Perturbational Blowup Solutions to the 2-Component Camassa-Holm Equations

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Abstract

In this article, we study the perturbational method to construct the non-radially symmetric solutions of the compressible 2-component Camassa-Holm equations. In detail, we first combine the substitutional method and the separation method to construct a new class of analytical solutions for that system. In fact, we perturb the linear velocity:

$$u = c(t)x + b(t), \tag{1}$$

and substitute it into the system. Then, by comparing the coefficients of the polynomial, we can deduce the functional differential equations involving $(c(t), b(t), \rho^2(0,t))$. Additionally, we could apply the Hubble's transformation

$$c(t) = \frac{\dot{a}(3t)}{a(3t)}, \tag{2}$$

to simplify the ordinary differential system involving $(a(3t), b(t), \rho^2(0,t))$. After proving the global or local existences of the corresponding dynamical system, a new class of analytical solutions is shown. And the corresponding solutions in radial symmetry are also given. To determine that the solutions exist globally or blow up, we just use the qualitative properties

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about the well-known Emden equation:

$$\begin{cases} \frac{d^2}{dt^2}a(3t) = \frac{\xi}{a^{\frac{1}{3}}(3t)}, \\ a(0) = a_0 > 0,\ \dot{a}(0) = a_1 \end{cases} \tag{3}$$

Our solutions obtained by the perturbational method, fully cover the previous known results in "M.W. Yuen, *Self-Similar Blowup Solutions to the 2-Component Camassa-Holm Equations,* J. Math. Phys., **51** (2010) 093524, 14pp." by the separation method.

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1 Introduction

The 2-component Camassa-Holm equations of shallow water system can be expressed by

$$\begin{cases} \rho_t + u\rho_x + \rho u_x = 0,\ x \in R \\ m_t + 2u_x m + u m_x + \sigma\rho\rho_x = 0 \end{cases} \tag{4}$$

with

$$m = u - \alpha^2 u_{xx}. \tag{5}$$

Here $u = u(x,t) \in R$ and $\rho = \rho(x,t) \geq 0$ are the velocity and the density of fluid respectively. The constant σ is equal to 1 or -1. If $\sigma = -1$, the gravity acceleration points upwards [2], [3], [8], [10] and [9]. For $\sigma = 1$, the researches regarding the corresponding models could be referred by [4], [6], [10] and [8]. When $\rho \equiv 0$, the system returns to the Camassa-Holm equation [1]. The searching of Camassa-Holm equation can capture breaking waves. Peaked traveling waves is a long-standing open problem [17].

In 2010, Yuen used the separation method to obtain a class of blowup or global solutions of the Camassa-Holm equations [22] and Degasperis-Procesi equations [23]. In particular, for the integrable system of the Camassa-Holm equations with $\sigma = 1$, we have the global solutions:

$$\begin{cases} \rho(x,t) = \max\left\{\frac{f(\eta)}{a(3t)^{1/3}}, 0\right\},\ u(x,t) = \frac{\dot{a}(3t)}{a(3t)}x \\ \ddot{a}(s) - \frac{\xi}{3a(s)^{1/3}} = 0,\ a(0) = a_0 > 0,\ \dot{a}(0) = a_1 \\ f(\eta) = \xi\sqrt{-\frac{\eta^2}{\xi} + (\xi\alpha)^2} \end{cases} \tag{6}$$

where $\eta = \frac{x}{a(s)^{1/3}}$ with $s = 3t$; $\xi > 0$ and $\alpha \geq 0$ are arbitrary constants. [22]

Meanwhile, the isentropic compressible Euler equations can be written in the following form:

$$\begin{cases} \rho_t + \nabla \cdot \rho u = 0 \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u) + \nabla P = 0. \end{cases} \tag{7}$$

As usual, $\rho = \rho(x,t)$ and $u = u(x,t) \in \mathbf{R}^N$ are the density and the velocity respectively with $x = (x_1, x_2, ..., x_N) \in R^N$. For some fixed $K > 0$, we have a γ-law on the pressure $P = P(\rho)$, i.e.

