Yang-Baxter $\breve{R}$ matrix, Entanglement and Yangian

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Abstract

We present a method to construct "X" form unitary Yang-Baxter $\breve{R}$ matrices, which act on the tensor product space $V_i^{j_1} \otimes V_{i+1}^{j_2}$. We can obtain a set of entangled states for $(2j_1+1)\times(2j_2+1)$-dimensional system with these Yang-Baxter $\breve{R}$ matrices. By means of Yang-Baxter approach, a 8×8 Yang-Baxter Hamiltonian is constructed. Yangian symmetry and Yangian generators as shift operators for this Yang-Baxter system are investigated in detail.

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I. INTRODUCTION

Quantum entanglement[1–4], which is a bizarre of quantum theory, has been recognized as an important resource for applications in quantum information and quantum computation processing. Quantum gates[5] are represented by unitary matrices, and they are building blocks of a quantum computer. On the other hand, the topological quantum computation(TQC) also has been studied by researchers[6]. Thus quantum computation is one of the important approaches to achieve a fault-tolerant quantum computer. This proposal relies on the existence of topological states of matter, whose quasiparticle excitations are non-Abelian anyons. Thus quasiparticles obey non-Abelian braiding statistics, and quantum gate operators are implemented by braiding quasiparticles.

Recently, Kauffman *et.al.* have shown that topological entanglement and quantum entanglement have deep relations[7–9]. The authors propose that it is more fundamental to view braid matrices(or solutions to Quantum Yang-Baxter Equation[10, 11]), which can implement topological entanglement, as universal quantum gates. For example, the authors showed that the Bell matrix is nothing, but a braid matrix, and thus braid matrix local equivalent to a Control-Not(CNOT) gate[8]. This motivated a novel way to study quantum entanglement by means of Yang-Baxter approach[12–18].

The Yangian theory established by Drinfeld offer a mathematic method for the studies about the symmetry of quantum integrable models in physics[19]. Many researchers have explored the role of Yangian operators in physics[20–22]. For example, by means of Yangian, we can investigate the symmetry for the integrable systems and shift operators. But many researchers worked on complex systems, this motivated us to search a simple system with Yangian symmetry to investigate the role of Yangian operators in this system.

In Sec. II, we will present a method for constructing the “X” form Yang-Baxter $\breve{R}$ matrices, and then we will investigate the entanglement properties in Sec. III. In Sec.IV, we construct Yang-Baxter Hamiltonian with a 8×8 “X” form Yang-Baxter $\breve{R}$ matrix, then Yangian symmetry and shift operators are studied in this Yang-Baxter system.

II. THE “X” FORM YANG-BAXTER $\breve{R}$ MATRICES

In this paper, Yang-Baxter $\breve{R}^{j_1 j_2}(\theta)$ matrix and $M^{j_1 j_2}$ matrix are $(2j_1+1)\times(2j_2+1)$-dimensional matrices acting on the tensor product $V^{j_1} \otimes V^{j_2}$, where V^{j_1} and V^{j_2} are $(2j_1 + 1)$ and $(2j_2 + 1)$

dimensional vector space, respectively. As Yang-Baxter $\breve{R}^{j_1 j_2}(\theta)$ matrix and $M^{j_1 j_2}$ matrix acting on the tensor product $V_i^{j_1} \otimes V_{i+1}^{j_2}$, we denote them by $\breve{R}_i^{j_1 j_2}(\theta)$ and $M_i^{j_1 j_2}$, respectively. The notation $I^{j_1 j_2}$ denotes $(2j_1 + 1) \times (2j_2 + 1)$-dimension identity matrix.

