

Surface Plasmons in Thin Metallic Films for the Case of Antisymmetrical Configuration of Magnetic Field

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Abstract

For the first time it is shown that for thin metallic films thickness of which does not exceed thickness of skin – layer, the problem of description of surface plasma oscillations allows analytical solution under arbitrary ratio of length of electron mean free path and thickness of a film. The dependance of frequency of surface plasma oscillations on wave number is deduced. We consider a case of specular – diffusive boundary value problems.

Key words: degenerate collisional plasma, surface plasma oscillations, thin metallic film.

PACS numbers: 73.50.-h Electronic transport phenomena in thin films, 73.50.Mx High-frequency effects; plasma effects, 73.61.-r Electrical properties of specific thin films.

Introduction

Electromagnetic properties of metal films has been being a subject of great interest for a long time already [1] – [6]. Problem of surface plasma oscillations has been a problem of special interest in recent time [7] – [17]. It is connected as with theoretical interest to this problem, and with numerous practical appendices as well. Thus the majority of researches is founded on the description of properties of films with use of methods of macroscopical electrodynamics. Such approach is inadequate for thin films, since macroscopical electrodynamics is inapplicable for the description of films in the thickness of an

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order of length of mean free path of electrons and less than this length. The electrons scattering on a surface demands kinetic consideration. It complicates the problem significantly.

In the present work it is shown that for thin films, a thickness of which does not exceed a thickness of a skin – layer, the problem of description of surface plasma oscillations allows analytical solution under arbitrary ratio between length of mean free path of electrons and thickness of a film. The given work is a continuation of our work [18] in which the case when values of a magnetic field on top and bottom surface of films coincide was considered. Now the situation, when the signs of these values differ is investigated.

Let us note, that the most part of reasonings carried out below is true for more general case of conductive (in particular, semiconductor) film.

Problem statement

Let us consider a thin metal film.

We take Cartesian coordinate system with origin of coordinates on one of the surfaces of a slab, with axis x , directed deep into the slab and perpendicularly to the surface of the film. The axis z we will direct along the direction of propagation of the surface electromagnetic wave. We will note, that in this case magnetic field is directed along the axis y .

Under such choice of system of coordinates the electric field vector and magnetic field vector have the following structure

$$\mathbf{E} = \{E_x(x, z, t), 0, E_z(x, z, t)\}, \quad \mathbf{H} = \{0, H_y(x, z, t), 0\}.$$

The origin of coordinates we will place at the bottom plane limiting a film. Let us designate a thickness of the film through d .

Out of the film the electromagnetic field is described by the equa-

tions

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \Delta \mathbf{E} = 0$$

and

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} - \Delta \mathbf{H} = 0.$$

Here c is the velocity of light, Δ is the Laplace operator.

The solution of these equations decreasing at infinity point has the following form

$$\mathbf{E} = \begin{cases} \mathbf{E}_1 e^{-i\omega t + \alpha x + ikz}, & x < 0, \\ \mathbf{E}_2 e^{-i\omega t + \alpha(d-x) + ikz}, & x > d, \end{cases} \quad (1a)$$

and

$$\mathbf{H} = \begin{cases} \mathbf{H}_1 e^{-i\omega t + \alpha x + ikz}, & x < 0, \\ \mathbf{H}_2 e^{-i\omega t + \alpha(d-x) + ikz}, & x > d. \end{cases} \quad (1b)$$

Here ω is the wave frequency, k is the wave number, damping parameter α is connected with these quantities by relation

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{c^2}}, \quad (2)$$

\mathbf{E}_j and \mathbf{H}_j ($j = 1, 2$) are constant amplitudies.

