

Perturbational Blowup Solutions to the 1-dimensional Compressible Euler Equations

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Abstract

We study the construction of analytical non-radially solutions for the 1-dimensional compressible adiabatic Euler equations in this article. We could design the perturbational method to construct a new class of analytical solutions. In details, we perturb the linear velocity:

$$u = c(t)x + b(t) \tag{1}$$

and substitute it into the compressible Euler equations. By comparing the coefficients of the polynomial, we could deduce the corresponding functional differential system of $(c(t), b(t), \rho^{\gamma-1}(0, t))$.

Then by skillfully applying the Hubble's transformation:

$$c(t) = \frac{\dot{a}(t)}{a(t)}, \tag{2}$$

the functional differential equations can be simplified to be the system of $(a(t), b(t), \rho^{\gamma-1}(0, t))$.

After proving the existence of the corresponding ordinary differential equations, a new class of blowup or global solutions can be shown. Here, our results fully cover the previous known ones by choosing $b(t) = 0$.

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1 Introduction

The isentropic compressible Euler equations can be written in the following form:

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0 \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u) + \nabla P = 0. \end{cases} \quad (3)$$

As usual, $\rho = \rho(x, t)$ and $u = u(x, t) \in \mathbf{R}^N$ are the density and the velocity respectively with $x = (x_1, x_2, \dots, x_N) \in \mathbf{R}^N$. For some fixed $K > 0$, we have a γ -law on the pressure $P = P(\rho)$, i.e.

$$P(\rho) = K\rho^\gamma \quad (4)$$

which is a common hypothesis. The constant $\gamma = c_P/c_v > 1$, where c_P, c_v are the specific heats per unit mass under constant pressure and constant volume respectively, is the ratio of the specific heats, that is, the adiabatic exponent. For the solutions in radially symmetry:

$$\rho(x, t) = \rho(r, t) \text{ and } u(x, t) = \frac{x}{r}V(r, t) =: \frac{x}{r}V \quad (5)$$

where the radial $r = \sum_{i=1}^N x_i^2$,

the compressible Euler equations become,

$$\begin{cases} \rho_t + V\rho_r + \rho V_r + \frac{N-1}{r}\rho V = 0 \\ \rho(V_t + VV_r) + \nabla P = 0 \end{cases} \quad (6)$$

For the studies of the compressible Euler equations, please see [1] and [2]. Recently, there are some research concerning the construction of solutions of the compressible Euler or Navier-Stokes equations by the substitutional method [3], [4], [5] and [6]. Li and Wang assumed the linear velocity

$$u(x, t) = c(t)x \quad (7)$$

and substituted it into the system to derive the dynamic system about the time function $c(t)$ [4]. Then they used the standard argument of phase diagram to drive the blowup or global existence

of the ordinary differential equation $c(t)$.

On the other hand, the separation method can be governed to seek the radial symmetric solutions by the functional form

$$\rho(r, t) = \frac{f\left(\frac{r}{a(t)}\right)}{a^N(t)} \text{ and } V(r, t) = \frac{\dot{a}(t)}{a(t)} r \quad (8)$$

([7], [8], [9], [3], [5] and [10]).

It is natural to consider the more general linear velocity

$$u(x, t) = c(t)x + b(t) \quad (9)$$

to construct analytical solutions. In this article, we can combine the two conventional approaches (substitutional method and separation method) to derive the corresponding solutions. In fact, the main idea is to substitute the linear velocity into the compressible Euler equations and compare the coefficients of the different polynomial degree to deduce the corresponding functional differential equations $(c(t), b(t), \rho^{\gamma-1}(0, t))$. We apply the Hubble's transformation

$$c(t) = \frac{\dot{a}(t)}{a(t)} \quad (10)$$

to simply the system of ordinary differential equations $(a(t), b(t), \rho^{\gamma-1}(0, t))$. After proving the existences of the corresponding ordinary differential equations, we can show the below theorem:

Theorem 1 *There exists a family of solutions for the 1-dimensional compressible Euler equations*