Let matrices $M^{j_1 j_2}$ and $M^{j_2 j_1}$ satisfying the following relations,

$$
\begin{aligned}
&[M^{j_1 j_2}]^2 = [M^{j_2 j_1}]^2 = I^{j_1 j_2} \\
&M_{12}^{j_1 j_2} M_{23}^{j_2 j_1} = M_{23}^{j_2 j_1} M_{12}^{j_1 j_2}, \quad (i.e.[M_{12}^{j_1 j_2}, M_{23}^{j_2 j_1}] = 0) \\
&M_{12}^{j_2 j_1} M_{23}^{j_1 j_2} = M_{23}^{j_1 j_2} M_{12}^{j_2 j_1}, \quad (i.e.[M_{12}^{j_2 j_1}, M_{23}^{j_1 j_2}] = 0).
\end{aligned}
\tag{1}
$$

In this paper, we set $[M^{j_1 j_2}]_{b\beta}^{a\alpha} = [M^{j_2 j_1}]_{\beta b}^{\alpha a}(-j_1 \le a, b \le j_1$ and $-j_2 \le \alpha, \beta \le j_2)$for convenience. Then two spectral-dependent Yang-Baxter $\breve{R}$ matrices via Yang-Baxterization[23–25] is obtained to be,

$$
\begin{aligned}
\breve{R}^{j_1 j_2}(\theta) = e^{-i\frac{\theta}{2}M^{j_1 j_2}} = cos\tfrac{\theta}{2} I^{j_1 j_2} - isin\tfrac{\theta}{2} M^{j_1 j_2}, \\
\breve{R}^{j_2 j_1}(\theta) = e^{-i\frac{\theta}{2}M^{j_2 j_1}} = cos\tfrac{\theta}{2} I^{j_1 j_2} - isin\tfrac{\theta}{2} M^{j_2 j_1}.
\end{aligned}
\tag{2}
$$

Here we used Tayloy expansion to derive the right hand of Eq. 2. If the matrices $M^{j_1 j_2}$ and $M^{j_2 j_1}$ are Hermitian matrices(*i.e.* $[M^{j_1 j_2}]^\dagger = M^{j_1 j_2}$ and $[M^{j_2 j_1}]^\dagger = M^{j_2 j_1}$), then we can verify that the matrices $\breve{R}^{j_1 j_2}$ and $\breve{R}^{j_2 j_1}$ are unitary(*i.e.* $\breve{R}^{j_1 j_2}(\theta)^\dagger \breve{R}^{j_1 j_2}(\theta) = \breve{R}^{j_1 j_2}(\theta) \breve{R}^{j_1 j_2}(\theta)^\dagger = I^{j_1 j_2}$ and $\breve{R}^{j_2 j_1}(\theta)^\dagger \breve{R}^{j_2 j_1}(\theta) = \breve{R}^{j_2 j_1}(\theta) \breve{R}^{j_2 j_1}(\theta)^\dagger = I^{j_2 j_1}$).

We can easily prove that $\breve{R}^{j_1 j_2}(\theta)$ and $\breve{R}^{j_2 j_1}(\theta)$ satisfy the following Yang-Baxter equation(YBE),

$$
\begin{aligned}
\breve{R}_{12}^{j_1 j_2}(\theta_1) \breve{R}_{23}^{j_2 j_1}(\theta_1 + \theta_2) \breve{R}_{12}^{j_1 j_2}(\theta_2) = \breve{R}_{23}^{j_2 j_1}(\theta_2) \breve{R}_{12}^{j_1 j_2}(\theta_1 + \theta_2) \breve{R}_{23}^{j_2 j_1}(\theta_1), \\
\breve{R}_{12}^{j_2 j_1}(\theta_1) \breve{R}_{23}^{j_1 j_2}(\theta_1 + \theta_2) \breve{R}_{12}^{j_2 j_1}(\theta_2) = \breve{R}_{23}^{j_1 j_2}(\theta_2) \breve{R}_{12}^{j_2 j_1}(\theta_1 + \theta_2) \breve{R}_{23}^{j_1 j_2}(\theta_1).
\end{aligned}
\tag{3}
$$