Further we search components of intensity vectors of electric and magnetic fields in the following form

$$E_x(x, z, t) = E_x(x) e^{-i\omega t + ikz}, \quad E_z(x, z, t) = E_z(x) e^{-i\omega t + ikz},$$

and

$$H_y(x, z, t) = H_y(x) e^{-i\omega t + ikz}.$$

Then the behaviour of electric and magnetic fields of the wave within the film is described by the following system of the differential

equations ([4] and [19])

$$\begin{cases} \frac{dE_z}{dx} - ikE_x + \frac{i\omega}{c}H_y = 0, \\ \frac{i\omega}{c}E_x - ikH_y = \frac{4\pi}{c}j_x, \\ \frac{dH_y}{dx} + \frac{i\omega}{c}E_z = \frac{4\pi}{c}j_z. \end{cases} \quad (3)$$

Here \mathbf{j} is the current density.

The equations (3) are satisfied out of the slab under the condition $\mathbf{j} = 0$ as well.

Then impedance at the bottom surface of the layer (film) is defined as follows

$$Z^{(j)} = \frac{E_z(-0)}{H_y(-0)}, \quad j = 1, 2. \quad (4)$$

Quantities $Z^{(j)}$ ($j = 1, 2$) correspond to an impedance on the bottom layer surface. At the same time the quantity $Z^{(1)}$ corresponds to magnetic field–symmetrical configuration of an external field. This is the case $j = 1$, when

$$H_y(0) = H_y(d), \quad E_x(0) = E_x(d), \quad E_z(0) = -E_z(d). \quad (5)$$

This case has been considered in [18].

The quantity $Z^{(2)}$ corresponds to configuration of an external field antisymmetric by magnetic field. It is the case $j = 2$, when

$$H_y(0) = -H_y(d), \quad E_x(0) = -E_x(d), \quad E_z(0) = E_z(d). \quad (6)$$

It is required to find a spatial dispersion of the surface plasmon, i.e. to find dependence of frequency of oscillations of eigen mode of the system (3) on quantity of the wave vector $\omega = \omega(k)$.

Let us consider a case when the width of the layer d is less than the depth of the skin – layer δ . We will note, that the depth of a skin – layer depends essentially on frequency of radiation, monotonously

decreasing in the process of growth of the last quantity. The quantity δ possesses minimum value in so-called infra-red case [20] $\delta_0 = \frac{c}{\omega_p}$, where ω_p is the plasma frequency.

For typical metals [20] $\delta_0 \sim 10^{-5}$ cm.

Thus for the films which thickness d is less δ_0 , our assumption is true for any frequencies.

Quantities H_y and E_z vary a little on distances smaller than the depth of a skin – layer. Therefore at performance of the given assumption ($d < \delta_0$) these fields will vary inside of slab.

Surface plasmon. Antisymmetric configuration of magnetic field

Let us consider the case 2 when $E_z(0) = E_z(d)$. We can assume that in this layer z – projection of the electric field E_z is constant. Then magnetic field change on the width of a layer can be defined from the third equation of the system (1)

$$H_y(d) - H_y(0) = -\frac{i\omega}{c}E_zd + \frac{4\pi}{c} \int_0^d j_z(x)dx. \quad (7)$$

Thus

$$j_z(x) = \sigma(x)E_z,$$

where $\sigma(x)$ is the conductivity which in general case depends on coordinate x .

We introduce conductivity averaged by the slab thickness

$$\sigma_d = \frac{1}{E_zd} \int_0^d j_z(x)dx = \frac{1}{d} \int_0^d \sigma(x)dx.$$

Now we can rewrite the relation (7) in the form

$$H_y(d) - H_y(0) = -\frac{i\omega}{c}E_zd + \frac{4\pi\sigma_d d}{c}E_z.$$

Considering symmetry of a magnetic field, from here we have

$$H_y(0) = \frac{i\omega d E_z}{2c} \left(1 + i \frac{4\pi\sigma_d}{\omega}\right).$$

Then for impedance (4) we have

$$Z^{(2)} = -\frac{2ic}{\omega d \left(1 + i \frac{4\pi\sigma_d}{\omega}\right)}. \quad (8)$$

In the same way as well as in (9), we receive

$$Z^{(2)} = \frac{i\alpha c}{\omega}. \quad (9)$$

The dispersive equation for a surface plasma wave can be derived from the expressions (2), (8) and (9)

$$\frac{2c}{\omega d + 4\pi i \sigma_d d} = -\frac{\sqrt{c^2 k^2 - \omega^2}}{\omega}. \quad (10)$$