(3),

$$\left\{ \begin{array}{l} \rho^{\gamma-1}(x, t) = \max \left\{ \rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} \left[\dot{b}(t) + b(t) \frac{\dot{a}(t)}{a(t)} \right] x - \frac{(\gamma-1)\xi}{2K\gamma a^{\gamma+1}(t)} x^2, 0 \right\} \\ u(x, t) = \frac{\dot{a}(t)}{a(t)} x + b(t) \\ \ddot{a}(t) = \frac{\xi}{a^{\gamma}(t)}, \quad a(0) = a_0 > 0, \quad \dot{a}(0) = a_1 \\ \ddot{b}(t) + \frac{(1+\gamma)\dot{a}(t)}{a(t)} \dot{b}(t) + \left[\frac{2\xi}{a^{\gamma+1}(t)} + (\gamma-1) \frac{\dot{a}^2(t)}{a^2(t)} \right] b(t) = 0, \quad b(0) = b_0, \quad \dot{b}(0) = b_1 \\ \frac{\partial}{\partial t} \rho^{\gamma-1}(0, t) + \rho^{\gamma-1}(0, t) \frac{\dot{a}(t)}{a(t)} - \frac{\gamma-1}{K\gamma} \left[\dot{b}(t) + b(t) \frac{\dot{a}(t)}{a(t)} \right] b(t) = 0, \quad \rho(0, 0) = \alpha \end{array} \right. \quad (11)$$

where a_0, a_1, b_1, b_2 and α are arbitrary constants.

Our solutions (11) fully cover the previous known ones for the 1-dimensional case in [3] and [4] with $b_0 = b_1 = 0$.

2 Perturbational Method

Our method does not rely on the phase diagram to show the blow or global existence. The main tool is to only use the previous well known properties of the Emden equation $a(t)$ (11)₃.

Proof. First, we perturb the velocity as this form:

$$u(x, t) = c(t)x + b(t). \quad (12)$$

The 1-dimensional momentum equation (3)₂ becomes for the non-trivial solutions:

$$(u_t + uu_x) + K \frac{1}{\rho} \frac{\partial \rho^\gamma}{\partial x} = 0 \quad (13)$$

for $\gamma > 1$,

$$\dot{c}(t)x + \dot{b}(t) + [c(t)x + b(t)]c(t) + \frac{K\gamma}{\gamma-1} \frac{\partial}{\partial x} \rho^{\gamma-1} = 0 \quad (14)$$

$$\frac{K\gamma}{\gamma-1} \frac{\partial}{\partial x} \rho^{\gamma-1} = -[\dot{b}(t) + b(t)c(t)] - [\dot{c}(t) + c^2(t)]x. \quad (15)$$

We take integration from $[0, x]$ to have:

$$\frac{K\gamma}{\gamma-1} \int_0^x \frac{\partial}{\partial s} \rho^{\gamma-1} ds = -[\dot{b}(t) + b(t)c(t)] \int_0^x ds - [\dot{c}(t) + c^2(t)] \int_0^x s ds \quad (16)$$

$$\frac{K\gamma}{\gamma-1} [\rho^{\gamma-1}(x, t) - \rho^{\gamma-1}(0, t)] = -[\dot{b}(t) + b(t)c(t)]x - \frac{\dot{c}(t) + c^2(t)}{2} x^2 \quad (17)$$

$$\rho^{\gamma-1}(x, t) = \rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} [\dot{b}(t) + b(t)c(t)]x - \frac{\gamma-1}{2K\gamma} [\dot{c}(t) + c^2(t)]x^2. \quad (18)$$

On the other hand, for the 1-dimensional mass equation (3)₁, we obtain

$$\rho_t + \rho_x u + \rho u_x = 0 \quad (19)$$

$$\rho_t + [c(t)x + b(t)] \rho_x + \rho c(t) = 0. \quad (20)$$

We multiple $\rho^{\gamma-2}$ on both sides to have

$$\left(\frac{\rho^{\gamma-1}}{\gamma-1} \right)_t + [c(t)x + b(t)] \left(\frac{\rho^{\gamma-1}}{\gamma-1} \right)_x + \rho^{\gamma-1} c(t) = 0. \quad (21)$$