where parameters θ_1 and θ_2 are called as spectral parameters. For convenience, we take $M^{j_1 j_2}$ and $M^{j_2 j_1}$ as $[M^{j_2 j_1}]_{\beta b}^{\alpha a} = [M^{j_1 j_2}]_{b\beta}^{a\alpha} = e^{-i\varphi_{a\alpha}} \delta_{a,-b} \delta_{\alpha,-\beta}$. Considering the first equation in Eqs. 1, we set $\varphi_{a\alpha} = -\varphi_{-a-\alpha}$. Substituting $M^{j_1 j_2}$ and $M^{j_2 j_1}$ into the second and the third relations in Eqs. 1, we can obtain the following conditions,

$$
\begin{aligned}
\varphi_{a\alpha} + \varphi_{-a\alpha} = \varphi_{b\alpha} + \varphi_{-b\alpha}, \\
\varphi_{a\alpha} + \varphi_{a-\alpha} = \varphi_{a\beta} + \varphi_{a-\beta}.
\end{aligned}
\tag{4}
$$

With this method, we can obtain high dimentional Yang-Baxter $\breve{R}^{j_1 j_2}$ matrices easily. By means of these Yang-Baxter $\breve{R}^{j_1 j_2}$ matrices, we can investigate quantum entanglement consequently.

III. THE "X" FORM $\check{R}$ MATRICES AS QUANTUM GATES

In this section, three examples are shown to illustrate this method in detail. The case $j_1 = j_2 = 1/2$ gives us a 4×4 unitary Yang-Baxter $\check{R}^{1/2,1/2}(\theta)$ matrix. Thus we can view the $\check{R}^{1/2,1/2}(\theta)$ matrix as a quantum gate for two-qubit system. If $j_1 = 1$ and $j_2 = 1/2$, we can obtain a 6×6 Yang-Baxter $\check{R}^{1,1/2}$ matrix. This unitary $\check{R}^{1,1/2}$ can entangle quantum states in system with one qubit and one qutrit. When $j_1 = 3/2$ and $j_2 = 1/2$, a three-qubit quantum gate $\check{R}^{3/2,1/2}$ can be obtained. For quantify the entanglement of bi-particle system states, we use the negativity[26, 27] defined by,

$$N(\rho) = \frac{||\rho^{T_B}||_1 - 1}{d - 1}. \qquad (5)$$

where ρ^{T_B} is the partial transpose of a state ρ in $d \times d'(d \leq d')$ quantum system, and the notation $||A||_1 = Tr\sqrt{A^\dagger A}$ denotes the trace norm of A. It should be noted that the negativity criterion is necessary and sufficient only for $2 \otimes 2$ and $2 \otimes 3$ quantum systems.

A. The 4×4 "X" form $\check{R}$ matrix

If $j_1 = j_2 = 1/2$, the equations in Eqs. (1) can be simplified as $[M^{1/2,1/2}]^2 = I^{1/2,1/2}$ and $[M_{12}^{1/2,1/2}, M_{23}^{1/2,1/2}] = 0$. Then we can obtain a matrix $M^{1/2,1/2}$ as following,

$$M^{1/2,1/2} = e^{-i(\varphi+\frac{\pi}{2})} s_1^+ s_2^+ + s_1^+ s_2^- + s_1^- s_2^+ + e^{i(\varphi+\frac{\pi}{2})} s_1^- s_2^-.$$

The Yang-Baxter $\check{R}^{1/2,1/2}$ matrix can be obtained as follows,

$$\check{R}^{1/2,1/2}(\theta) = e^{-i\frac{\theta}{2}M^{1/2,1/2}} = \cos\frac{\theta}{2} I^{1/2,1/2} - i\sin\frac{\theta}{2} M^{1/2,1/2},$$

or in matrix form,

$$\check{R}^{1/2,1/2}(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & 0 & 0 & -\sin\frac{\theta}{2}e^{-i\varphi} \\ 0 & \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} & 0 \\ 0 & -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2}e^{i\varphi} & 0 & 0 & \cos\frac{\theta}{2} \end{pmatrix}.$$

