Negative frequency communication

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Abstract

Spectrum is the most valuable resource in the communication system, but unfortunately, so far, a half of the spectrum has been wasted, that is negative frequency spectrum. In fact, the negative frequency, just as positive frequency, has a physical meaning, the complete description of a frequency signal is a rotating complex signal, in complete description, positive and negative frequency signals are two distinguishable and independent frequency signals, they can carry different information. Because the current carrier modulation and demodulation do not distinguish positive and negative frequencies, so half of the spectrum resources and signal energies are wasted. The complex-carrier modulation and demodulation, proposed by this paper, use the complex frequency signal as carrier signal, the negative frequency can carry different information from the positive frequency, so the spectrum resources are full used, the signal energies carried by complex-carrier modulation is focused on a certain band, so the signal energies will not be lost by the complex-carrier demodulation.

1 Introduction

According to Shannon formula:

$$C = W * \log_2^{(1+S/N)} \tag{1}$$

Where C is the channel capacity, W is the channel bandwidth, S is the signal power, and N is the noise power. We can see that the most effective way to increase the channel capacity is increasing bandwidth, and enhance the signal to noise ratio can also increase the channel capacity.

In current communication system, to take full advantage of the spectrum, carrier modulation technology is used. The principle of current carrier modulation shown in Fig. 1, the real part of the baseband complex signal multiplies $cos(\omega t)$ and the imaginary part multiplies $-sin(\omega t)$, then add, as the following formula:

$$s_{CB}(t) = Re\{s_{BB}(t) * e^{i\omega t}\} \tag{2}$$

Where $s_{CB}(t)$ is the modulation signal, $e^{i\omega t}$ is the carrier signal, $s_{BB}(t)$ is the baseband signal, Re is to take the real part. Obviously in the current carrier modulation, although the baseband signal is complex, the carrier signal is complex, but the modulation signal is taken the real part, so a real signal is sent. In this paper it is called real-carrier modulation.

In fact, the real-carrier modulation technology wastes a half of the spectrum resources and the signal energy, that is because of the incorrect understanding and using of negative frequency. So far, in all current communication systems, including wireless, cable and fiber the available bandwidth is defined in the positive spectrum. For example, the bandwidth defined in the latest version of LTE protocol, see [3], negative spectrum is not used. So to make full use of spectrum resources, we must first correct understanding of negative frequency.

2 Negative frequency and the completeness of frequency signal

The negative frequency has a physical meaning, see [1], [2]. Shown in Fig. 2, we define the angle of counterclockwise rotation as $+\theta$, the angle of clockwise rotation as $-\theta$, so the definition of angular frequency is:

$$\omega = \frac{d\theta}{dt} \tag{3}$$

$$-\omega = \frac{d(-\theta)}{dt} \tag{4}$$

By (3), (4) we can see that the positive angular frequency corresponding to the speed of counterclockwise rotation, the negative angular frequency corresponding to the speed of clockwise rotation, so "negative angular frequency" does not due to the "negative time", but due to "negative angle", positive and negative frequency represent two different directions of rotation in the plane, there are two kinds of frequencies is essentially because of the angle is defined in a plane, a plane has two sides.

Understand the physical meaning of negative frequency, then how to distinguish between positive and negative frequencies, or how to describe these two directions of rotations in the plane? That is the Euler formula:

$$e^{\pm i\omega t} = \cos(\omega t) \pm i * \sin(\omega t) \tag{5}$$

Shown in Fig. 3, $e^{-i\omega t}$ and $e^{i\omega t}$ represent the clockwise and counterclockwise rotation curves, corresponding to the negative and positive frequency signals.

Although in the "time-complex" direct product space, it is easy to distinguish positive and negative frequency signals, but in "time-real" direct product space, the projection of positive and negative frequency signals are the same real signal $cos(\omega t)$, that is:

$$Re\{e^{-i\omega t}\} = Re\{e^{i\omega t}\} = \cos(\omega t) \tag{6}$$

So when we see a real signal, we can not distinguish it between positive or negative frequency signals from its projection, the probability of the frequency signal being negative or positive is equal, it is 1/2, that is:

$$\cos(\omega t) = (e^{-i\omega t} + e^{i\omega t})/2 \tag{7}$$

Similarly:

$$\sin(\omega t) = i * (e^{-i\omega t} - e^{i\omega t})/2 \tag{8}$$

Therefore, a frequency signal described by a real signal $cos(\omega t)$ or $sin(\omega t)$ is not complete, the complete description of a frequency signal must be a rotating complex signal $e^{\pm i\omega t}$, in complete description, the negative frequency signal $e^{i\omega t}$ and positive frequency signal $e^{\imath\omega t}$ are two distinguishable and independent frequency signals, they can carry different information.

In this paper, we define the positive frequency, which direction of rotation meet the right hand rule as the R-frequency. We define the negative frequency, which direction of rotation meet the left hand rule as the L-frequency. Unless otherwise cited, this paper will use the terms L-frequency and R-frequency, L-band and R-band instead of the positive frequency and negative frequency, positive band and negative band.

3 Real-carrier modulation and demodulation

In this section, we will see, the current real-carrier modulation technology occupies all the L-band and R-band in the spectrum, so a half of the spectrum resources are wasted, and the current real-carrier demodulation technology only receives one of the L-band or R-band, so a half of the signal energies are lost.