$$P(\rho) = K\rho^\gamma \tag{8}$$

with a constant $\gamma \geq 1$. For solutions in radially symmetry:

$$\rho(x,t) = \rho(r,t) \text{ and } u(x,t) = \frac{x}{r}V(r,t) =: \frac{x}{r}V \tag{9}$$

where the radial $r = \sum_{i=1}^{N} x_i^2$,

the compressible Euler equations (7) become,

$$\begin{cases} \rho_t + V\rho_r + \rho V_r + \frac{N}{r}\rho V = 0 \\ \rho(V_t + VV_r) + \nabla P = 0. \end{cases} \tag{10}$$

Recently, there are some researches concerning the construction of solutions of the compressible Euler and Navier-Stokes equations by the substitutional method [12], [13], [18] and [14]. They assume that the velocity is linear:

$$u(x,t) = c(t)x \tag{11}$$

and substitute it into the system to derive the dynamic system about the function $c(t)$. Then they use the standard argument of phase diagram to drive the blowup or global existence of the ordinary differential equation involving $c(t)$.

On the other hand, the separation method can be governed to seek the radial symmetric solutions by the functional form:

$$\rho(r,t)=\frac{f(\frac{r}{a(t)})}{a^N(t)} \text{ and } V(r,t)=\frac{\dot{a}(t)}{a(t)}r. \tag{12}$$

([7], [15], [5], [12], [18], [19], [20] and [21])

It is natural to consider the more general linear velocity:

$$u(x,t)=c(t)x+b(t) \tag{13}$$

to construct new solutions. In this article, we can first combine the two conventional approaches (substitutional method and separation method) to derive the corresponding solutions for the system. In fact, the main theme of this article is to substitute the linear velocity (13) into the Camassa-Holm equations (4) and compare the coefficient of the different polynomial degrees for deducing the functional differential equations involving $(c(t), b(t), \rho^2(0,t))$. Then, we can apply the Hubble's transformation

$$c(t)=\frac{\dot{a}(3t)}{a(3t)} \tag{14}$$

with $\dot{a}(3t):=\frac{da(3t)}{dt}$, to simplify the functional differential system involving $(a(3t), b(t), \rho^2(0,t))$. After proving the local existences of the corresponding dynamical system, we can show the results below:

Theorem 1 *For the 2-component Camassa-Holm equations (4), there exists a family of solutions,*

$$\begin{cases} \rho^2(x,t)=\max\left\{\rho^2(0,t)-\frac{2}{\sigma}[\dot{b}(t)+3b(t)\frac{\dot{a}(t)}{a(t)}]x-\frac{3\xi}{\sigma a^{\frac{4}{3}}(t)}x^2,\ 0\right\} \\ u(x,t)=\frac{\dot{a}(3t)}{a(3t)}x+b(t) \\ \frac{d^2}{dt^2}a(3t)=\frac{\xi}{a^{\frac{1}{3}}(3t)},\ a(0)=a_0>0,\ \dot{a}(0)=a_1 \\ \frac{d^2}{dt^2}b(t)+\frac{6\dot{a}(3t)}{a(3t)}\frac{d}{dt}b(t)+\frac{12\xi}{a^{\frac{4}{3}}(3t)}b(t)=0, b(0)=b_0,\ \dot{b}(0)=b_1 \\ \frac{d}{dt}\left[\rho^2(0,t)\right]=\rho^2(0,t)-\frac{2}{\sigma}[\dot{b}(t)+3b(t)\frac{\dot{a}(3t)}{a(3t)}]x-\frac{3\xi}{\sigma a^2(3t)}x^2,\ \rho^2(0,0)=\alpha^2 \end{cases} \tag{15}$$

where a_0, a_1, b_1, b_2 and α are arbitrary constants.

We remark that the above solutions (15) fully cover the previous known results [22] by the separation method by choosing $b_0=b_1=0$.