In this section, we choose $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as standard bases. Acting this Yang-Baxter $\check{R}^{1/2,1/2}$ matrix on the standard bases, we can obtain a set entangled states $\{|e_i\rangle, i = 1, 2, 3, 4\}$£

$$\begin{pmatrix} |e_1\rangle \\ |e_2\rangle \\ |e_3\rangle \\ |e_4\rangle \end{pmatrix} = \check{R}^{1/21/2}(\theta) \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2}|00\rangle - \sin\frac{\theta}{2}e^{-i\varphi}|11\rangle \\ \cos\frac{\theta}{2}|01\rangle - i\sin\frac{\theta}{2}|10\rangle \\ -i\sin\frac{\theta}{2}|01\rangle + \cos\frac{\theta}{2}|10\rangle \\ \sin\frac{\theta}{2}e^{i\varphi}|00\rangle + \cos\frac{\theta}{2}|11\rangle \end{pmatrix}.$$

Let us find the entanglement degree of the above states by using negativity. For a pure two qubit state, $|\psi\rangle = a|00\rangle + b|11\rangle$ or $|\phi\rangle = a|01\rangle + b|10\rangle$, the negativity can be find to be $N(|\psi\rangle) = N(|\phi\rangle) = 2|ab|$. We can easily obtain the negativity for the above entangled states as $N(|e_i\rangle) = |\sin\theta|$, where $i = 1, 2, 3, 4$. With the Yang-Baxter $\check{R}$ acting on the standard bases, we can obtain a set of entangled states, and these states possess the same entanglement degree which depends on the parameter θ. This character of the Yang-Baxter $\check{R}$ matrices has revealed in the Refs.. For the 2-qubit quantum system, there is good entanglement measure concurrence[28, 29], $C(\rho_{12}) = Max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$. Here $\{\lambda_i\}$ denotes the eigenvalues of the matrix $\rho_{12}\sigma_1^y\sigma_2^y\rho_{12}^*\sigma_1^y\sigma_2^y$. The notations ρ_{12} and ρ_{12}^* are biqubit density matrix and its complex conjugate, correspondingly. The notations $\sigma_{1,2}^y$ are pauli matrices. We can verify that concurrence is equivalence to negativity for two-qubit "X" state(which density matrices are "X" form).

B. The 6×6 "X" form $\check{R}$ matrix

When $j_1 = 1$ and $j_2 = 1/2$, with the relations in Eqs. (1), we can determine two matrices $M^{1,1/2}$ and $M^{1/2,1}$. In this section, the bases for the tensor product space $V^{j_1} \otimes V^{j_2}$ are given by $\{|a\alpha\rangle : a = 1, 0, -1; \alpha = 1/2, -1/2\}$. In this case, the Eqs. (4) gives the following relation,

$$2\varphi_{0,1/2} = \varphi_{1,1/2} - \varphi_{1,-1/2}. \tag{6}$$

If we set $\varphi_{1,1/2} = \varphi_1$ and $\varphi_{1,-1/2} = \varphi_2$, then $\varphi_{0,1/2} = (\varphi_1 - \varphi_2)/2$. Then a 6-dimensional $M^{1,1/2}$ matrix is given as follows,

$$\begin{aligned} M^{1,1/2} &= (e^{-i\varphi_1}|1, 1/2\rangle\langle -1, -1/2| + e^{-i\varphi_2}|1, -1/2\rangle\langle -1, 1/2| \\ &+ e^{-i(\varphi_1-\varphi_2)}|0, 1/2\rangle\langle 0, -1/2|) + H.C \end{aligned} \tag{7}$$

The $M^{1,1/2}$ matrix takes the following matrix form,

$$M^{1,1/2}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & e^{-i\varphi_1} \\ 0 & 0 & 0 & 0 & e^{-i\varphi_2} & 0 \\ 0 & 0 & 0 & e^{-i(\varphi_1-\varphi_2)} & 0 & 0 \\ 0 & 0 & e^{i(\varphi_1-\varphi_2)} & 0 & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 & 0 & 0 & 0 \\ e^{i\varphi_1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