We substitute back to the above equation with equation (18):

$$\left(\frac{\rho^{\gamma-1}}{\gamma-1} \right)_t + [c(t)x + b(t)] \left(\frac{\rho^{\gamma-1}}{\gamma-1} \right)_x + \rho^{\gamma-1} c(t) \quad (22)$$

$$= \frac{1}{\gamma-1} \left(\frac{\partial}{\partial t} \rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} \frac{\partial}{\partial t} [\dot{b}(t) + b(t)c(t)]x - \frac{\gamma-1}{2K\gamma} \frac{\partial}{\partial t} [\dot{c}(t) + c^2(t)]x^2 \right) \quad (23)$$

$$+ [c(t)x + b(t)] \cdot \frac{1}{\gamma-1} \left(-\frac{\gamma-1}{K\gamma} \frac{\partial}{\partial x} [\dot{b}(t) + b(t)c(t)]x - \frac{\gamma-1}{2K\gamma} \frac{\partial}{\partial x} [\dot{c}(t) + c^2(t)]x^2 \right) \quad (24)$$

$$+ c(t) \left[\rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} [\dot{b}(t) + b(t)c(t)]x - \frac{\gamma-1}{K\gamma} \frac{[\dot{c}(t) + c^2(t)]}{2} x^2 \right] \quad (25)$$

$$= \frac{1}{\gamma-1} \left(\frac{\partial}{\partial t} \rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} \frac{\partial}{\partial t} [\dot{b}(t) + b(t)c(t)]x - \frac{\gamma-1}{2K\gamma} \frac{\partial}{\partial t} [\dot{c}(t) + c^2(t)]x^2 \right) \quad (26)$$

$$+ [c(t)x + b(t)] \left(-\frac{1}{K\gamma} [\dot{b}(t) + b(t)c(t)] - \frac{1}{K\gamma} [\dot{c}(t) + c^2(t)]x \right) \quad (27)$$

$$+ c(t) \left[\rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} [\dot{b}(t) + b(t)c(t)]x - \frac{\gamma-1}{2K\gamma} [\dot{c}(t) + c^2(t)]x^2 \right] \quad (28)$$

$$= \frac{1}{\gamma-1} \frac{\partial}{\partial t} \rho^{\gamma-1}(0, t) + c(t) \rho^{\gamma-1}(0, t) - \frac{b(t)}{K\gamma} [\dot{b}(t) + b(t)c(t)] \quad (29)$$

$$+ \left\{ \begin{array}{l} -\frac{1}{K\gamma} \frac{\partial}{\partial t} [\dot{b}(t) + b(t)c(t)] - \frac{c(t)}{K\gamma} [\dot{b}(t) + b(t)c(t)] \\ -\frac{b(t)}{K\gamma} [\dot{c}(t) + c^2(t)] - \frac{(\gamma-1)c(t)}{K\gamma} [\dot{b}(t) + b(t)c(t)] \end{array} \right\} x \quad (30)$$

$$+ \left\{ \begin{array}{l} -\frac{1}{2K\gamma} \frac{\partial}{\partial t} [\dot{c}(t) + c^2(t)] - \frac{c(t)}{K\gamma} [\dot{c}(t) + c^2(t)] \\ -\frac{(\gamma-1)c(t)}{2K\gamma} [\dot{c}(t) + c^2(t)] \end{array} \right\} x^2 \quad (31)$$

By comparing the coefficients of the polynomial, we require the functional differential equations involving $(c(t), b(t), \rho^{\gamma-1}(0, t))$:

$$\left\{ \begin{array}{l} \frac{d}{dt} \rho^{\gamma-1}(0, t) + (\gamma-1)c(t)\rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} b(t) [\dot{b}(t) + b(t)c(t)] = 0 \\ \frac{d}{dt} [\dot{b}(t) + b(t)c(t)] + \gamma c(t) [\dot{b}(t) + b(t)c(t)] + b(t) [\dot{c}(t) + c^2(t)] = 0 \\ \frac{d}{dt} [\dot{c}(t) + c^2(t)] + (\gamma+1) [\dot{c}(t) + c^2(t)] c(t) = 0 \end{array} \right. \quad (32)$$

to solve the 1-dimensional compressible Euler system.

For details (existence, uniqueness and continuous dependence) about theories of functional differential equations, the interested reader could see the classical literatures [11] and [12].