2 Perturbational Method

The proof for Theorem 1 requires the standard manipulation of algebraic computation only:

Proof. The momentum equation $(4)_2$, becomes

$$(u-u_{xx})_t + 2u_x(u-u_{xx}) + u(u-u_{xx})_x + \sigma\rho\rho_x = 0. \tag{16}$$

First, we perturb the velocity with this following functional form:

$$u(x,t) = c(t)x + b(t) \tag{17}$$

where $b(t)$ and $c(t)$ are the time functions determined later.

As the velocity u (17) is linear:

$$u_{xx} = 0, \tag{18}$$

it can be simplified to be

$$u_t + 3uu_x + \sigma\rho\rho_x = 0 \tag{19}$$

$$\dot{c}(t)x + \dot{b}(t) + 3[c(t)x + b(t)]c(t) + \frac{\sigma}{2}\frac{\partial}{\partial x}\rho^2 = 0 \tag{20}$$

$$\frac{\sigma}{2}\frac{\partial}{\partial x}\rho^2 = -[\dot{b}(t) + 3b(t)c(t)] - [\dot{c}(t) + 3c^2(t)]x. \tag{21}$$

Then, we take integration from $[0, x]$ to have:

$$\frac{\sigma}{2}\int_0^x \frac{\partial}{\partial s}\rho^2 ds = -[\dot{b}(t) + 3b(t)c(t)]\int_0^x ds - [\dot{c}(t) + 3c^2(t)]\int_0^x s ds \tag{22}$$

$$\frac{\sigma}{2}\left[\rho^2(x,t) - \rho^2(0,t)\right] = -[\dot{b}(t) + 3b(t)c(t)]x - \frac{[\dot{c}(t) + 3c^2(t)]}{2}x^2 \tag{23}$$

$$\rho^2(x,t) = \rho^2(0,t) - \frac{2}{\sigma}[\dot{b}(t) + 3b(t)c(t)]x - \frac{[\dot{c}(t) + 3c^2(t)]}{\sigma}x^2. \tag{24}$$

On the other hand, for the 1-dimensional mass equation $(4)_1$, we obtain

$$\rho_t + [c(t)x + b(t)]\,\rho_x + \rho c(t) = 0. \tag{25}$$

Here, we multiple ρ on both sides to have

$$\frac{1}{2}\left(\rho^2\right)_t + \frac{[c(t)x + b(t)]}{2}\left(\rho^2\right)_x + \rho^2 c(t) = 0. \tag{26}$$

After that, we can substitute equation (24) into equation (26):

$$\frac{1}{2}\left(\frac{\partial}{\partial t}\left[\rho^2(0,t)\right]-\frac{2}{\sigma}\frac{\partial}{\partial t}[\dot{b}(t)+3b(t)c(t)]x-\frac{\partial}{\partial t}\frac{[\dot{c}(t)+3c^2(t)]}{\sigma}x^2\right) \tag{27}$$

$$+[c(t)x+b(t)]\left(-\frac{1}{\sigma}[\dot{b}(t)+3b(t)c(t)]-\frac{1}{\sigma}[\dot{c}(t)+3c^2(t)]x\right) \tag{28}$$

$$+c(t)\left[\rho^2(0,t)-\frac{2}{\sigma}[\dot{b}(t)+3b(t)c(t)]x-\frac{[\dot{c}(t)+3c^2(t)]}{\sigma}x^2\right] \tag{29}$$

$$=\frac{1}{2}\frac{\partial}{\partial t}\left[\rho^2(0,t)\right]+c(t)\rho^2(0,t)-\frac{b(t)}{\sigma}[\dot{b}(t)+3b(t)c(t)] \tag{30}$$

$$+\left\{\begin{array}{c}-\frac{1}{\sigma}\frac{\partial}{\partial t}[\dot{b}(t)+3b(t)c(t)]-\frac{c(t)}{\sigma}[\dot{b}(t)+3b(t)c(t)]\\ -\frac{b(t)}{\sigma}[\dot{c}(t)+3c^2(t)]-\frac{2c(t)}{\sigma}[\dot{b}(t)+3b(t)c(t)]\end{array}\right\}x \tag{31}$$

$$+\left\{\begin{array}{c}-\frac{1}{2\sigma}\frac{\partial}{\partial t}[\dot{c}(t)+3c^2(t)]-\frac{1}{\sigma}[\dot{c}(t)+3c^2(t)]c(t)\\ -\frac{c(t)[\dot{c}(t)+3c^2(t)]}{\sigma}\end{array}\right\}x^2 \tag{32}$$