Then a 6-dimensional Yang-Baxter $\check{R}^{1,1/2}(\theta)$ can be construct as following,

$$\check{R}^{1,1/2}(\theta)=\cos\frac{\theta}{2}I^{1,1/2}-i\sin\frac{\theta}{2}M^{1,1/2} \tag{9}$$

When $\check{R}^{1,1/2}(\theta)$ act on the standard basis(product states),

$$\begin{pmatrix} |e_1\rangle \\ |e_2\rangle \\ |e_3\rangle \\ |e_4\rangle \\ |e_5\rangle \\ |e_6\rangle \end{pmatrix}=\check{R}^{1,1/2}(\theta)\begin{pmatrix} |1,1/2\rangle \\ |1,-1/2\rangle \\ |0,1/2\rangle \\ |0,-1/2\rangle \\ |-1,1/2\rangle \\ |-1,-1/2\rangle \end{pmatrix} \tag{10}$$

Then we obtain six entangled states,

$$\begin{aligned} |e_1\rangle &= \cos\tfrac{\theta}{2}|1,1/2\rangle - i\sin\tfrac{\theta}{2}e^{-i\varphi_1}|-1,-1/2\rangle \\ |e_2\rangle &= \cos\tfrac{\theta}{2}|1,-1/2\rangle - i\sin\tfrac{\theta}{2}e^{-i\varphi_2}|-1,1/2\rangle \\ |e_3\rangle &= \cos\tfrac{\theta}{2}|0,1/2\rangle - i\sin\tfrac{\theta}{2}e^{-i(\varphi_1-\varphi_2)}|0,-1/2\rangle \\ |e_4\rangle &= -i\sin\tfrac{\theta}{2}e^{i(\varphi_1-\varphi_2)}|0,1/2\rangle + \cos\tfrac{\theta}{2}|0,-1/2\rangle \\ |e_5\rangle &= -i\sin\tfrac{\theta}{2}e^{i\varphi_2}|1,-1/2\rangle + \cos\tfrac{\theta}{2}|-1,1/2\rangle \\ |e_6\rangle &= -i\sin\tfrac{\theta}{2}e^{i\varphi_1}|1,1/2\rangle + \cos\tfrac{\theta}{2}|-1,-1/2\rangle \end{aligned} \tag{11}$$

Using the formula of negativity, we can obtain the entanglement degree for the eigenstates of this Yang-Baxter system as $N(|e_i\rangle)=|\sin\theta|$. These eigenstates possess the same degree of entanglement.

C. The 8×8 Yang-Baxter system

When $j_1=3/2$ and $j_2=1/2$, we can obtain a 8×8 $M^{3/2,1/2}$ matrix which satisfying the relations Eqs.(1). For the following convenience, we introduce the notation $\{|i\rangle; i=1,2\cdots 8\}$ to denote the standard three-qubit basis.

$$M^{3/2,1/2} = i(e^{-i\varphi_1} s_1^+ s_2^+ s_3^+ + e^{-i\varphi_2} s_1^+ s_2^+ s_3^- + e^{-i\varphi_3} s_1^+ s_2^- s_3^+ + e^{-i\varphi_4} s_1^+ s_2^- s_3^-)$$
$$- i(e^{i\varphi_4} s_1^- s_2^+ s_3^+ + e^{i\varphi_3} s_1^- s_2^+ s_3^- + e^{i\varphi_2} s_1^- s_2^- s_3^+ + e^{i\varphi_1} s_1^- s_2^- s_3^-)$$