3.1 Real-carrier modulation

The principle of current real-carrier modulation is as formula (1), it is modulated by the R-frequency signal $e^{\imath\omega t}$, according to the formula (7) and (8):

$$\begin{aligned}
&s_{CB}(t)\\
&= Re\{s_{BB}(t) * e^{i\omega t}\}\\
&= Re\{s_{BB}(t)\} * \cos(\omega t) - Im\{s_{BB}(t)\} * \sin(\omega t)\\
&= Re\{s_{BB}(t)\} * (e^{-i\omega t} + e^{i\omega t})/2 - Im\{s_{BB}(t)\} * i * (e^{-i\omega t} - e^{i\omega t})/2\\
&= (Re\{s_{BB}(t)\} - Im\{s_{BB}(t)\} * i) * e^{-i\omega t}/2 + (Re\{s_{BB}(t)\} + Im\{s_{BB}(t)\} * i) * e^{i\omega t}/2\\
&= s_{BB}(t)^* * e^{-i\omega t}/2 + s_{BB}(t) * e^{i\omega t}/2
\end{aligned} \tag{9}$$

Where $s_{CB}(t)$ is the modulation signal, $s_{BB}(t)$ is the baseband signal, $s_{BB}(t)^*$ is the conjugated baseband signal, $e^{i\omega t}$ is the R-frequency signal. According to the signals multiplication in time domain is equivalent to the signals convolution in frequency domain, it can be seen from the above equation that the real-carrier modulation will move the baseband to the R-band and L-band, and the signal energy is divided equally.

Therefore, after modulated by the R-frequency signal $e^{i\omega t}$ and take the real part, the R-band signal is the same as the baseband signal, and the amplitude is a half, the L-band signal is conjugated symmetry to the baseband signal, and the amplitude is also a half.

Similarly, if it is modulated by the L-frequency signal $e^{-i\omega t}$:

$$s_{CB}(t) = Re\{s_{BB}(t) * e^{-i\omega t}\} = s_{BB}(t) * e^{-i\omega t}/2 + s_{BB}(t)^* * e^{i\omega t}/2 \tag{10}$$

Where $s_{CB}(t)$ is the modulation signal, $s_{BB}(t)$ is the baseband signal, $s_{BB}(t)^*$ is the conjugated baseband signal, $e^{-i\omega t}$ is the L-frequency signal.

Therefore, after modulated by the L-frequency signal $e^{-i\omega t}$ and take the real part, the L-band signal is the same as the baseband signal, and the amplitude is a half, the R-band signal is conjugated symmetry to the baseband signal, and the amplitude is also a half.

Summary, the real-carrier modulation occupies all the L-band and R-band, and the information on the L-band and R-band are conjugated symmetric, not independent, and the signal energy is a half on each side.

A demo of band move of the real-carrier modulation is shown in Fig. 4, modulated by the R-frequency signal, amplitude spectrum.

3.2 Real-carrier demodulation

The current real-carrier demodulation also assumed to receive a real signal, so it only multiplies with a real signal. As the real-carrier modulation does not distinguish L-frequency and R-frequency, so the modulation signal maybe modulated by the L-frequency signal or the R-frequency signal, here assumes the modulation signal is modulated by the R-frequency signal, as formula (9).

If demodulated by the L-frequency signal $e^{-i\omega t}$, according to the formula (9):

$$\begin{aligned} s_{RBB}(t) &= s_{CB}(t) * e^{-i\omega t} \\ &= (s_{BB}(t)^* * e^{-i\omega t}/2 + s_{BB}(t) * e^{i\omega t}/2) * e^{-i\omega t} \\ &= s_{BB}(t)^* * e^{-2*i\omega t}/2 + s_{BB}(t)/2 \end{aligned} \tag{11}$$

Where $s_{RBB}(t)$ is the demodulation signal, $s_{CB}(t)$ is the modulation signal, $s_{BB}(t)$ is the baseband signal, $s_{BB}(t)^*$ is the conjugated baseband signal, $e^{-i\omega t}$ is the L-frequency signal. After demodulation, the L-band $s_{BB}(t)^*/2$ is moved to two times far away and the R-band

$s_{BB}(t)/2$ is moved to the baseband, after a low-pass filter, the L-band signal energy is discarded.

If demodulated by the R-frequency signal $e^{i\omega t}$, according to the formula (9):

$$\begin{aligned} & s_{RBB}(t) = s_{CB}(t) * e^{i\omega t} \\ & = (s_{BB}(t)^* * e^{-i\omega t}/2 + s_{BB}(t) * e^{i\omega t}/2) * e^{i\omega t} \\ & = s_{BB}(t) * e^{2*i\omega t}/2 + s_{BB}(t)^*/2 \end{aligned} \tag{12}$$

Where $s_{RBB}(t)$ is the demodulation signal, $s_{CB}(t)$ is the modulation signal, $s_{BB}(t)$ is the baseband signal, $s_{BB}(t)^*$ is the conjugated baseband signal, $e^{i\omega t}$ is the R-frequency signal. After demodulation, the R-band $s_{BB}(t)/2$ is moved to two times far away and the L-band $s_{BB}(t)^*/2$ is moved to the baseband, after a low-pass filter, the R-band signal energy is discarded. But the reserved L-band signal is conjugated to original signal.

Although information on the L-band and R-band is conjugated, but filtering out one of them will lost a half of signal energy. Fortunately, just because the information on the L-band and R-band is conjugated, so even the demodulation side confused the L-band and R-band, and the received information is conjugated does not matter, exchange real part and imaginary part can conjugate the signal back.

A demo of band move of the real-carrier demodulation is shown in Fig. 5, demodulated by the L-frequency signal, amplitude spectrum.

4 Complex-carrier modulation and demodulation

As mentioned earlier, the complete description of a frequency signal is a rotating complex signal, in complete descriptions, the L-frequency signal $e^{-i\omega t}$ and R-frequency signal $e^{i\omega t}$ are two distinguishable and independence frequency signals, so they can carry different information. Therefore, we can modulate the baseband signals by the L-frequency or R-frequency signal, in order to distinguish from the real-carrier modulation, it is called complex-carrier modulation in this paper. Because there are two kinds of frequencies, so there are two kinds of complex-carrier modulations, in this paper the modulation using the L-frequency signal is called L-complex modulation, the modulation using the R-frequency signal is called R-complex modulation.