By comparing the coefficients of the polynomial, we require the functional differential equations involving $(c(t),b(t),\rho^2(0,t))$:

$$\left\{\begin{array}{c}\frac{d}{dt}\left[\rho^2(0,t)\right]+2c(t)\rho^2(0,t)-\frac{2}{\sigma}b(t)[\dot{b}(t)+3b(t)c(t)]=0\\ \frac{d}{dt}[\dot{b}(t)+3b(t)c(t)]+3c(t)[\dot{b}(t)+3b(t)c(t)]+b(t)[\dot{c}(t)+3c^2(t)]=0\\ \frac{d}{dt}[\dot{c}(t)+3c^2(t)]+4[\dot{c}(t)+3c^2(t)]c(t)=0\end{array}\right. \tag{33}$$

For details (existence, uniqueness and continuous dependence) about general functional differential equations, the interested reader may refer to the classical literatures [11] and [16].

For solving the above ordinary differential system (33), we initially solve equation $(33)_3$ about the function $c(t)$. Here we let the function $c(t)$ be expressed with the Hubble's transformation:

$$c(t)=\frac{\dot{a}(3t)}{a(3t)} \tag{34}$$

where $\dot{a}(3t):=\frac{da(3t)}{dt}$ and the function $a(3t)$ could be determined later.

It is transformed to be

$$\frac{d}{dt}[\frac{3\ddot{a}(3t)}{a(3t)}-\frac{3\dot{a}^2(3t)}{a^2(3t)}+\frac{3\dot{a}^2(3t)}{a^2(3t)}]+4[\frac{3\ddot{a}(3t)}{a(3t)}-\frac{3\dot{a}^2(3t)}{a^2(3t)}+\frac{3\dot{a}^2(3t)}{a^2(3t)}]\frac{\dot{a}(3t)}{a(3t)}=0 \tag{35}$$

$$\begin{cases} \frac{d}{dt}\left(\frac{\ddot{a}(3t)}{a(3t)}\right) + \frac{4\ddot{a}(3t)}{a(3t)}\frac{\dot{a}(3t)}{a(3t)} = 0 \\ a(0) = a_0 > 0,\ \dot{a}(0) = a_1,\ \ddot{a}(0) = a_2 \end{cases} \tag{36}$$

$$\frac{3\dddot{a}(3t)}{a(3t)} - \frac{3\dot{a}(3t)\ddot{a}(3t)}{a^2(3t)} + \frac{4\dot{a}(3t)\ddot{a}(3t)}{a^2(3t)} = 0 \tag{37}$$

$$\frac{\dddot{a}(3t)}{a(3t)} + \frac{\dot{a}(3t)\ddot{a}(3t)}{3a^2(3t)} = 0. \tag{38}$$

Then, we multiple $a^2(3t)$ on both sides to have:

$$a(3t)\dddot{a}(3t) + \frac{\dot{a}(3t)\ddot{a}(3t)}{3} = 0. \tag{39}$$

It can be reduced to the second-order Emden equation:

$$\begin{cases} \frac{d^2}{dt^2}a(3t) = \frac{\xi}{a^{\frac{1}{3}}(3t)} \\ a(0) = a_0 > 0,\ \dot{a}(0) = a_1 \end{cases} \tag{40}$$

where $\xi := a_0^{\frac{1}{3}}a_2$ is an arbitrary constant by choosing a_2.

We remark that the well-known Emden equation is well studied in astrophysics and mathematics.