Here, we solve equation (32)₃ with the Hubble's expression for $c(t)$:

$$c(t) = \frac{\dot{a}(t)}{a(t)} \quad (33)$$

$$\frac{d}{dt} \left[\frac{d}{dt} \left(\frac{\dot{a}(t)}{a(t)} \right) + \frac{\dot{a}^2(t)}{a^2(t)} \right] + (\gamma+1) \left[\frac{d}{dt} \left(\frac{\dot{a}(t)}{a(t)} \right) + \frac{\dot{a}^2(t)}{a^2(t)} \right] \frac{\dot{a}(t)}{a(t)} = 0 \quad (34)$$

$$\frac{d}{dt} \left[\frac{\ddot{a}(t)}{a(t)} - \frac{\dot{a}^2(t)}{a^2(t)} + \frac{\dot{a}^2(t)}{a^2(t)} \right] + (\gamma + 1) \left[\frac{\ddot{a}(t)}{a(t)} - \frac{\dot{a}^2(t)}{a^2(t)} + \frac{\dot{a}^2(t)}{a^2(t)} \right] \frac{\dot{a}(t)}{a(t)} = 0 \quad (35)$$

$$\begin{cases} \frac{d}{dt} \left[\frac{\ddot{a}(t)}{a(t)} \right] + (\gamma + 1) \frac{\ddot{a}(t)}{a(t)} \frac{\dot{a}(t)}{a(t)} = 0 \\ a(0) = a_0 > 0, \dot{a}(0) = a_1, \ddot{a}(0) = a_2 \end{cases} \quad (36)$$

$$\frac{\ddot{a}(t)}{a(t)} - \frac{\dot{a}(t)\ddot{a}(t)}{a^2(t)} + (\gamma + 1) \frac{\dot{a}(t)\ddot{a}(t)}{a^2(t)} = 0 \quad (37)$$

$$\frac{\ddot{a}(t)}{a(t)} + \gamma \frac{\dot{a}(t)\ddot{a}(t)}{a^2(t)} = 0. \quad (38)$$

We multiple $a^{\gamma+1}(t)$ on both sides:

$$a^\gamma(t)\ddot{a}(t) + \gamma a^{\gamma-1}(t)\dot{a}(t)\ddot{a}(t) = 0. \quad (39)$$

Here, we can observe it can reduce to the Emden equation:

$$\begin{cases} \ddot{a}(t) = \frac{\xi}{a^\gamma(t)} \\ a(0) = a_0 > 0, \dot{a}(0) = a_1 \end{cases} \quad (40)$$

where $\xi = a_0^\gamma a_2$ are an arbitrary constant by choosing a_2 .

We remark that the Emden equation (40) was well studied in the literature of astrophysics and mathematics. The local existence of the Emden equation (40) can be promised by the fixed point theorem.

For the second equation (32)₂ of the dynamic system, we have

$$\frac{d}{dt} \left[\dot{b}(t) + b(t) \frac{\dot{a}(t)}{a(t)} \right] + \gamma \left[\dot{b}(t) + b(t) \frac{\dot{a}(t)}{a(t)} \right] \frac{\dot{a}(t)}{a(t)} + \frac{\ddot{a}(t)}{a(t)} b(t) = 0 \quad (41)$$

$$\ddot{b}(t) + \dot{b}(t) \frac{\dot{a}(t)}{a(t)} + b(t) \frac{d}{dt} \frac{\dot{a}(t)}{a(t)} + \gamma \frac{\dot{a}(t)}{a(t)} \dot{b}(t) + \gamma b(t) \frac{\dot{a}^2(t)}{a^2(t)} + \frac{\ddot{a}(t)}{a(t)} b(t) = 0 \quad (42)$$

$$\ddot{b}(t) + \frac{(1+\gamma)\dot{a}(t)}{a(t)} \dot{b}(t) + \left[\frac{\ddot{a}(t)}{a(t)} - \frac{\dot{a}^2(t)}{a^2(t)} + \frac{\gamma\dot{a}^2(t)}{a^2(t)} + \frac{\ddot{a}(t)}{a(t)} \right] b(t) = 0 \quad (43)$$

$$\begin{cases} \ddot{b}(t) + \frac{(1+\gamma)\dot{a}(t)}{a(t)} \dot{b}(t) + \left[\frac{2\xi}{a^{\gamma+1}(t)} + (\gamma-1) \frac{\dot{a}^2(t)}{a^2(t)} \right] b(t) = 0 \\ b(0) = b_0, \dot{b}(0) = b_1 \end{cases} \quad (44)$$

with the Emden equation (40) for $a(t)$.