From the dispersive equation (10) we find the required spatial dispersion

$$k(\omega) = \frac{\omega}{c} \sqrt{1 + \frac{4c^2}{\omega^2 d^2 \left(1 + \frac{4\pi i}{\omega} \sigma_d\right)^2}}. \quad (11)$$

Let us assume, that boundary conditions are specular – diffusive, p is the specularity coefficient. Let the relation $kd \ll 1$ be true. Then in a low-frequency case, when $\omega \rightarrow 0$, the quantity σ_d can be presented in the form [8]

$$\sigma_d = \frac{w}{\Phi(w)} \sigma_0, \quad w = \frac{d}{l}, \quad (12)$$

and

$$\frac{1}{\Phi(w)} = \frac{1}{w} - \frac{3}{2w^2} (1-p) \int_1^\infty \left(\frac{1}{t^3} - \frac{1}{t^5}\right) \frac{1 - e^{-wt}}{1 - pe^{-wt}} dt. \quad (13)$$

Here l is the mean free path of electrons, p is the coefficient of specular reflection (specularity coefficient), $\sigma_0 = \omega_p^2 \tau / (4\pi)$ is the static conductivity of volume pattern, $\tau = l/v_F$ is the electron-transit time, v_F is the Fermi velocity.

For arbitrary frequencies the expression (12) and (13) will hold true under the condition, that it is necessary to use the following expression $l \rightarrow (v_F\tau)/(1 - i\omega\tau)$ as quantity l , and instead of σ_0 it's necessary to use the expression $\sigma_0 \rightarrow \sigma_0/(1 - i\omega\tau)$.

Let us reduce the formula (11) to the form convenient for numerical calculations. We introduce the dimensionless parameters $\varepsilon = \frac{\nu}{\omega_p}$ and $\Omega = \frac{\omega}{\omega_p}$. Then we can transform the formula (19) to the following form

$$k(\Omega, \varepsilon) = \sqrt{\omega_p^2 \frac{\Omega^2}{c^2} + \frac{4}{d^2} \left(1 - \frac{\varphi(\omega)}{\Omega(\Omega - i\varepsilon)}\right)^{-2}}, \quad (14)$$

where

$$\varphi(\omega) = 1 - \frac{1.5}{\omega}(1-p) \int_1^{\infty} \left(\frac{1}{t^3} - \frac{1}{t^5}\right) \frac{1 - e^{-\omega t}}{1 - pe^{-\omega t}} dt.$$

In the case, when electrons reflect under specular condition from the film surface (i.e. at $p = 1$), the formula (14) becomes simpler and looks like

$$k(\omega) = \frac{\omega}{c} \sqrt{1 + \frac{c^2(\nu - i\omega)^2}{d_0^2(\nu\omega - i\omega^2 + i\omega_p^2)^2}}, \quad (15)$$

where $d_0 = \frac{d}{2}$ is the half of a film thickness.

In dimensionless parameters the formula (15) can be written as follows

$$k(\Omega) = \frac{\omega_p}{c} \Omega \sqrt{1 + \frac{c^2(\Omega + i\varepsilon)^2}{(\omega_p d)^2(\Omega^2 - 1 + i\varepsilon\Omega)^2}}. \quad (16)$$

Under $\varepsilon = 0$ we receive from here that

$$k(\Omega) = \frac{\omega_p}{c} \Omega \sqrt{1 + \left(\frac{c}{\omega_p d_0}\right)^2 \left(\Omega - \frac{1}{\Omega}\right)^{-2}}.$$

It is clear that under $\varepsilon = 0$ from the last formula it follows that $\text{Im } k(\Omega) = 0$, i. e. in collisionless plasma plasmon damping is absent.

From the formula (16) it is visible, that there exists such critical frequency $\omega_0 = \omega_0(\varepsilon, d)$, that under $\omega < \omega_p$ $\text{Im } k(\omega) > \text{Re } k(\omega)$, i.e. in field of subcritical frequencies surface plasmons do not exist.