In this section, we will see, compared with the real-carrier modulation, the complex-carrier modulation uses the distinguishable and independence L-frequency signal $e^{-i\omega t}$ and R-frequency signal $e^{i\omega t}$ modulating the baseband signal, they can carry different information, so the spectrum resources are full used, the signal energies carried by complex-carrier modulation is focused on a certain band, so the signal energies will not be lost by the demodulation.

4.1 Complex-carrier modulation

The principle of L-complex modulation is as the following formula:

$$\begin{aligned}
&s_{CB}(t) = s_{BB}(t) * e^{-i\omega t}\\
&= (Re\{s_{BB}(t)\} + i * Im\{s_{BB}(t)\}) * (\cos(\omega t) - i * \sin(\omega t))\\
&= (Re\{s_{BB}(t)\} * \cos(\omega t) + Im\{s_{BB}(t)\} * \sin(\omega t)) + i * (Im\{s_{BB}(t)\} * \cos(\omega t) - Re\{s_{BB}(t)\} * \sin(\omega t))
\end{aligned} \tag{13}$$

Where $s_{CB}(t)$ is the complex-carrier modulation signal, $s_{BB}(t)$ is baseband signal, $e^{-i\omega t}$ is the L-frequency signal.

The principle of R-complex modulation is as the following formula:

$$\begin{aligned}
&s_{CB}(t) = s_{BB}(t) * e^{i\omega t}\\
&= (Re\{s_{BB}(t)\} + i * Im\{s_{BB}(t)\}) * (\cos(\omega t) + i * \sin(\omega t))\\
&= (Re\{s_{BB}(t)\} * \cos(\omega t) - Im\{s_{BB}(t)\} * \sin(\omega t)) + i * (Im\{s_{BB}(t)\} * \cos(\omega t) + Re\{s_{BB}(t)\} * \sin(\omega t))
\end{aligned} \tag{14}$$

Where $s_{CB}(t)$ is the complex-carrier modulation signal, $s_{BB}(t)$ is baseband signal, $e^{i\omega t}$ is the R-frequency signal.

Band move of the L-complex modulation is shown in Fig. 6. Band move of the R-complex modulation is similar and omitted.

The L-frequency and R-frequency are two distinguishable and independence frequency signals, they can carry different information. The band move of carry two different information shown in Fig. 7, the baseband A and B are moved to L-band and R-band respectively.

According to the formula (13) the principle of L-complex modulation shown in Fig. 8:

It can be seen from the figure, the real part and the imaginary part of the L-complex modulation signal are modulated respectively, they are two orthogonal signals in space, so the L-complex modulation signals in the transmission medium are the left rotating complex signals.

The principle of R-complex modulation is similar and omitted.

4.2 Complex-carrier demodulation

In essence, the principle of complex-carrier modulation and demodulation are the same, they are band move, but in opposite directions. So the L-complex modulation signal is demodulated by the R-frequency signal, and the R-complex modulation signal is demodulated by the L-frequency signal.

The L-complex modulation signal is demodulated by the R-frequency signal as the following:

$$s_{RBB}(t) = s_{CB}(t) * e^{i\omega t} = (s_{BB}(t) * e^{-i\omega t}) * e^{i\omega t} = s_{BB}(t) \tag{15}$$

Where is $s_{RBB}(t)$ the demodulation signal, $e^{i\omega t}$ is the R-frequency signal, $s_{CB}(t)$ is the modulation signal, $s_{BB}(t)$ is baseband signal. The band move of the demodulation for the L-complex modulation signal is shown in Fig. 9.

The R-complex modulation signal is demodulated by the L-frequency signal as the following:

$$s_{RBB}(t) = s_{CB}(t) * e^{-i\omega t} = (s_{BB}(t) * e^{i\omega t}) * e^{-i\omega t} = s_{BB}(t) \tag{16}$$

Where is $s_{RBB}(t)$ the demodulation signal, $e^{-i\omega t}$ is the L-frequency signal, $s_{CB}(t)$ is the modulation signal, $s_{BB}(t)$ is baseband signal. The band move of the demodulation for the R-complex modulation signal is is similar and omitted.

The different information on the L-band and R-band can be demodulated respectively, as shown in Fig. 10.

5 Complex signal and electromagnetic signal

According to Euler formula (5), shown in Fig. 3, $cos(\omega t)$ and $sin(\omega t)$ are two orthogonal signals in space, so the physical meaning of "i" is orthogonal in space, therefore, the real part and the imaginary part constitute a rotating complex signal. The superposition of the real part and imaginary part matches the superposition of the electromagnetic vector, so the rotating complex signal is the rotating electromagnetic signal, as shown in Figure 11, two kinds of rotating circularly polarized electromagnetic signals corresponding to the two kinds of rotating complex signals.

In fact, the circularly polarized light in optical fiber communication and the circularly polarized signal in satellite communication is the rotating complex signal. Similarly, the linearly polarized signal in wireless communication is the real signal. So far, although it is well known that the circularly polarized signal has two kinds of directions, but nobody has ever corresponded the circularly polarized signals to the rotating complex signals, because they have not realized that the frequency signal has two kinds of directions.

