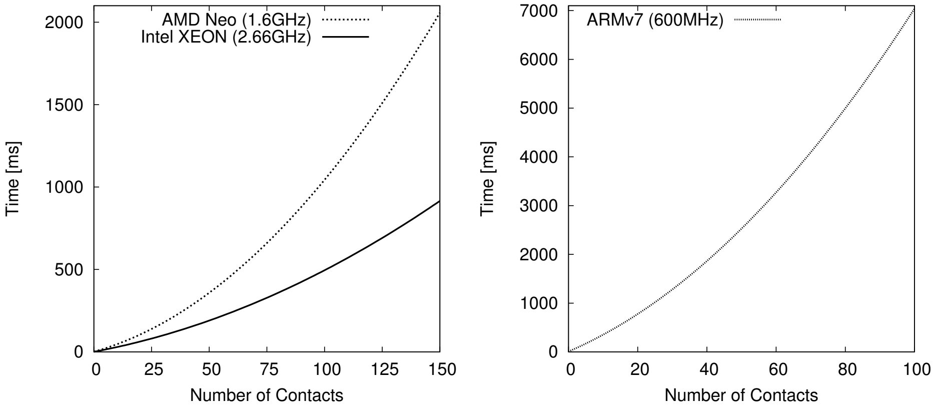
gorithms AddContact and RevokeContact only once for each added or removed contact, respectively, and can, furthermore, be performed off-line.

Computational Complexity and Bandwidth Requirements. The computational complexity of the Discover protocol is essentially related to the number of (relatively more expensive) exponentiations, executed for each contact in lines 5 and 18. Any user V needs to compute $2|\mathsf{CL}|$ modular exponentiations with modulus size $2\kappa'$, where $|\mathsf{CL}|$ denotes the number of contacts. If the polynomial-based IHME constructions from [30] or [31] are used to encode messages of the Discover protocol, the polynomial interpolation would only require inexpensive operations, such as multiplications in $\mathbb{F}$. (Specifically, the number of multiplications in $\mathbb{F}$ would amount to $O(|\mathsf{CL}|^2)$, which can be considered small in practice, as discussed in [30,31]).

The overall communication complexity of the Discover protocol (including the IHME-encoded transmission) is linear in the number of contacts. More precisely, each user sends and receives approximately $4|\mathsf{CL}|(2\kappa' + \kappa)$ bits. Observe that this value can be lowered to $4|\mathsf{CL}|(\kappa' + \kappa)$ by shortening confirmation messages c_0, c_1 to κ bits, in lines 19 and 20 (see also [31]).

Note that users need to keep revocation lists of their contacts up-to-date. In practice, this does not impose a significant overhead, as revocation lists grow incrementally and include only (short) identifiers of revoked contacts. Thus, the related communication overhead is negligible compared to that of an actual Discover session.

Experimental Analysis. In addition to our asymptotic analysis, we also measured the performance of our scheme experimentally. To this end, we conducted several experiments involving laptops and mobile devices. Our prototypes use the recent optimizations to IHME scheme, proposed by Manulis and Poettering [31].



(a) AMD Neo and Intel Xeon

(b) ARMv7

Fig. 2. Running times of our Discover protocol on different CPUs with an increasing number of contacts. Measurements are performed for 80-bit (symmetric) security and 1024-bit RSA moduli.

Following [31], for $|\mathsf{CL}| < 100$, the overall costs for running the protocol is dominated by the time consumed in the exponentiations. If certain precomputations are allowed [31], then this bound increases to $|\mathsf{CL}| < 250$. Therefore, we argue that, in practice, the computational overhead of our CDS construction is almost linear in the number of contacts.

Figure 2 presents running times of our Discover protocol, using different CPUs: a single core of an Intel XEON CPU, an AMD NEO processor (often found in Netbook computers), and an ARMv7 CPU (typically installed on today's smartphones). All measurements were performed using the GMP library [14], thus, execution on smartphones can be even speeded up using different cryptographic libraries optimized for mobile environments. We observe that our protocol for Private Contact Discovery scales fairly well. For security level $(\kappa, \kappa') = (80, 1024/2)$, i.e., 80-bit symmetric security and 1024-bit RSA moduli, on laptops and server machines, a full protocol execution requires less than a second, even for 100 or more contacts per user. On cores with smaller footprint, e.g., on recent smartphones like Nokia's N900 (equipped with the ARMv7 processor), protocol execution with 70 contacts requires about 4 seconds, which can be considered an acceptable overhead. Note that smartphones' CPU speeds are envisioned to increase rapidly in the near future (e.g., the iPhone 4G is now equipped with a 1GHz processor). Finally, we measured that each user sends and receives around 71KB for a contact list of size 70.