We adduce the table of critical frequencies, referring to plasma (Langmuir) frequencies, $\Omega_0(\varepsilon, d) = \frac{\omega_0(\varepsilon, d)}{\omega_p}$, under $\varepsilon = 10^{-1}$ for the case of specular boundary conditions ($p = 1$).

Table 1 (critical frequencies)

Film thickness d , nm		1	2	3	4
Critical frequency Ω_0		0.101	0.100	0.097	0.092
Film thickness d , nm	5	6	7	8	9
Critical frequency Ω_0	0.086	0.078	0.067	0.051	0.023

Let us adduce graphics on Figs. 1 – 8 of dependencies of real and imaginary parts of the wave vector on the ratio of frequencies ω/ω_p under various values of frequency of electron collisions ν , thickness of a film d and coefficient of specular reflection p . We will consider the case of sodium films, i.e. we take $\omega_p = 6.5 \cdot 10^{15} \text{ sec}^{-1}$, $v_F = 8.52 \cdot 10^7 \text{ cm/sec}$.

Depending on quantity of parameters ν, d, p the quantities $\text{Re } k$ and $\text{Im } k$ can essentially differ. So, at $\nu = 10^{-5}\omega_p$, $d = 10$ nanometer and $p = 1$ (Fig. 1) the quantity $\text{Re } k$ surpasses $\text{Im } k$ on some orders.

From Fig. 1 it is visible, that if to enter quantity $Z = \frac{\text{Re } k}{\text{Im } k}$, then $Z(0.1, 10^{-5}, 10, 1) = 2.1 \cdot 10^4$, $Z(0.5, 10^{-5}, 10, 1) = 3.8 \cdot 10^4$.

Besides, the quantity $\text{Re } k$ is always positive under all values of parameters ω, ν, d, p , while the quantity $\text{Im } k$ can be negative in the field of superhigh frequencies as well.

Let us stop on existence of surface plasma waves (see Figs. 2 and 3).

Depending on quantities ε, d, p two critical frequencies ω_0 and ω_1 , such, that the inequality $\text{Im } k < \text{Re } k$ is true under $\omega_0 < \omega < \omega_1$, and the inequality $\text{Im } k > \text{Re } k$ is true under $0 < \omega < \omega_p$ or $\omega_1 < \omega < \omega_p$ can exist.

In the last case of surface plasma waves do not exist. Let us consider the case $\nu = 10^{-1}\omega_p, p = 0.1$. Results calculations we will present in the following table.

Table 2 (critical frequencies)

Film thickness, nm d	First critical frequency, ω_0	Second critical frequency, ω_1
1	0.168	0.904
2	0.130	0.924
3	0.116	0.929
4	0.107	0.932
5	0.098	0.934
6	0.089	0.935
7	0.077	0.935
8	0.063	0.936
9	0.041	0.936
10	0.000	0.937

The behaviour of the real and imaginary parts of a wave vector in dependence on a thickness of a film is presented on Figs. 4 and 5. The real part has a sharp maximum nearby the plasma resonance $\omega \sim \omega_p$.

Let us note, the more is thickness of a film the less are values of the real part under each value of frequency of oscillations of an electromagnetic field. The imaginary part has the same behaviour. But its values are less than values of real parts significantly. Unlike the real part in the field of the superhigh frequencies the values of the imaginary part become negative.

On Figs. 6 and 7 dependences of the real and imaginary parts of wave vector on quantity of collision frequencies of electrons are presented. The less is the quantity ν , the more are the values of the real part. For the imaginary part inverse relation takes place. Namely, the less is the quantity of collision frequencies, the less is the value of an imaginary part. It means, that at electron collisions frequency increase the attenuation of surface plasma waves becomes stronger (in the field of subcritical frequencies).

On Fig. 8 dependence of an imaginary part of the wave vector on

quantity of coefficient of specular reflection in the field of subcritical frequencies is presented. Graphics show, that with the growth of coefficient of specular reflection the values of an imaginary part decrease. It means, that damping of plasma waves by that becomes the stronger, the less are quantities of coefficient of specular reflection.