Next, for the second equation $(33)_2$ about $b(t)$ of the functional differential system, we could further simply it in terms of the known function $a(3t)$:

$$\frac{d}{dt}[\dot{b}(t) + 3b(t)\frac{\dot{a}(3t)}{a(3t)}] + 3\frac{\dot{a}(3t)}{a(3t)}[\dot{b}(t) + 3b(t)\frac{\dot{a}(3t)}{a(3t)}] + \frac{3\ddot{a}(3t)}{a(3t)}b(t) = 0 \tag{41}$$

$$\ddot{b}(t) + 6\frac{\dot{a}(3t)}{a(3t)}\dot{b}(t) + \left[9\frac{\ddot{a}(3t)}{a(3t)} - 9\frac{\dot{a}^2(3t)}{a^2(3t)} + \frac{9\dot{a}^2(3t)}{a^2(3t)} + \frac{3\ddot{a}(3t)}{a(3t)}\right]b(t) = 0 \tag{42}$$

$$\begin{cases} \ddot{b}(t) + 6\frac{\dot{a}(3t)}{a(3t)}\dot{b}(t) + \frac{12\xi}{a^{\frac{4}{3}}(3t)}b(t) = 0, \\ b(0) = b_0,\ \dot{b}(0) = b_1 \end{cases} \tag{43}$$

with the Emden equation (40).

We denote $f_1(t) = 6\frac{\dot{a}(3t)}{a(3t)}$ and $f_2(t) = \frac{12\xi}{a^{\frac{4}{3}}(3t)}$ to have

$$\begin{cases} \ddot{b}(t) + f_1(t)\dot{b}(t) + f_2(t)b(t) = 0 \\ b(0) = b_0,\ \dot{b}(0) = b_1. \end{cases} \tag{44}$$

Therefore, when the functions $f_1(t)$ and $f_2(t)$ are bounded, that is

$$|f_1(t)| \le F_1 \text{ and } |f_2(t)| \le F_2 \tag{45}$$

with some constants F_1 and F_2, provided that the functions $\frac{1}{a(3t)}$ and $\dot{a}(3t)$ exist, the functions $b(t)$ and $\dot{b}(t)$ can be guaranteed for existing by the comparison theorem of ordinary differential equations [17].

Lastly, for the first equation $(33)_1$, we denote $H(t) = \frac{2\dot{a}(3t)}{a(3t)}$ and $G(t) = \frac{2b(t)}{\sigma}[\dot{b}(t) + 3b(t)\frac{\dot{a}(3t)}{a(3t)}]$ in terms of functions $\frac{1}{a(3t)}$, $a(3t)$, $b(t)$ and $\dot{b}(t)$ provided that they exists, to solve

$$\begin{cases} \frac{d}{dt}\left[\rho^2(0,t)\right] + \rho^2(0,t)H(t) = G(t) \\ \rho^2(0,0) = \alpha^2. \end{cases} \tag{46}$$

The formula of the first-order ordinary differential equation (46) is

$$\rho^2(0,t) = \frac{\int_0^t \mu(s)G(s)ds + k}{\mu(t)} \tag{47}$$

where

$$\mu(t) = e^{\int_0^t H(s)ds}. \tag{48}$$

Therefore, we have the density function from equation (24):

$$\rho^2(x,t) = \rho^2(0,t) - \frac{2}{\sigma}[\dot{b}(t) + 3b(t)\frac{\dot{a}(3t)}{a(3t)}]x - \frac{3\xi}{\sigma a^{\frac{4}{3}}(3t)}x^2. \tag{49}$$

For $\rho(x,t) \geq 0$, we may set

$$\rho^2(x,t) = \max\left\{\rho^2(0,t) - \frac{2}{\sigma}[\dot{b}(t) + 3b(t)\frac{\dot{a}(3t)}{a(3t)}]x - \frac{3\xi}{\sigma a^{\frac{4}{3}}(3t)}x^2,\ 0\right\}. \tag{50}$$

In conclusion, we have the corresponding functional differential equations (15) to be the solutions of Camassa-Holm equations.