If parameters φ_i's satisfy the relation $\varphi_1 + \varphi_4 = \varphi_2 + \varphi_3$, then the $M^{\frac{3}{2}\frac{1}{2}}$ satisfy the relations in Eqs.(1). Then we can obtain a 8×8 unitary Yang-Baxter $\breve{R}$−matrix,

$$\breve{R}^{3/2,1/2}(\theta) = \cos\frac{\theta}{2} I^{3/2,1/2} - i \sin\frac{\theta}{2} M^{3/2,1/2}$$

We can verify that the Yang-Baxter $\breve{R}^{3/2,1/2}(\theta)$ matrix is unitary(*i.e.* $\breve{R}(\theta)^\dagger \breve{R}(\theta) = \breve{R}(\theta)\breve{R}(\theta)^\dagger = I$). Let $H_0 = s_1^3 \otimes I_2 \otimes I_3$. With this Yang-Baxter $\breve{R}$−matrix and this simple Hamiltonian, we can derive a hamiltonian as $H = \breve{R}(\theta)^\dagger H_0 \breve{R}(\theta) = \sum_{i=1}^4 \mathbf{B}_i \cdot \mathbf{S}_i$, where $\mathbf{B}_i = (sin\theta cos\varphi_i, sin\theta sin\varphi_i, cos\theta)$ and

$$S_1^+ = |1\rangle\langle 8|,\ S_1^- = |8\rangle\langle 1|,\ S_1^3 = \tfrac{1}{2}(|1\rangle\langle 1| - |8\rangle\langle 8|);$$
$$S_2^+ = |2\rangle\langle 7|,\ S_2^- = |7\rangle\langle 2|,\ S_2^3 = \tfrac{1}{2}(|2\rangle\langle 2| - |7\rangle\langle 7|);$$
$$S_3^+ = |3\rangle\langle 6|,\ S_3^- = |6\rangle\langle 3|,\ S_3^3 = \tfrac{1}{2}(|3\rangle\langle 3| - |6\rangle\langle 6|);$$
$$S_4^+ = |4\rangle\langle 5|,\ S_4^- = |5\rangle\langle 4|,\ S_4^3 = \tfrac{1}{2}(|4\rangle\langle 4| - |5\rangle\langle 5|).$$

After some algebra, we can obtain the eigenvalues $\{E_i^\alpha\}$ and eigenvectors $\{|e_i^\alpha\rangle\}$ ($\alpha = +, -; i = 1, 2, 3, 4$) for Hamiltonian H as following,

$$E_i^+ = -E_i^- = 1/2,$$

and corresponding eigenvectors,

$$|e_1^+\rangle = cos\tfrac{\theta}{2}|1\rangle + sin\tfrac{\theta}{2}e^{i\varphi_1}|8\rangle,\ |e_1^-\rangle = -sin\tfrac{\theta}{2}e^{-i\varphi_1}|1\rangle + cos\tfrac{\theta}{2}|8\rangle;$$
$$|e_2^+\rangle = cos\tfrac{\theta}{2}|2\rangle + sin\tfrac{\theta}{2}e^{i\varphi_2}|7\rangle,\ |e_2^-\rangle = -sin\tfrac{\theta}{2}e^{-i\varphi_2}|2\rangle + cos\tfrac{\theta}{2}|7\rangle;$$
$$|e_3^+\rangle = cos\tfrac{\theta}{2}|3\rangle + sin\tfrac{\theta}{2}e^{i\varphi_3}|6\rangle,\ |e_3^-\rangle = -sin\tfrac{\theta}{2}e^{-i\varphi_3}|3\rangle + cos\tfrac{\theta}{2}|6\rangle;$$
$$|e_4^+\rangle = cos\tfrac{\theta}{2}|4\rangle + sin\tfrac{\theta}{2}e^{i\varphi_4}|5\rangle,\ |e_4^-\rangle = -sin\tfrac{\theta}{2}e^{-i\varphi_4}|4\rangle + cos\tfrac{\theta}{2}|5\rangle.$$

In fact, the Hamiltonian H can be recast as following,

$$H = \sum_{i=1}^{4}(|e_i^+\rangle\langle e_i^+| - |e_i^-\rangle\langle e_i^-|) \tag{12}$$