Conclusion

In the present work the dispersion relation for surface plasmons is deduced. We consider the case of an antisymmetric configuration of x – component of the electric field and y – component of magnetic field, and symmetric z – component of electric field. We consider the case of specular – diffusive boundary value conditions.

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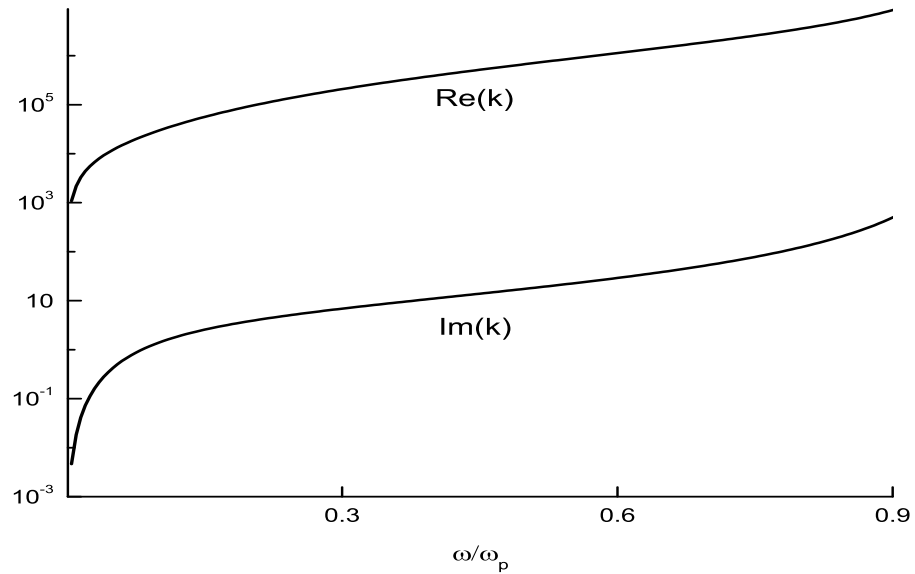


Figure 1. Real and imaginary parts of wave number, film thickness $d = 10$ nm, collision electron frequency $\nu = 10^{-5}\omega_p$ 1/sec, specular reflection, $p = 1$.

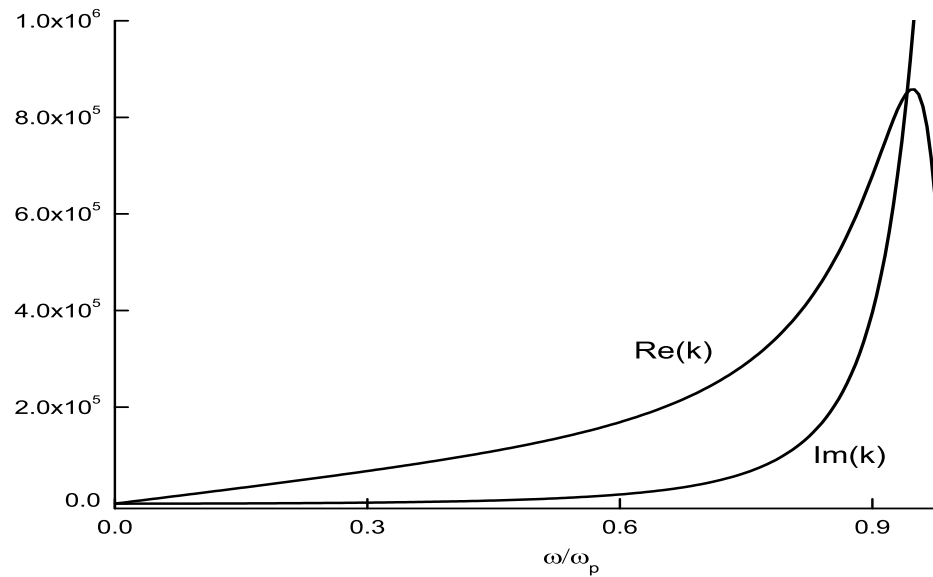


Figure 2. Real and imaginary parts of wave number, film thickness $d = 100$ nm, collision electron frequency $\nu = 0.1\omega_p$ 1/sec, coefficient of specular reflection, $p = 0.1$.