We denote $f_1(t) = \frac{(1+\gamma)\dot{a}(t)}{a(t)}$ and $f_2(t) = \left[\frac{2\xi}{a^{\gamma+1}(t)} + (\gamma-1) \frac{\dot{a}^2(t)}{a^2(t)} \right]$ to have

$$\begin{cases} \ddot{b}(t) + f_1(t)\dot{b}(t) + f_2(t)b(t) = 0 \\ b(0) = b_0, \dot{b}(0) = b_1. \end{cases} \quad (45)$$

Therefore, when the functions $f_1(t)$ and $f_2(t)$ are bounded, that is

$$|f_1(t)| < F_1 \text{ and } |f_2(t)| < F_2 \quad (46)$$

with the constant F_1 and F_2 , provided that $a(t) \neq 0$ and $\dot{a}(t)$ exist for $0 \leq t < T$, the functions $b(t)$ and $\dot{b}(t)$ exist and are bounded by the comparison theorem of ordinary differential equations.

For the first equation (32)₁, as it is a first order ordinary differential equations only, we can solve direct

$$\frac{d}{dt}\rho^{\gamma-1}(0, t) + (\gamma - 1)\rho^{\gamma-1}(0, t)\frac{\dot{a}(t)}{a(t)} - \frac{\gamma - 1}{K\gamma} \left[\dot{b}(t) + b(t)\frac{\dot{a}(t)}{a(t)} \right] b(t) = 0. \quad (47)$$

Denote $H(t) = (\gamma - 1)\frac{\dot{a}(t)}{a(t)}$ and $G(t) = \frac{\gamma-1}{K\gamma} \left[\dot{b}(t) + b(t)\frac{\dot{a}(t)}{a(t)} \right] b(t)$ to solve

$$\frac{d}{dt}\rho^{\gamma-1}(0, t) + \rho^{\gamma-1}(0, t)H(t) = G(t) \quad (48)$$

with the bounded $a(t) \neq 0$ and $\dot{a}(t)$ for $0 \leq t < T$.

The formula of the first order ordinary differential equation is

$$\rho^{\gamma-1}(0, t) = \frac{\int_0^t \mu(s)G(s)ds + k}{\mu(t)} \quad (49)$$

with

$$\mu(t) = e^{\int_0^t H(s)ds} \quad (50)$$

and a constant k .

Therefore, we have the density function by equation (18)

$$\rho^{\gamma-1}(x, t) = \rho^{\gamma-1}(0, t) - \frac{\gamma - 1}{K\gamma} \left[\dot{b}(t) + b(t)\frac{\dot{a}(t)}{a(t)} \right] x - \frac{\gamma - 1}{2K\gamma} \frac{\ddot{a}(t)}{a(t)} x^2 \quad (51)$$

$$\rho^{\gamma-1}(x, t) = \rho^{\gamma-1}(0, t) - \frac{\gamma - 1}{K\gamma} \left[\dot{b}(t) + b(t)\frac{\dot{a}(t)}{a(t)} \right] x - \frac{(\gamma - 1)\xi}{2K\gamma a^{\gamma+1}(t)} x^2. \quad (52)$$

For the non-negative density solutions $\rho(x, t)$, we must set

$$\rho^{\gamma-1}(x, t) = \max \left\{ \rho^{\gamma-1}(0, t) - \frac{\gamma - 1}{K\gamma} \left[\dot{b}(t) + b(t)\frac{\dot{a}(t)}{a(t)} \right] x - \frac{(\gamma - 1)\xi}{2K\gamma a^{\gamma+1}(t)} x^2, 0 \right\}. \quad (53)$$

The proof is completed. ■

We notice that the above solutions are not radially symmetric for the function $b(t) \neq 0$. Therefore, the above density solutions ρ , cannot be obtained by separation method of the self-similar

functional, as

$$\rho(x, t) \neq f\left(\frac{x^2}{a(t)}\right)g(a(t)) \text{ and } u(x, t) = \frac{\dot{a}(t)}{a(t)}x + b(t). \quad (54)$$