The proof is completed. ■

We notice that the above solutions are not radially symmetric for the density function ρ with $b(t) \neq 0$. Thus, the above solutions, cannot be obtained by the separation method of the self-similar functional [22], as

$$\rho(x,t) \neq f(\frac{x}{a(3t)})g(a(3t)) \text{ and } u(x,t) = \frac{\dot{a}(3t)}{a(3t)}x + b(t). \tag{51}$$

On the other hand, for the 2-component Camassa-Holm equations in radial symmetry with linear velocity $u(r,t)$:

$$\begin{cases} \rho_t + V\rho_r + \rho V_r = 0 \\ V_t + 3VV_r + \sigma\rho\rho_r = 0, \end{cases} \tag{52}$$

we may replace equation (22) to have the corresponding step by taking the integration from [0, r]

$$\frac{\sigma}{2}\int_0^r \frac{\partial}{\partial s}\rho^2 ds = -[\dot{b}(t) + b(t)c(t)]\int_0^r ds - 3[\dot{c}(t) + c^2(t)]\int_0^r sds. \tag{53}$$

It is clear for that the rest of proof is similar to have the corresponding result for the solutions in radial symmetry:

Theorem 2 *For the 2-component Camassa-Holm equations in radial symmetry (4), there exists a family of solutions,*

$$\begin{cases} \rho^2(r,t) = \max\left\{\rho^2(0,t) - \frac{2}{\sigma}[\dot{b}(t) + 3b(t)\frac{\dot{a}(t)}{a(t)}]r - \frac{3\xi}{\sigma a^{\frac{4}{3}}(t)}r^2,\ 0\right\} \\ u(r,t) = \frac{\dot{a}(3t)}{a(3t)}r + b(t) \\ \frac{d}{dt}\left[\rho^2(0,t)\right] = \rho^2(0,t) - \frac{2}{\sigma}[\dot{b}(t) + 3b(t)\frac{\dot{a}(3t)}{a(3t)}]r - \frac{3\xi}{\sigma a^2(3t)}r^2,\ \rho^2(0,0) = \alpha^2 \end{cases} \tag{54}$$

where $a(3t)$ and $b(t)$ are the solutions of equations (15)$_3$ and (15)$_4$.

3 Blowup or Global Solutions

To determine that the solutions are global or local only, we can use the corresponding lemma about the Emden equation:

Lemma 3 *For the Emden equation (15)$_3$,*

$$\begin{cases} \ddot{a}(3t) = \frac{\xi}{a^{\frac{1}{3}}(3t)} \\ a(0) = a_0 > 0,\ \dot{a}(0) = a_1, \end{cases} \tag{55}$$

(1) if $\xi < 0$, there exists a finite time T, such that

$$\lim_{t \to T^-} a(3t) = 0. \tag{56}$$

(2) if $\xi = 0$, with $a_1 < 0$, the solution $a(t)$ blows up in the finite time:

$$T = \frac{-a_0}{a_1}, \tag{57}$$

(3) otherwise, the solution $a(t)$ exists globally.

We observe that it is the same lemma for the function $a(s)$ with $s = 3t$ by the separation methods in [22]. Therefore, the proofs can be found in Lemma 3 of [22].

The gradient of the velocity in solutions (15) and (54), is

$$\frac{\partial}{\partial x}u(x,t) = \frac{\partial}{\partial r}u(r,t) = \frac{\dot{a}(3t)}{a(3t)}. \tag{58}$$

When the function $a(t)$ blows up with a finite time T, $\frac{\partial}{\partial x}u(x,T)$ also blows up at every point x. And based on the above lemma about the Emden equation for $a(t)$, it is clear to have the corollary below:

Corollary 4 *(1a) For $\xi < 0$, solutions (15) and (54) blow up in a finite time T;*

(1b) For $\xi = 0$, with $a_1 < 0$, solutions (15) and (54) blow up in the finite time:

$$T = \frac{-a_0}{a_1}. \tag{59}$$

(2) otherwise, solutions (15) and (54) exist globally.

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