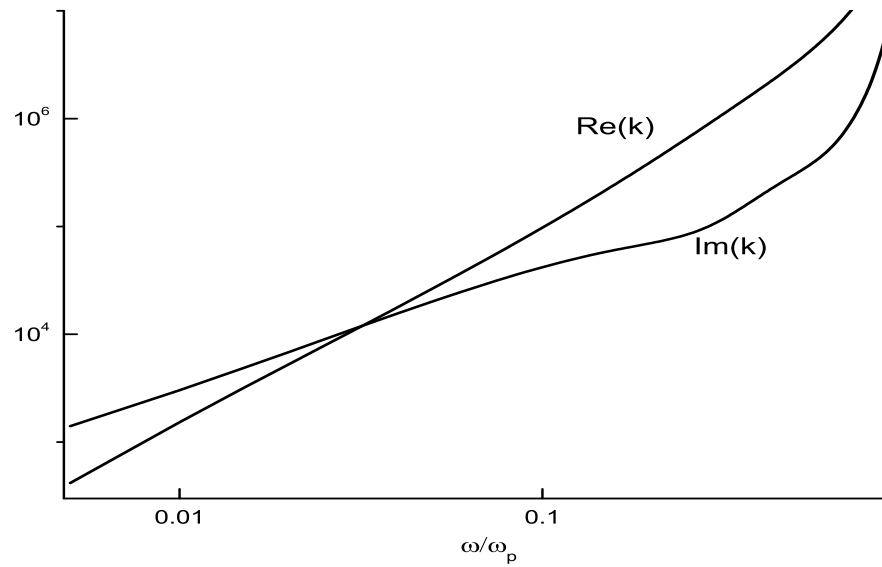


Figure 3. Real and imaginary parts of wave number, film thickness $d = 100$ nm, collision electron frequency $\nu = 0.02\omega_p$ 1/sec, coefficient of specular reflection, $p = 0.1$.

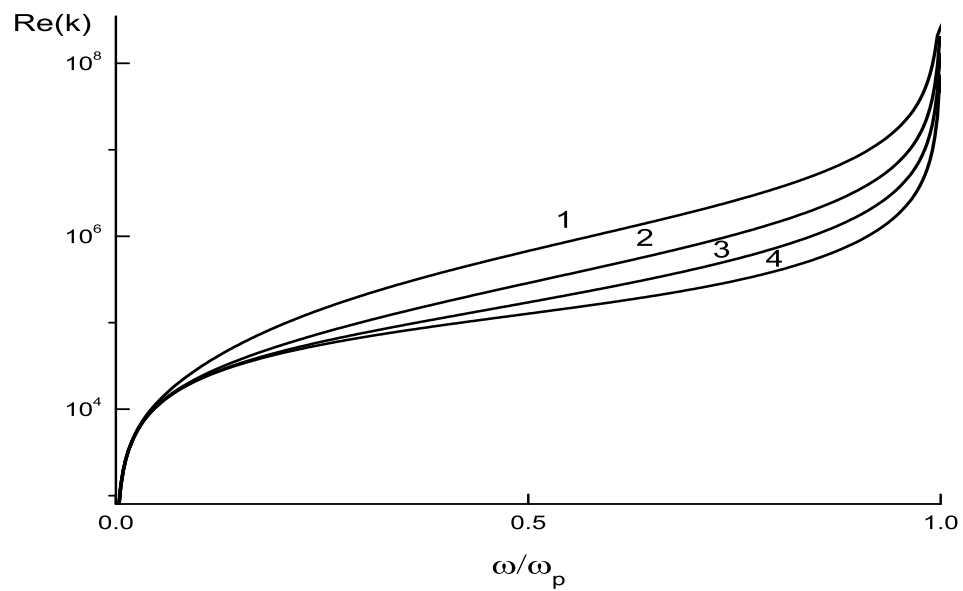


Figure 4. Real and imaginary parts of wave number, collision electron frequency $\nu = 10^{-3}\omega_p$ 1/sec, $p = 0.5$, curves 1, 2, 3, 4 correspond to values of the film thickness $d = 10, 25, 50, 100$ nm.