On the other hand, for the 1-dimensional compressible Euler system in radially symmetry (6), we may replace equation (16) to have the corresponding equation by taking the integration from $[0, r]$:

$$\frac{K\gamma}{\gamma-1} \int_0^r \frac{\partial}{\partial s} \rho^{\gamma-1} ds = -[\dot{b}(t) + b(t)c(t)] \int_0^r ds - [c(t) + c^2(t)] \int_0^r s ds. \quad (55)$$

It is clear to have the corresponding result in radial symmetry:

Theorem 2 *There exists a family of solutions for the 1-dimensional compressible Euler equations in radial symmetry (3):*

$$\begin{cases} \rho^{\gamma-1}(r, t) = \max \left\{ \rho^{\gamma-1}(0, t) - \frac{\gamma-1}{K\gamma} \left[\dot{b}(t) + b(t) \frac{\dot{a}(t)}{a(t)} \right] r - \frac{(\gamma-1)\xi}{2K\gamma a^{\gamma+1}(t)} r^2, 0 \right\} \\ u(r, t) = \frac{\dot{a}(t)}{a(t)}r + b(t). \end{cases} \quad (56)$$

It is clear to see that the solutions (11) and (56) are also the solutions of the 1-dimensional compressible Navier-Stokes equations:

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0 \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u) + \nabla P = \mu \Delta u \end{cases} \quad (57)$$

with a positive constant μ .

3 Blowup or Global Solutions

To determine if the solutions are global or local only, we could use the following lemma about the Emden equation (11)₃.

Lemma 3 *For the Emden equation*

$$\begin{cases} \ddot{a}(t) = \frac{\xi}{a^\kappa(t)} \\ a(0) = a_0 > 0, \dot{a}(0) = a_1 \end{cases} \quad (58)$$

with the constant $\kappa > 1$,

(1) if $\xi < 0$

$$a_1 < \sqrt{\frac{-2\xi}{\kappa-1}} a_0^{\frac{(-\kappa+1)}{2}}, \quad (59)$$

there exists a finite time T , such that

$$\lim_{t \rightarrow T^-} a(t) = 0, \tag{60}$$

otherwise, the solution $a(t)$ exists globally, such that

$$\lim_{t \rightarrow +\infty} a(t) = +\infty. \tag{61}$$

(2) if $\xi = 0$, with $a_1 < 0$, the solution $a(t)$ blows up in

$$T = \frac{-a_0}{a_1}, \tag{62}$$

otherwise, the solution $a(t)$ exists globally.

(3) if $\xi > 0$, the solution $a(t)$ exists globally, such that

$$\lim_{t \rightarrow +\infty} a(t) = +\infty. \tag{63}$$

All the proof can be shown by the standard energy method of classical mechanics. The particular proofs can be found in [9] with $\kappa > 1$ for blowup cases. Therefore, we can omit the proof here.

We observe that the gradient of the velocity is

$$\frac{\partial}{\partial x} u(x, t) = \frac{\dot{a}(t)}{a(t)}. \tag{64}$$

When the function $a(T) = 0$ with a finite time T , $\frac{\partial}{\partial x} u(x, T)$ blows up at every space-point x .

And based on the above lemma about the Emden equation for $a(t)$, (11)₃, it is clear to have the corollary:

Corollary 4 (1a) For $\xi < 0$ and

$$a_1 < \sqrt{\frac{-2\xi}{\kappa - 1}} a_0^{\frac{(-\kappa+1)}{2}}, \tag{65}$$

the solutions (11) and (56) blow up in a finite time T ;

(1b) For $\xi = 0$, with $a_1 < 0$, the solutions (11) blow up in

$$T = \frac{-a_0}{a_1}. \tag{66}$$

(2) otherwise, the solutions (11) and (56) exist globally.

We remark that we also apply this perturbational method to handle the 2-component Camassa-Holm equations

$$\begin{cases} \rho_t + u\rho_x + \rho u_x = 0, & x \in \mathbb{R} \\ m_t + 2u_x m + um_x + \sigma\rho\rho_x = 0 \end{cases} \quad (67)$$

with

$$m = u - \alpha^2 u_{xx} \quad (68)$$

[13] and [14].

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