In conclusion, we argue that our Private Contact Discovery solution is efficient and practical enough for actual deployment, also on smartphones widely available today. Yet, our technique does not give up solid privacy guarantees, as we show in the next section.

4 Security Model for Contact Discovery Protocols

In this section, we introduce our security model for a Contact Discovering Scheme (CDS). We formalize this notion by describing adversarial capabilities and defining Contact-Hiding security. Finally, we analyze the properties of our scheme with respect to this model.

4.1 Adversary Model

The adversary $\mathcal{A}$ is modeled as a PPT machine interacting with protocol participants and having access to the following set of queries ($\mathcal{U}$ denotes the set of honest users in the system):

$\mathsf{Discover}(U, \mathsf{role}, \mathsf{CL}, \mathsf{partner})$: This query results in initiating, on behalf of user $U \in \mathcal{U}$, a new session π of Discover. Query's input is a role identifier $\mathsf{role} \in \{\mathsf{init}, \mathsf{resp}\}$, a contact list $\mathsf{CL} \subseteq \mathcal{U}$ of users, and an identifier partner of the protocol partner. Query's output is a first protocol message M (if available).

$\mathsf{Send}(\pi, M)$: With this query, message M is delivered to session π. After processing M, the output (if any) is given to $\mathcal{A}$. The query is ignored if π is not waiting for input.

$\mathsf{Reveal}(\pi)$: This query is ignored if $\pi.\mathsf{state} = \mathsf{running}$. Otherwise, the query returns $(\pi.\mathsf{state}, \pi.\mathsf{SCL})$.

$\mathsf{RevealCC}(V, U)$: This query gives the adversary contact certificate $\mathsf{cc}_{U \to V}$ of user V for contact U. It models the possibility of selective contact corruptions.

$\mathsf{Revoke}(U, V)$: This query lets user U include user V in its contact revocation list $U.\mathsf{crl}$.

4.2 Contact-Hiding Security

Informally, the Contact-Hiding property protects users from disclosing non-matching contacts to other participants. We model CH-security with a game, following the indistinguishability approach. The goal of the adversary is to decide which of two contact lists, CL_0^* or CL_1^*, is used by some challenge session π^*. The adversary can also invoke any number of $\mathsf{Discover}$ sessions, and perform Reveal and $\mathsf{RevealCC}$ queries at will.

Definition 4 (Contact-Hiding Security). *Let* $\mathsf{CDS} = \{\mathsf{Init}, \mathsf{AddContact}, \mathsf{RevokeContact}, \mathsf{Discover}\}$*, b be a randomly chosen bit, and* $\mathcal{Q} = \{\mathsf{Discover}, \mathsf{Send}, \mathsf{Reveal}, \mathsf{RevealCC}, \mathsf{Revoke}\}$ *denote the set of queries the adversary $\mathcal{A}$ has access to. We consider the following game between a challenger and the adversary $\mathcal{A}$:*

$\mathsf{Game}^{\mathsf{ch},b}_{\mathcal{A},\mathsf{CDS}}(\kappa, n)$:

- *The challenger creates n users, denoted by $\mathcal{U} = \{U_1, \ldots, U_n\}$. The adversary $\mathcal{A}$ specifies a set $\mathcal{U}^c \subseteq \mathcal{U}$ of initially corrupted users. Let $\mathcal{U}^h = \mathcal{U} \setminus \mathcal{U}^c$.* $\mathsf{Init}(1^\kappa)$ *is run for all $U \in \mathcal{U}^h$, and, for all combinations $(U, V) \in \mathcal{U}^h \times \mathcal{U}^h$, contact certificates* $\mathsf{cc}_{U \to V}$ *are created by respective user U and given to V, each time by running the* $\mathsf{AddContact}(U, V)$ *algorithm.*
  For all $U \in \mathcal{U}^c$, the adversary sets up all parameters himself, including $U.\mathsf{crl}$*. He then specifies a list $\mathcal{L} \subseteq \mathcal{U}^h \times \mathcal{U}^c$, and for all $(U, V) \in \mathcal{L}$, algorithm* $\mathsf{AddContact}(U, V)$ *is run, and the respective certificate* $\mathsf{cc}_{U \to V}$ *is given to $\mathcal{A}$.*
- $\mathcal{A}^\mathcal{Q}$ *interacts with all (honest) users using the queries in $\mathcal{Q}$; at some point* $\mathcal{A}^\mathcal{Q}$ *outputs a tuple* $(U^*, \mathsf{role}^*, \mathsf{CL}_0^*, \mathsf{CL}_1^*, \mathsf{partner}^*)$ *where* $U^* \in \mathcal{U}^h$, $\mathsf{role}^* \in \{\mathsf{init}, \mathsf{resp}\}$, $\mathsf{CL}_0^*, \mathsf{CL}_1^* \subseteq \mathcal{U}^h$ *with* $|\mathsf{CL}_0^*| = |\mathsf{CL}_1^*|$*, and* $\mathsf{partner}^*$ *is any user id (in $\mathcal{U}$). Set* $\mathcal{D}^* = (\mathsf{CL}_0^* \setminus \mathsf{CL}_1^*) \cup (\mathsf{CL}_1^* \setminus \mathsf{CL}_0^*) = (\mathsf{CL}_0^* \cup \mathsf{CL}_1^*) \setminus (\mathsf{CL}_0^* \cap \mathsf{CL}_1^*)$ *is called the* distinguishing set*;*
- *the challenger invokes a* $\mathsf{Discover}(U^*, \mathsf{role}^*, \mathsf{CL}_b^*, \mathsf{partner}^*)$ *session π^* (and provides all needed credentials);*
- $\mathcal{A}^\mathcal{Q}$ *continues interacting via queries (including on session π^*) until it terminates and outputs bit b';*
- *the output of the game is 1 if all of the following hold; else the output is 0:*
  (a) $b = b'$,
  (b) if there is a $\mathsf{Discover}$ *session* π' *with* $\mathcal{D}^* \cap \pi'.\mathsf{CL} \neq \emptyset$ *and* $(\pi'.\mathsf{id}, \pi'.\mathsf{partner}) = (\pi^*.\mathsf{partner}, \pi^*.\mathsf{id})$ *which was in state* $\mathsf{running}$ *while π^* was in state* $\mathsf{running}$*, then neither* $\mathsf{Reveal}(\pi^*)$ *nor* $\mathsf{Reveal}(\pi')$ *was asked,*
  (c) no pair $(U, \mathsf{partner}^*)$ *for* $U \in \mathcal{D}^*$ *is contained in $\mathcal{L}$, i.e. the adversary did not ask for a contact certificate for* $\mathsf{partner}^*$ *issued by any user in the distinguishing set.*

We define

$$\mathsf{Adv}^{\mathsf{ch}}_{\mathcal{A},\mathsf{CDS}}(\kappa, n) := \left|\Pr[\mathsf{Game}^{\mathsf{ch},0}_{\mathcal{A},\mathsf{CDS}}(\kappa, n) = 1] - \Pr[\mathsf{Game}^{\mathsf{ch},1}_{\mathcal{A},\mathsf{CDS}}(\kappa, n) = 1]\right|$$

and denote with $\mathsf{Adv}^{\mathsf{ch}}_{\mathsf{CDS}}(\kappa, n)$ *the maximum advantage over all PPT adversaries $\mathcal{A}$. We say that* CDS *is* CH-secure *if this advantage is negligible in κ (for all n polynomially dependent on κ).*

Conditions (b) and (c) exclude some trivial attacks on contact hiding. Condition (b) thwarts the attack where $\mathcal{A}$ starts a $\mathsf{Discover}(U', \mathsf{role}', \mathsf{CL}', \mathsf{partner}')$ session π' with $\mathsf{CL}' \cap \mathcal{D}^* \neq \emptyset$ and $(\pi'.\mathsf{id}, \pi'.\mathsf{partner}) = (\pi^*.\mathsf{partner}, \pi^*.\mathsf{id})$, relays all messages between π^* and π', and finally asks $\mathsf{Reveal}(\pi^*)$ or $\mathsf{Reveal}(\pi')$. By protocol correctness, $\pi^*.\mathsf{SCL} = \pi'.\mathsf{SCL}$ would contain elements from $\mathcal{D}^*$, and it would be trivial to correctly decide about b. Condition (c) prevents $\mathcal{A}$ to ask for a contact certificate issued by a user $U \in \mathcal{D}^*$ for a user $V \in \mathcal{U}^c$, to simulate a protocol session on behalf of V, to relay all messages between that session and π^*, and to decide about bit b from the results.