6 The group properties of band-move and the relativity of frequency

As mentioned earlier, in essence, the principle of complex-carrier modulation and demodulation are the same, they are band move, so in this paper, using a single word band-move instead of the modulation and demodulation. Furthermore, band-move is a transformation, it is the rotational speed transform in time domain, the move transform in frequency domain. The transformation has the following properties:

1: The additive property, one band-move equivalent to the sum of several band-moves, as the following formula:

$$s_{CB}(t) = s_{BB}(t) * e^{i\omega_1 t} * e^{i\omega_2 t} = s_{BB}(t) * e^{i(\omega_1+\omega_2)t} \tag{17}$$

Where $s_{CB}(t)$ is the band-move signal, $s_{BB}(t)$ is the original signal.

2: The commutative property, the result of several band-moves has nothing to do with the orders, as the following formula:

$$s_{CB}(t) = s_{BB}(t) * e^{i\omega_1 t} * e^{i\omega_2 t} = s_{BB}(t) * e^{i\omega_2 t} * e^{i\omega_1 t} \tag{18}$$

Where $s_{CB}(t)$ is the band-move signal, $s_{BB}(t)$ is the original signal.

Furthermore, the transformations of band-move make up a continuous Abel group.

In addition, from the transformation properties, we can also see that the frequency is a relative value, its value is related with the reference frequency, as the following formula:

$$e^{i\omega^* t} = e^{i\omega t} * e^{i\omega_c t} = e^{i(\omega+\omega_c)t} \tag{19}$$

Where $e^{i\omega^* t}$ is the frequency after transform, $e^{i\omega t}$ is the frequency before transform, $e^{i\omega_c t}$ is the reference frequency.

Thus, a "negative frequency" can become a "positive frequency" after the band-move transform, which also confirmed on the other hand, "negative frequency" has a physical meaning, the "sign" of a frequency is related with the reference frequency.

7 Conclusions

In summary, we draw the following conclusions:

1: At present, do not use of "negative frequency" wasted a half of the spectrum.

2: The current carrier modulation is also flawed.

3: Negative frequency has a physical meaning, the complete description of a frequency signal is a rotating complex signal, in complete description, "positive frequency" signal and "negative frequency" signal are two distinguishable and independent frequency signals.

4: The real frequency signal is not complete, it can not determine the "positive and negative".

5: Frequency is a relative value, relates with the reference frequency, therefore the "sign" of the frequency is relative.

6: Real-carrier modulation is not complete, it will occupy all the L-band and R-band, so the spectrum is not sufficiently used.

7: Real-carrier demodulation discards a half of the signal energy, so reduces the channel capacity.

8: The complex-carrier modulation can carry different information by L-band and R-band, the spectrum resources is full used.

9: Complex-carrier demodulation can receive the whole signal energy, so the channel capacity is high.

10: The rotating circular polarization signal is the rotating complex signal, the linear polarization signal is the real signal.

11: Modulation and demodulation is essentially a transformation, make up a continuous Abel group.

Due to the complex-carrier modulation and demodulation technology can make full use of the spectrum, and do not discard the signal energy, so be sure, complex-carrier modulation and demodulation technology will become the mainstream of the next-generation communication technology. For example, in the current LTE technology, if the two code-words are modulated by the L-band and R-band respectively, then clearly the channel capacity will be greatly improved.

References

[1] Chen, Huaichen, Fang Haiyan, "On The Physical Meaning of Negative Frequency in Spectrum", Journal of Electrical & Electronic Education, Jan. 2008

[2] Richard Lyons, "Quadrature Signals: complex, but not complicated", http://www.dspguru.com/info/tutor/quadsig2.htm, Nov. 2000