Consider the state $|\psi\rangle$ in a three-qubit Hilbert space $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$. Its coefficients with respect to a basis of product states (the 'computational basis') are $\psi_i = \langle i|\psi\rangle$, $i \in \{0, 1 \cdots 8\}$. An important measure for the entanglement in pure three-qubit states is the three-tangle (or residual tangle) introduced in Ref.[30]. The three-tangle of $|\psi\rangle$ is a so-called polynomial invariant and can be written in terms of the coefficients ψ_i as

$$\begin{aligned}
\tau_3(\psi) &= 4|d_1 - 2d_2 + 4d_3| \qquad (13)\\
d_1 &= \psi_1^2\psi_8^2 + \psi_2^2\psi_7^2 + \psi_3^2\psi_6^2 + \psi_5^2\psi_4^2\\
d_2 &= \psi_1\psi_8\psi_4\psi_5 + \psi_1\psi_8\psi_6\psi_3 + \psi_1\psi_8\psi_7\psi_2\\
&+ \psi_4\psi_5\psi_6\psi_3 + \psi_4\psi_5\psi_7\psi_2 + \psi_6\psi_3\psi_7\psi_2\\
d_3 &= \psi_1\psi_7\psi_6\psi_4 + \psi_8\psi_2\psi_3\psi_5 \ .
\end{aligned}$$

Then we can obtain three-tangle for the Eigenstates are as following,

$$\tau_3(|e_i^\alpha\rangle) = sin^2\theta.$$

By using the definition of concurrence we can obtain the

$$C_{AB}(|e_i^\alpha\rangle) = C_{AC}(|e_i^\alpha\rangle) = C_{BC}(|e_i^\alpha\rangle) = 0$$

where $i = 1, 2, 3, 4$ and $\alpha = +, -$. When the parameter $\theta = \pi/2$, $\tau_3(|e_i^\alpha\rangle) = 1$ and $C_{XY}(|e_i^\alpha\rangle) = 0 (XY = AB, BC, AC)$. Then we can say these eigenstates are GHZ type states.

IV. YANGIAN SYMMETRY AND SHIFT OPERATORS

In the Sec.III, we construct a Hamiltonian(*i.e.* Eq.(12)) with the Yang-Baxter $\check{R}^{3/2,1/2}$ matrix. As is known to all, the Yangian is a very important tool to study symmetry and shift operators. Motivated this, we will investigate the symmetry to this Yang-Baxter Hamiltonian and Yangian generators as shift operators in detail.

In fact, with the eigenvectors $\{|e_i^\alpha\rangle\}$ we can construct a special Yangian Y(sl(2)) realization $\{I_\pm, I_3\}$ and $\{F_\pm, F_3\}$ as following,

$$\begin{aligned}
I_+ &= |e_1^+\rangle\langle e_2^+| + |e_3^+\rangle\langle e_4^+| + |e_1^-\rangle\langle e_2^-| + |e_3^-\rangle\langle e_4^-|\\
I_- &= |e_2^+\rangle\langle e_1^+| + |e_4^+\rangle\langle e_3^+| + |e_2^-\rangle\langle e_1^-| + |e_4^-\rangle\langle e_3^-|\\
I_3 &= \frac{1}{2}[(|e_1^+\rangle\langle e_1^+| + |e_3^+\rangle\langle e_3^+| + |e_1^-\rangle\langle e_1^-| + |e_3^-\rangle\langle e_3^-|)\\
&- (|e_2^+\rangle\langle e_2^+| + |e_4^+\rangle\langle e_4^+|) + |e_2^-\rangle\langle e_2^-| + |e_4^-\rangle\langle e_4^-|)],
\end{aligned}$$