Remark 2. One may also define a stronger notion of contact-hiding property by requiring that distinct sessions of the Discover protocol executed by the same user remain *unlinkable*. We observe that our Discover protocol in its plain form would not guarantee such unlinkability, since credentials (i.e., certified friendships) are re-used across multiple protocol executions, which also allows an efficient realization. Although linkable protocols may yield traceability concerns (with respect to eavesdropping adversaries) and leak sensitive information about users, we address this issue by executing the protocol over secure (encrypted and authenticated) channels. Nonetheless, it is an interesting open problem to design a Discover protocol, which would preserve the linear-complexity of our solution and simultaneously achieve such stronger property of unlinkability.

4.3 Security Analysis of our Protocol

Following the definitions in Section 4.2, we argue that our CDS protocol, described in Section 3, is Contact-Hiding secure. The proof is presented in Appendix B.

Theorem 1 (Contact-Hiding Security). *The* CDS *protocol in Section 3.2 is CH-secure under the RSA assumption on safe moduli, in the Random Oracle Model (ROM).*

5 Conclusion

This paper motivated the importance and introduced the concept of Private Contact Discovery. Following a cryptographic treatment of the problem, we presented an efficient and provably secure construction. During protocol design, we overcame several potential issues, such as the arbitrary expansion of contact lists, by using friendship certification. Our solution relies on Full-Domain-Hash RSA signatures and on the recent IHME primitive [30]. We also showed, through experimental evaluation, that our solution is practical enough to be deployed in real-world applications, including those running on mobile devices.

Private Contact Discovery provides a valuable privacy-preserving tool that can serve as building block for many collaborative applications, including popular social networks. Since this work represents an initial foray into Private Contact

Discovery protocols, much remains to be done. First, we plan to extend our techniques to privately discover *communities*: consider, for example, two smartphone users in proximity willing to find out whether or not they are member of the same social community (e.g., a Facebook group or Awarenet community [1]), in privacy-preserving manner. Users may receive (from a community manager) credentials for community membership, and execute our CDS protocol to discover common memberships. Also, we intend to address the (privacy-preserving) discovery of i-th grade contacts and cliques [35], which currently seems impossible without relying on some trusted third party.

References

1. A. Ahtiainen, K. Kallio, M. Kasslin, K. Leppanen, A. Richter, P. Ruuska, and C. Wijting. Awareness Networking in Wireless Environments: Means of Exchanging Information. In *IEEE Vehicular Technology Magazine*, 2009.
2. G. Ateniese, J. Kirsch, and M. Blanton. Secret Handshakes with Dynamic and Fuzzy Matching. In *NDSS*, pages 159–177, 2007.
3. D. Balfanz, G. Durfee, N. Shankar, D. K. Smetters, J. Staddon, and H.-C. Wong. Secret Handshakes from Pairing-Based Key Agreements. In *IEEE S&P*, pages 180–196, 2003.
4. M. Bellare and P. Rogaway. Random Oracles are Practical: A Paradigm for Designing Efficient Protocols. In *ACM CCS*, pages 62–73, 1993.
5. R. Bradshaw, J. Holt, and K. Seamons. Concealing Complex Policies with Hidden Credentials. In *CCS*, pages 146–157, 2004.
6. J. Camenisch and G. M. Zaverucha. Private Intersection of Certified Sets. In *Financial Cryptography*, pages 108–127, 2009.
7. C. Castelluccia, S. Jarecki, and G. Tsudik. Secret Handshakes from CA-Oblivious Encryption. In *Asiacrypt*, pages 293–307, 2004.
8. S. Chiou, S. Chang, and H. Sun. Common Friends Discovery with Privacy and Authenticity. In *IAS*, pages 337–340, 2009.
9. D. Dachman-Soled, T. Malkin, M. Raykova, and M. Yung. Efficient Robust Private Set Intersection. In *ACNS*, pages 125–142, 2009.
10. E. De Cristofaro, J. Kim, and G. Tsudik. Linear-Complexity Private Set Intersection Protocols Secure in Malicious Model. In *Asiacrypt*, pages 213–231, 2010.
11. E. De Cristofaro and G. Tsudik. Practical Private Set Intersection Protocols with Linear Complexity. In *Financial Cryptography*, pages 143–159, 2010.
12. Y. Desmedt. Securing Traceability of Ciphertexts — Towards a Secure Software Key Escrow System. In *Eurocrypt*, pages 147–157, 1995.
13. C. Diehl, G. Namata, and L. Getoor. Relationship Identification for Social Network Discovery. *AAAI*, 22(1):546–552, 2007.
14. Free Software Foundation. The GNU MP Bignum Library. `http://gmplib.org/`.
15. M. J. Freedman and A. Nicolosi. Efficient Private Techniques for Verifying Social Proximity. In *IPTPS*, 2007.
16. M. J. Freedman, K. Nissim, and B. Pinkas. Efficient Private Matching and Set Intersection. In *Eurocrypt*, pages 1–19, 2004.
17. O. Goldreich. *Foundations of Cryptography.* Cambridge Univ. Press, 2004.
18. C. Hazay and Y. Lindell. Efficient Protocols for Set Intersection and Pattern Matching with Security Against Malicious and Covert Adversaries. In *TCC*, 2008.