[3] 3GPP TS 36.101 V9.5.0, Oct. 2010

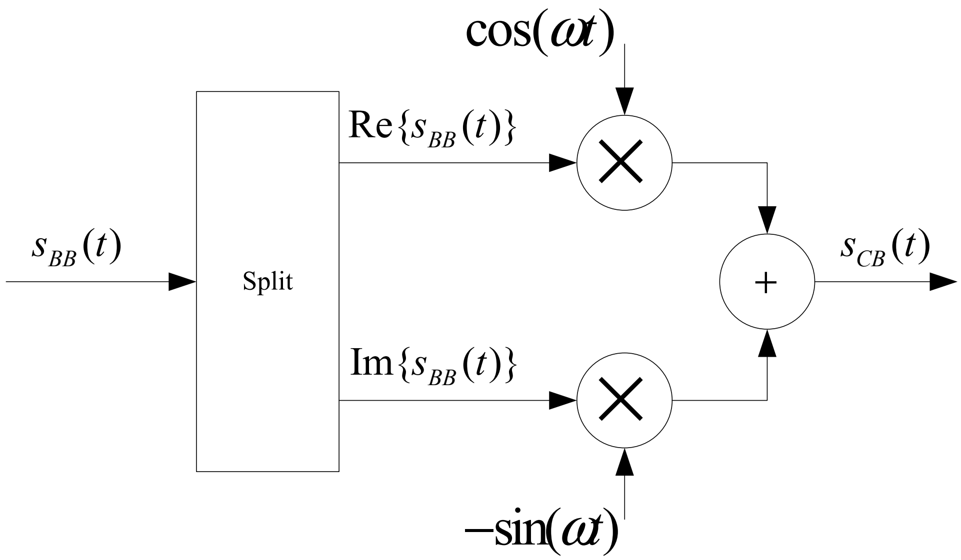

Figure 1: Principle of real-carrier modulation

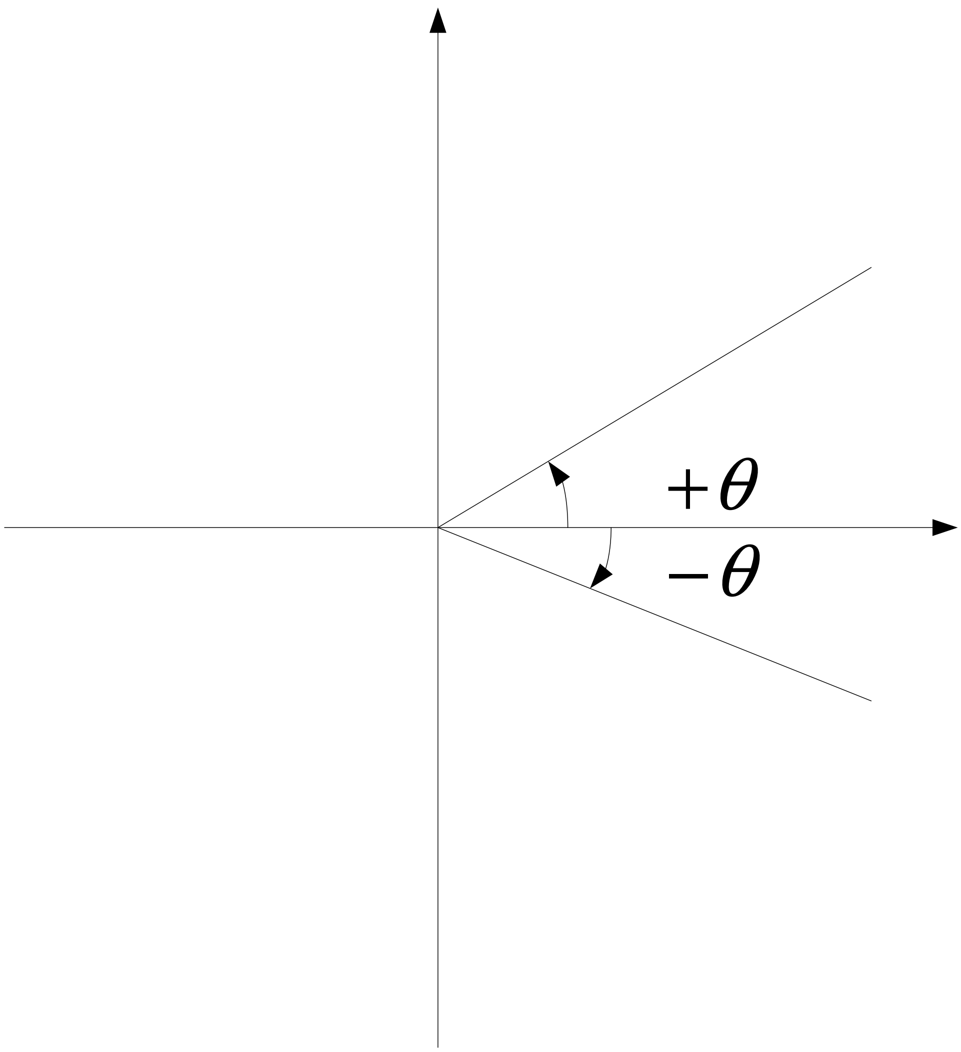

Figure 2: Definition of the angle

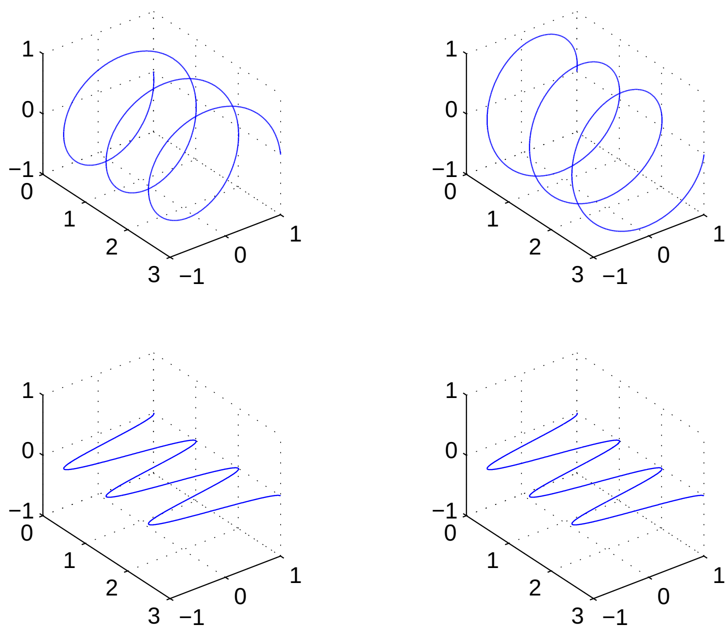