and

$$
\begin{aligned}
F_+ &= 2\alpha(|e_1^+\rangle\langle e_4^+| + \beta|e_3^+\rangle\langle e_2^+|) + 2\gamma(|e_1^-\rangle\langle e_4^-| + \delta|e_3^-\rangle\langle e_2^-|) \\
F_- &= 2\alpha(\beta|e_4^+\rangle\langle e_1^+| + |e_2^+\rangle\langle e_3^+|) + 2\gamma(\delta|e_4^-\rangle\langle e_1^-| + |e_2^-\rangle\langle e_3^-|) \\
F_3 &= \alpha(|e_1^+\rangle\langle e_3^+| - |e_2^+\rangle\langle e_4^+| + \beta|e_3^+\rangle\langle e_1^+| - \beta|e_4^+\rangle\langle e_2^+|) \\
&\quad +\gamma(|e_1^-\rangle\langle e_3^-| - |e_2^-\rangle\langle e_4^-| + \delta|e_3^-\rangle\langle e_1^-| - \delta|e_4^-\rangle\langle e_2^-|).
\end{aligned}
$$

It is not difficulty to verify that $\{I_\pm, I_3\}$ and $\{F_\pm, F_3\}$ satisfy the following Yanigian $Y(sl(2))$ relations,

$$
\begin{aligned}
&[I_3, I_\pm] = \pm I_\pm, \quad [I_+, I_-] = 2I_3 \\
&[I_3, F_\pm] = [F_3, I_\pm] = \pm F_\pm, \quad [I_\pm, F_\mp] = \pm 2F_3 \\
&[I_3, F_3] = [I_\pm, F_\pm] = 0,
\end{aligned}
$$

and

$$
\begin{aligned}
&[F_3, [F_+, F_-]] = 0, \quad [F_\pm, [F_3, F_\pm]] = 0 \\
&[F_\pm, [F_\pm, F_\mp]] \pm 2[F_3, [F_3, F_\pm]] = 0.
\end{aligned}
$$

We can verify that the Hamiltonian and Yangian operators satisfy the following relation,

$$[H, Y_\alpha] = 0,$$

where $Y = I, F$ and $\alpha = \pm, 3$. That is to say this Hamiltonian possess a Yangian Y$(sl(2))$ symmetry.

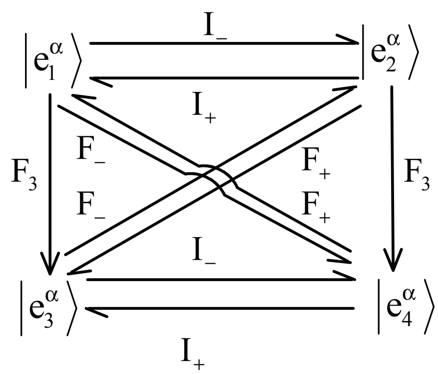

FIG. 1: The states transfer graph for the Yang-Baxter Hamiltonian($\alpha = \pm$).

This maybe the simplest Hamiltonian with Yangian $Y(sl(2))$ symmetry. In quantum physics, the

Yangian generators can be used to construct shift operators. Then we will construct shift operators for this Yang-Baxter Hamiltonian. When the Yangian operators $\{I_\pm, I_3\}$ and $\{F_\pm, F_3\}$ act on the eigenstates of this Yang-Baxter Hamiltonian, we can obtain a state transfer graph in Fig.(1).

V. SUMMARY

In this paper, we construct a set of $(2j_1+1)\times(2j_2+1)-$dimensional "X" form Yang-Baxter $\check{R}^{j_1 j_2}(\theta)$. We investigated this set unitary Yang-Baxter $\check{R}^{j_1 j_2}(\theta)$ as quantum gate in quantum computation processing. When these "X" form Yang-Baxter $\check{R}^{j_1 j_2}(\theta)$ matrices act on standard bases, we can obtain a set of entangled states, which possess the same degree of quantum entanglement. We also construct a Yang-Baxter Hamiltonian with Yangian Y(*sl(2)*) symmetry. And Yangian generators can be viewed as shift operators.

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