19. C. Hazay and K. Nissim. Efficient Set Operations in the Presence of Malicious Adversaries. In *PKC*, pages 312–331, 2010.
20. B. Huberman, M. Franklin, and T. Hogg. Enhancing Privacy and Trust in Electronic Communities. In *ACM Conference on Electronic Commerce*, 1999.
21. S. Jarecki, J. Kim, and G. Tsudik. Group Secret Handshakes or Affiliation-Hiding Authenticated Group Key Agreement. In *CT-RSA*, pages 287–308, 2007.
22. S. Jarecki, J. Kim, and G. Tsudik. Beyond Secret Handshakes: Affiliation-Hiding Authenticated Key Exchange. In *CT-RSA*, pages 352–369, 2008.
23. S. Jarecki and X. Liu. Unlinkable Secret Handshakes and Key-Private Group Key Management Schemes. In *ACNS*, pages 270–287, 2007.
24. S. Jarecki and X. Liu. Affiliation-Hiding Envelope and Authentication Schemes with Efficient Support for Multiple Credentials. In *ICALP (2)*, 2008.
25. S. Jarecki and X. Liu. Efficient Oblivious Pseudorandom Function with Applications to Adaptive OT and Secure Computation of Set Intersection. In *TCC*, pages 577–594, 2009.
26. S. Jarecki and X. Liu. Private Mutual Authentication and Conditional Oblivious Transfer. In *CRYPTO*, pages 90–107, 2009.
27. S. Jarecki and X. Liu. Fast Secure Computation of Set Intersection. In *SCN*, 2010.
28. L. Kissner and D. X. Song. Privacy-Preserving Set Operations. In *CRYPTO*, pages 241–257, 2005.
29. A. Korolova, R. Motwani, S. Nabar, and Y. Xu. Link Privacy in Social Networks. In *CIKM*, pages 289–298, 2008.
30. M. Manulis, B. Pinkas, and B. Poettering. Privacy-Preserving Group Discovery with Linear Complexity. In *ACNS*, pages 420–437, 2010.
31. M. Manulis and B. Poettering. Practical Affiliation-Hiding Authentication from Improved Polynomial Interpolation. In *To appear at ACM ASIACCS 2011*. Available at: `http://eprint.iacr.org/2010/659`.
32. M. Manulis, B. Poettering, and G. Tsudik. Affiliation-Hiding Key Exchange with Untrusted Group Authorities. In *ACNS*, pages 402–419, 2010.
33. M. Manulis, B. Poettering, and G. Tsudik. Taming Big Brother Ambitions: More Privacy for Secret Handshakes. In *PETS*, pages 149–165, 2010.
34. E. Okamoto. Key Distribution Systems Based on Identification Information. In *CRYPTO*, pages 194–202, 1987.
35. P. Pons and M. Latapy. Computing Communities in Large Networks using Random Walks. In *ISCIS*, pages 284–293, 2005.
36. G. Tsudik and S. Xu. A Flexible Framework for Secret Handshakes. In *PETS*, pages 295–315, 2006.
37. M. Von Arb, M. Bader, M. Kuhn, and R. Wattenhofer. Veneta: Serverless Friend-of-Friend Detection in Mobile Social Networking. In *WiMob*, pages 184–189, 2008.
38. S. Xu and M. Yung. k-Anonymous Secret Handshakes with Reusable Credentials. In *ACM CCS*, pages 158–167, 2004.
39. P. S. Yu, J. Han, and C. Faloutsos. *Link Mining: Models, Algorithms, and Applications*. Springer, 2010.
40. E. Zheleva, L. Getoor, J. Golbeck, and U. Kuter. Using Friendship Ties and Family Circles for Link Prediction. In *SNA-KDD*, pages 97–113, 2008.