Figure 3: Positive and negative frequency signal and its projection

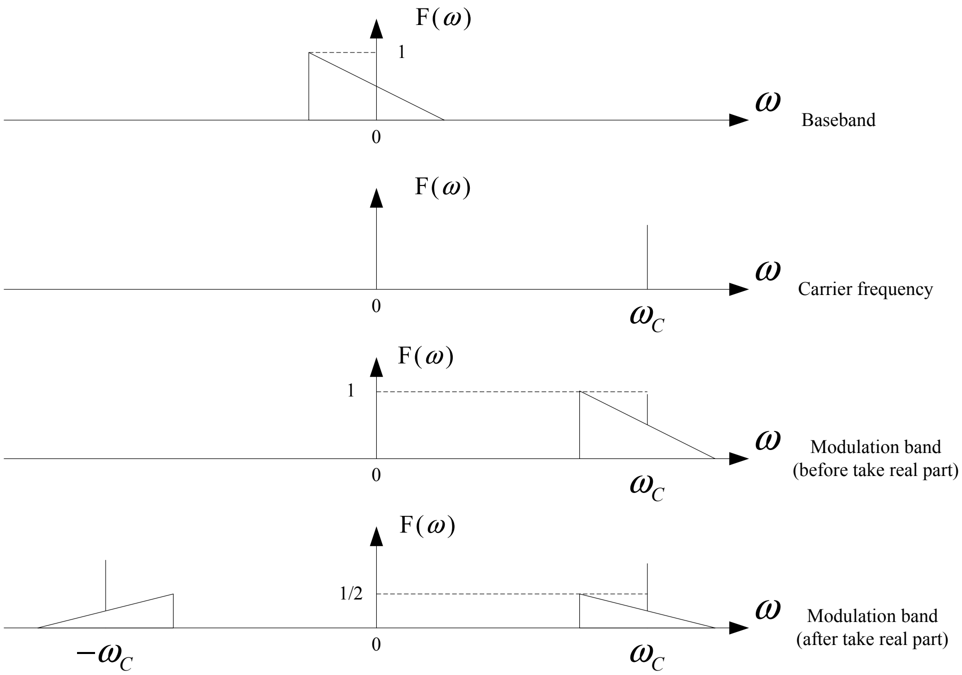

Figure 4: Band move of the real-carrier modulation

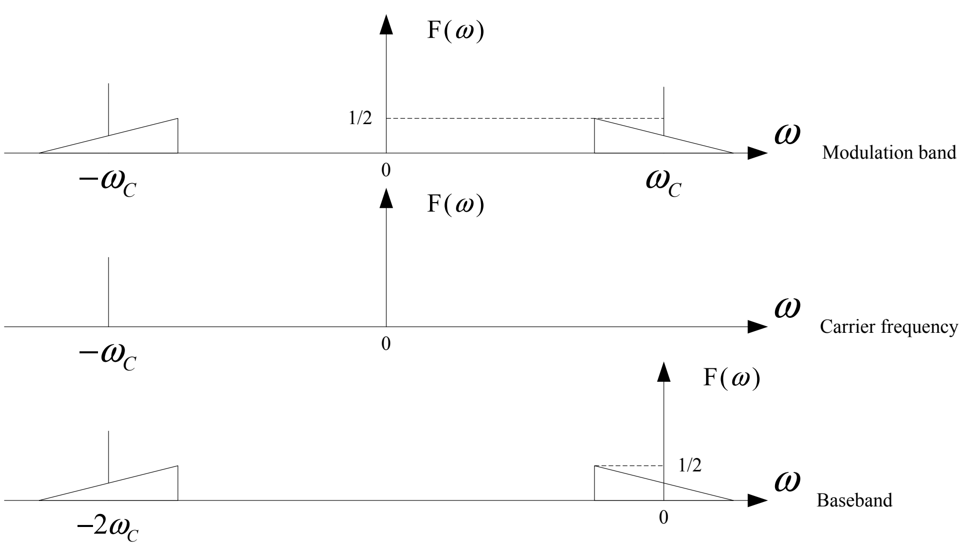

Figure 5: Band move of the real-carrier demodulation

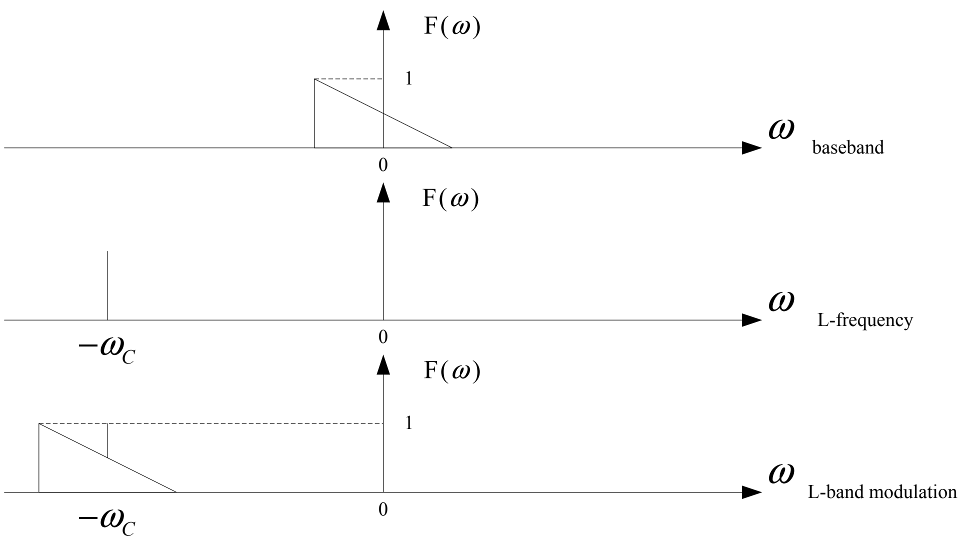

Figure 6: Band move of the L-complex modulation

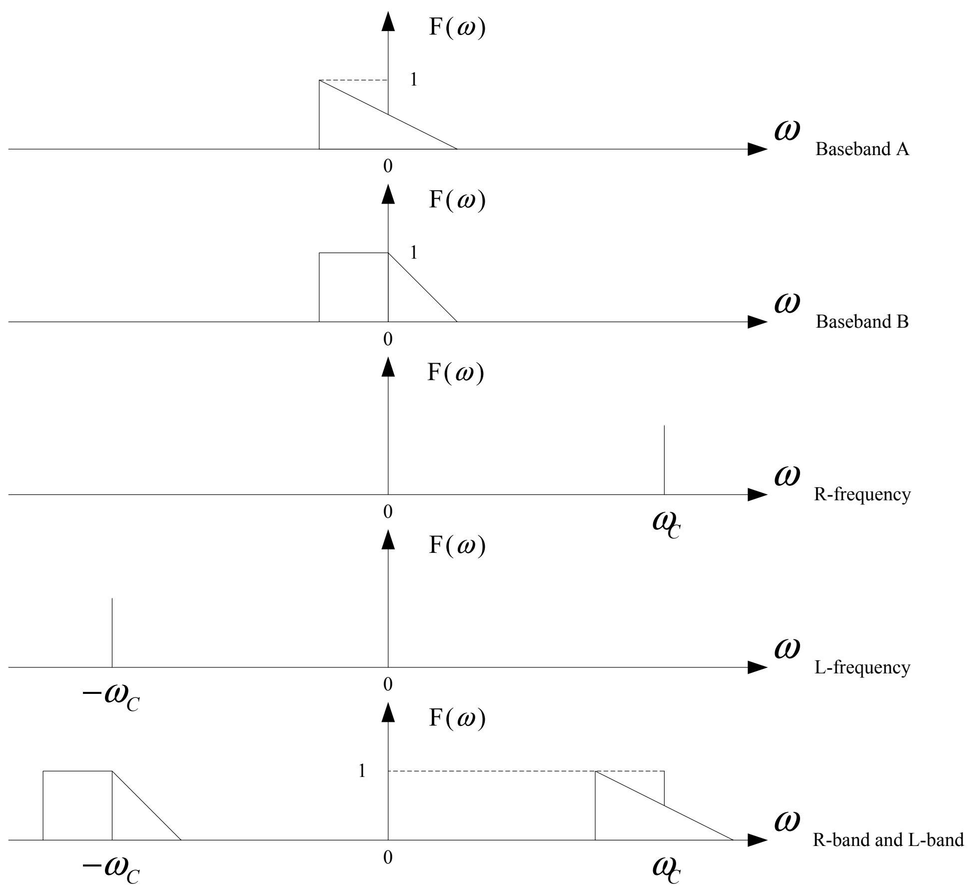

Figure 7: Band move of modulate two different information

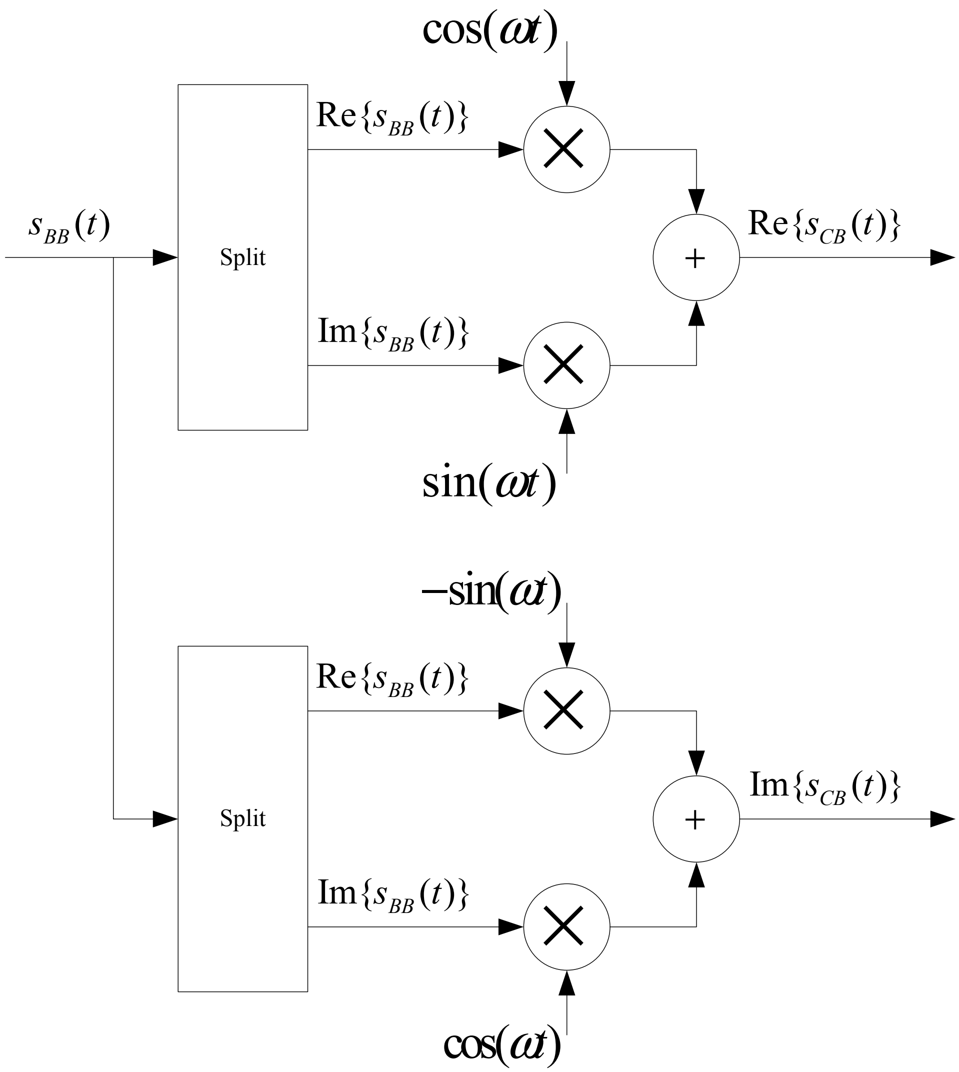

Figure 8: Principle of L-complex modulation

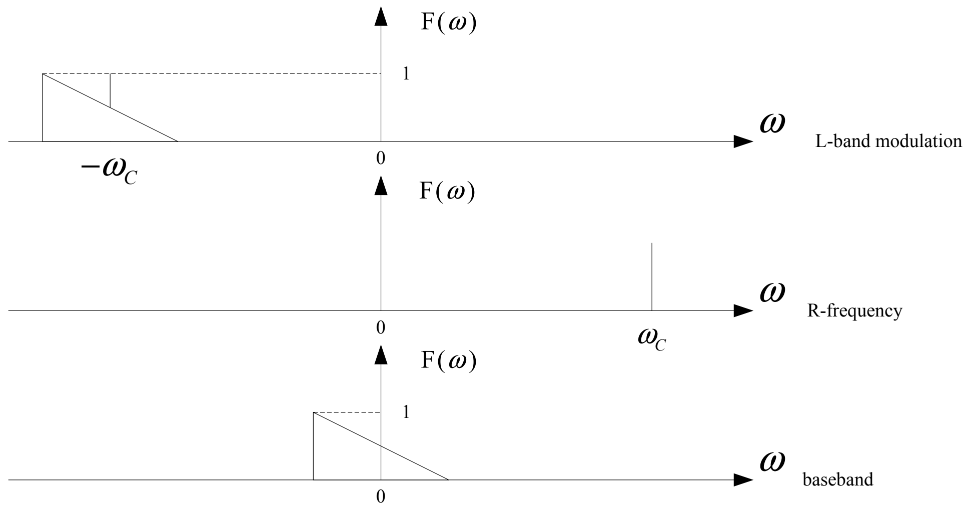