A Index-Hiding Message Encoding

This section recalls the formal definition of Index-Hiding Message Encoding (IHME) and the polynomial-based IHME construction from [30], for which the index-hiding property holds unconditionally. We refer to Section 2 for the definition of IBME.

Definition 5 (Index-Hiding Message Encoding (IHME)). *Let* $\mathsf{IHME} = (\mathsf{iEncode}, \mathsf{iDecode})$ *denote a correct index-based message encoding scheme (IBME) over index space* $\mathcal{I}$ *and message space* $\mathcal{M}$*. Let* $b \in \{0,1\}$ *be a randomly chosen bit and* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ *be a PPT adversary that participates in the following game.*

$\mathsf{Game}^{\mathsf{ihide},b}_{\mathcal{A},\mathsf{IHME}}(\kappa)$:

- $(I_0, I_1, M', St) \leftarrow \mathcal{A}_1(1^\kappa)$ *such that* $I_0, I_1 \subseteq \mathcal{I}$ *with* $|I_0| = |I_1| = \nu$*, and* $M' = (m'_1, \ldots, m'_{|I_0 \cap I_1|})$ *with each* $m'_j \in \mathcal{M}$*; (the adversary chooses two subsets of* ν *indices each, as well as a message* m'_j *for each index* i_j *in the intersection of the sets);*
- *denote the indices in* $I_b \setminus I_{1-b}$ *as* $\{i_1, \ldots, i_r\}$ *and define* $m_1, \ldots, m_r \xleftarrow{\$} \mathcal{M}$*, (additional* $r = \nu - |I_0 \cap I_1|$ *messages are chosen uniformly at random in the message space),*
  let $\mathcal{S} \leftarrow \mathsf{iEncode}(\{(i_j, m'_j) \mid i_j \in I_0 \cap I_1\} \cup \{(i_j, m_j) \mid i_j \in I_b \setminus I_{1-b}\})$*, (the messages are encoded for the indices in* I_b*);*
- $b' \leftarrow \mathcal{A}_2(St, \mathcal{S})$ *(the adversary is given* S *and attempts to find* b*);*
- *if* $b' = b$ *then return 1 else return 0.*

The advantage of $\mathcal{A}$ *is defined as*

$$\mathsf{Adv}^{\mathsf{ihide}}_{\mathcal{A},\mathsf{IHME}}(\kappa) := \left|\Pr[\mathsf{Game}^{\mathsf{ihide},0}_{\mathcal{A},\mathsf{IHME}}(\kappa) = 1] - \Pr[\mathsf{Game}^{\mathsf{ihide},1}_{\mathcal{A},\mathsf{IHME}}(\kappa) = 1]\right| .$$

$\mathsf{Adv}^{\mathsf{ihide}}_{\mathsf{IHME}}(\kappa)$ *denotes the maximum advantage over all PPT adversaries* $\mathcal{A}$*. We say that* IHME *is* index-hiding *if this advantage is negligible in* κ*. Moreover, if* $\mathsf{Adv}^{\mathsf{ihide}}_{\mathsf{IHME}}(\kappa) = 0$ *for all* κ*, the* IHME*-scheme is* perfectly index-hiding.

For completeness, we now review an efficient implementation of IHME, proposed in [30]. The scheme offers perfect index-hiding and is based on polynomial interpolation over arbitrary finite fields $\mathbb{F}$. After setting $\mathcal{I} = \mathcal{M} = \mathbb{F}$, the index-hiding message encoding scheme $\mathsf{IHME} = (\mathsf{iEncode}, \mathsf{iDecode})$ with index space $\mathcal{I}$ and message space $\mathcal{M}$ is constructed by the following algorithms. Note that various technical optimizations for this scheme are available in [31].

$\mathsf{iEncode}(\mathcal{P})$ On input of $\mathcal{P} = \{(i_1, m_1), \ldots, (i_n, m_n)\} \subseteq \mathcal{I} \times \mathcal{M} = \mathbb{F}^2$, the encoding is defined as the list $\mathcal{S} = (c_{n-1}, \ldots, c_0)$ of coefficients of the polynomial $f = \sum_{k=0}^{n-1} c_k x^k \in \mathbb{F}[x]$ that interpolates all points in $\mathcal{P}$, i.e. $f(i_j) = m_j$ for all $(i_j, m_j) \in \mathcal{P}$. Note that this polynomial exists uniquely, i.e., the $\mathsf{iEncode}$ algorithm is deterministic.

$\mathsf{iDecode}(\mathcal{S}, i)$ On input of $\mathcal{S} = (c_{n-1}, \ldots, c_0)$ and index $i \in \mathcal{I}$, this algorithm outputs the evaluation $m = f(i) = \sum_{k=0}^{n-1} c_k i^k$ of f at position i.

B CH-Security Proof of CDS in Fig. 1

We now prove CH-Security (Definition 4) of our CDS protocol (illustrated in Fig. 1), relying on the RSA assumption on safe moduli and on the security of the IHME scheme (Definitions 1 and 5). Note that our security arguments for our CDS construct have similarities with those of the multi-credential AHA protocol in [30], which proves the "affiliation hiding" property of the proposed AHA under the same assumptions. We only sketch the proof steps that mirror those in [30], as the reader can find more details in [30] and its predecessor [22]. In contrast, we present detailed description of the steps regarding the IHME scheme. We prove CH-Security of CDS by presenting a sequence of games $\mathbf{G}_0, \ldots, \mathbf{G}_5$.

Game $\mathbf{G}_0$. We start with $\mathbf{G}_0 = \mathsf{Game}^{\mathsf{ch},b}_{\mathcal{A},\mathsf{CDS}}(\kappa, n)$, in which $\mathcal{A}$ interacts with simulator $\mathcal{C}$, which answers all queries honestly according to the specification of the game (see Definition 4).

Game $\mathbf{G}_1$. Game $\mathbf{G}_1$ is like $\mathbf{G}_0$, except that the simulation is aborted (with bit b' set at random) if there exists a $\mathsf{Discover}$ session $\pi' \neq \pi^*$ that sends out the same $\mathcal{M}$ structure as π^* (see line 10 in Figure 1).

Note that $\mathcal{M}$ sent by π^* contains for each contact in CL^*_b a specific θ value. These are almost uniformly distributed in a set of size $p \approx 2^{2\kappa'+\kappa}$. Thus, the probability of finding a collision in the $\mathcal{M}$'s is upper-bounded by $q_q/2^{2\kappa'+\kappa}$ (where q_q denotes the number of $\mathsf{Discover}$ queries), and thus negligible in κ.

Game $\mathbf{G}_2$. Let $R = (r_1, \ldots, r_k)$ denote the list of r-values for the contacts in $\mathsf{CL}^*_b \setminus \mathsf{CL}^*_{1-b} = \mathsf{CL}^*_b \cap \mathcal{D}^*$ of session π^*, as computed in line 18 of the protocol. Game $\mathbf{G}_2$ is like $\mathbf{G}_1$ except that all confirmation messages c_0, c_1 of π^* (see lines 19 and 20), computed based on the values in R, are replaced by random elements in the respective range.

By the Random Oracle Model (ROM), the modification introduced in Game $\mathbf{G}_2$ can only be detected by adversaries that can compute and query the H oracle on at least one of the r-values in R. Let $r_t \in R$ be such a value (and assume that $\mathcal{C}$ guesses t correctly). By embedding an RSA challenge into user identifiers (by programming the H_N oracle) and public user parameters (by choosing N and g appropriately), the problem of computing r_t can be reduced to the hardness of the RSA problem (see [22, Section 3] for more details). We conclude that the computational distance between $\mathbf{G}_2$ and $\mathbf{G}_1$ is polynomially dependent on $\mathsf{Succ}^{\mathsf{rsa}}(\kappa')$, and is thus negligible in κ.

Remark. We have seen that for all contacts in $\mathsf{CL}^*_b \cap \mathcal{D}^*$, the adversary cannot distinguish correct confirmation messages for π^* from random ones. In particular, she cannot compute them on her own. Thus, the protocol's output set, $\pi^*.\mathsf{SCL}$, is disjoint with $\mathcal{D}^*$.