Figure 9: Band move of the demodulation for the L-complex modulation signal

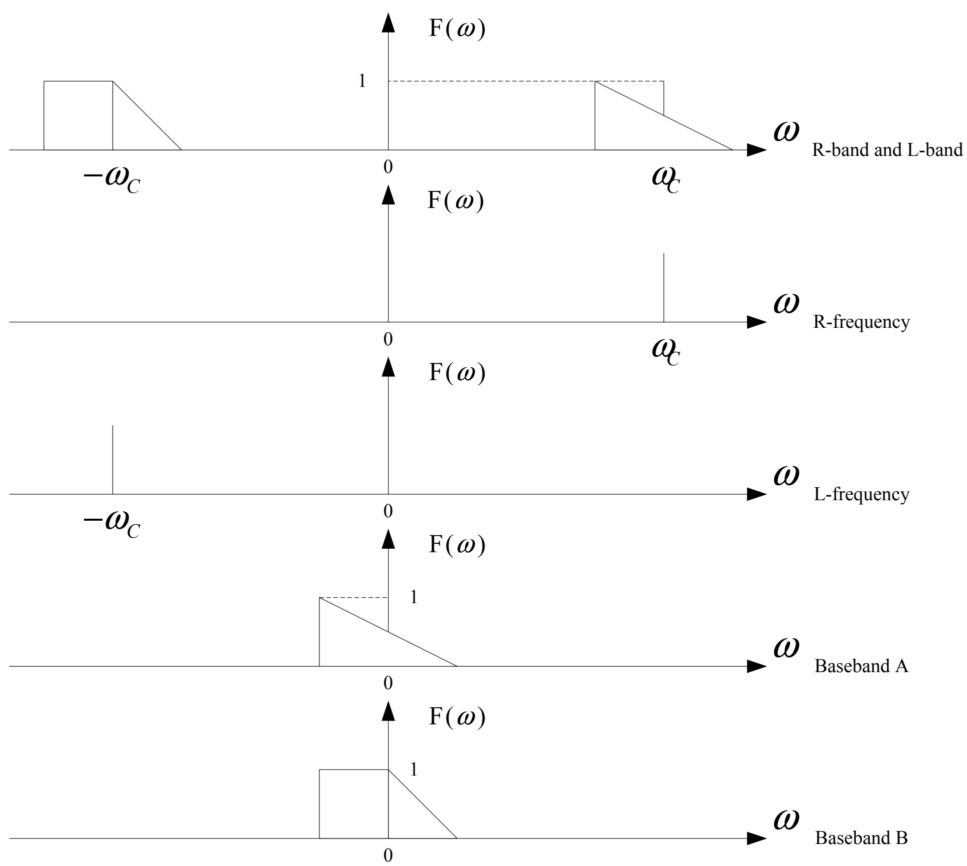

Figure 10: Band move of demodulate two different information

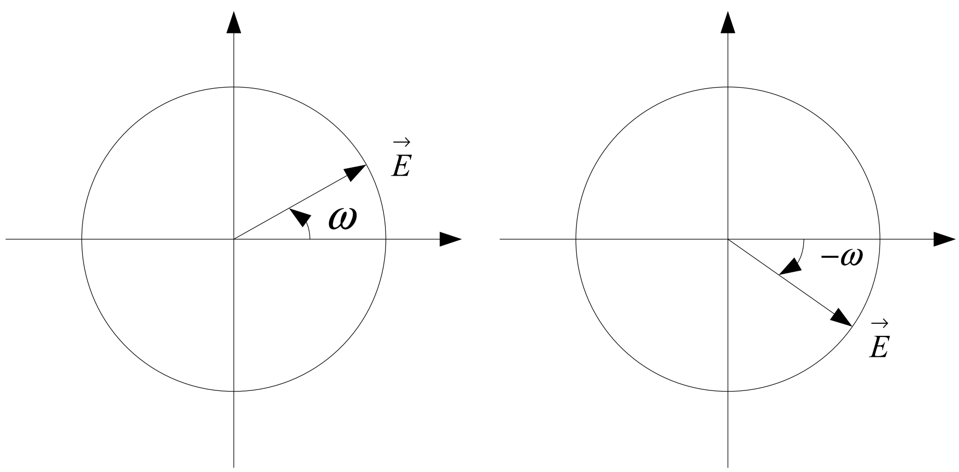

Figure 11: Two kinds of circularly polarized electromagnetic signals