Game $\mathbf{G}_3$. This game is like Game $\mathbf{G}_2$, except that, for session π^*, the θ-values for all contacts in $\mathsf{CL}^*_b \cap \mathcal{D}^*$ (as computed in line 7) are replaced by values uniformly random in $[0, p-1]$.

Observe from the protocol definition that the θ-values replaced in this game only affect the computation of the $r_t \in R$ (in Game $\mathbf{G}_2$), which cannot be computed and checked by the adversary anyway by a result of Game $\mathbf{G}_2$. Hence, the only detectable difference between $\mathbf{G}_2$ and $\mathbf{G}_3$ may arise from different distributions of the original and the modified θ-values. Although the original values are *not* uniformly distributed in $[0, p-1]$, their distribution is statistically indistinguishable from the uniform distribution. In [22], the corresponding statistical difference is proven to be bounded by $2^{-\kappa}$, what is negligible in κ.

Game $\mathbf{G}_4$. Game $\mathbf{G}_4$ is like $\mathbf{G}_3$, except that, for session π^*, in the IHME-encoding step in line 10, for all contacts in $\mathsf{CL}_b^* \cap \mathcal{D}^*$, the indices N are replaced by the indices N that correspond to the contacts in $\mathsf{CL}_0^* \cap D^*$. For all contacts in $\mathsf{CL}_b^* \cap \mathsf{CL}_{1-b}^*$ the indices remain unchanged.

As Games $\mathbf{G}_3$ and $\mathbf{G}_4$ are exactly the same in case $b = 0$ (as nothing is changed), in the following we assume $b = 1$. We show that, if an efficient distinguisher $\mathcal{D}^{3,4}$ that distinguishes between Game $\mathbf{G}_3$ and $\mathbf{G}_4$ exists, then we can use it to construct an adversary $\mathcal{A}^{\mathsf{ihide}}$ against IHME (see Definition 5) as follows. Adversary $\mathcal{A}^{\mathsf{ihide}}$ acts as a challenger for $\mathcal{D}^{3,4}$, i.e., $\mathcal{A}^{\mathsf{ihide}}$ sets up all users, and answers all protocol queries honestly (but following the rules of Game $\mathbf{G}_3$), with the only exception that the encoding step in line 10 for session π^* is performed by the IHME challenger (i.e., the latter receives (I_0, I_1, M') where I_0 and I_1 are the sets of indices N of CL_0^* and CL_1^*, respectively, and M' is the list of the θ-values honestly computed for the contacts in $\mathsf{CL}_0^* \cap \mathsf{CL}_1^*$), which returns an encoding $\mathcal{M}$ using either the indices corresponding to CL_0^* or CL_1^*. The bit outputted by $\mathcal{D}^{3,4}$ serves as output b' for $\mathcal{A}^{\mathsf{ihide}}$. We see that the success probability of $\mathcal{D}^{3,4}$ is bounded by $\mathsf{Adv}^{\mathsf{ihide}}_{\mathsf{IHME}}(\kappa)$, which is 0 (since the IHME construction is perfect). Hence, the computational difference between Games $\mathbf{G}_3$ and $\mathbf{G}_4$ is 0.

Game $\mathbf{G}_5$. The step between Game $\mathbf{G}_4$ and $\mathbf{G}_5$ is very similar to the previous transition: this time, it is the IHME-encoding in line 24 for which the indices N of all contacts in $\mathsf{CL}_b^* \cap D^*$ are replaced by the indices N that correspond to the contacts in $\mathsf{CL}_0^* \cap D^*$.

The difference between $\mathbf{G}_5$ and $\mathbf{G}_4$ is bounded by $\mathsf{Adv}^{\mathsf{ihide}}_{\mathsf{IHME}}(\kappa) = 0$, exactly for the same reason above.

We conclude that the computational difference between $\mathbf{G}_0$ and $\mathbf{G}_5$ is negligible in κ. Therefore, a CH-adversary cannot distinguish between $\mathsf{Game}^{\mathsf{ch},b}_{\mathcal{A},\mathsf{CDS}}$ and $\mathsf{Game}^{\mathsf{ch},0}_{\mathcal{A},\mathsf{CDS}}$ with non-negligible probability, either by analyzing the exchanged messages, or by interpreting the results of Reveal queries. As the latter game contains no information about bit b, it follows that $\mathsf{Adv}^{\mathsf{ch}}_{\mathsf{CDS}}(\kappa, n)$ is negligible in κ (for all N polynomially dependent on κ). □