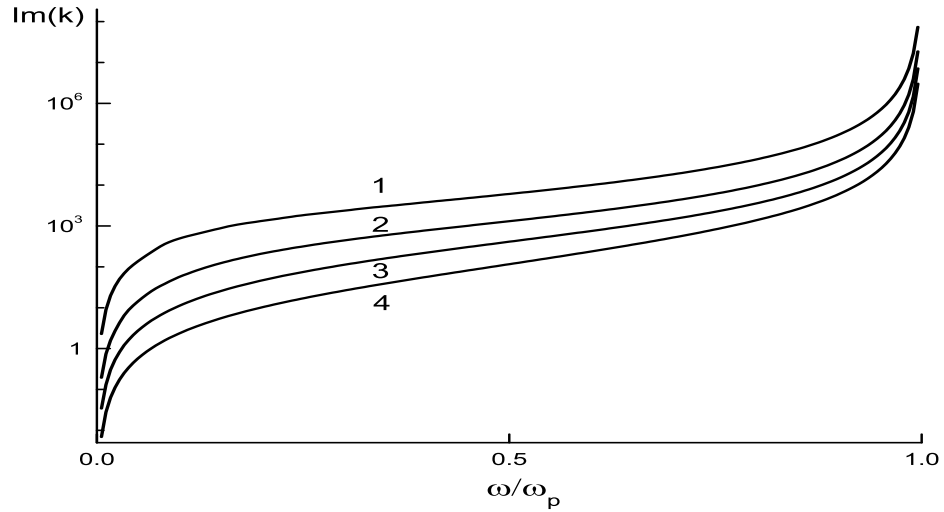


Figure 5. Imaginary part of wave number, $\nu = 10^{-3}\omega_p$ 1/sec, $p = 0.5$, curves 1, 2, 3, 4 correspond to values of the film thickness $d = 10, 25, 50, 100$ nm. Subcritical frequencies: $0 < \omega < \omega_p$ 1/sec.

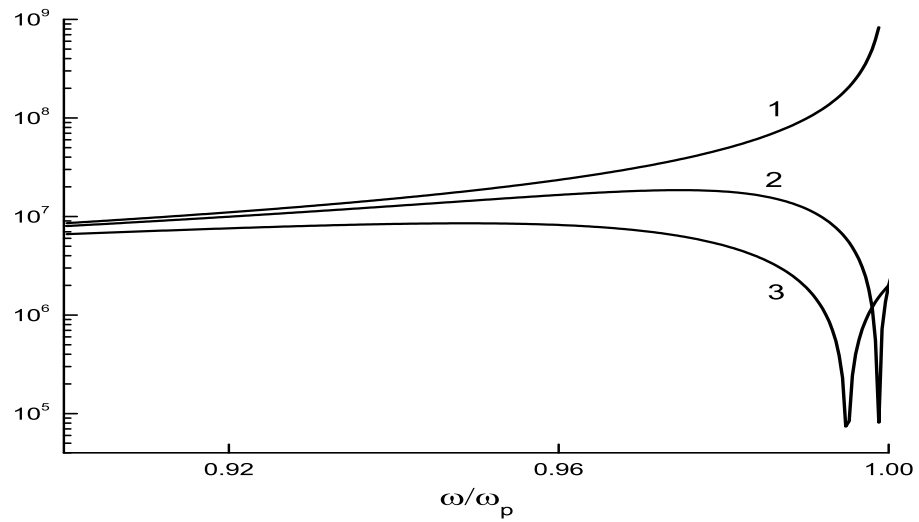


Figure 6. Real part of wave number, curves 1, 2, 3 correspond to values of electron collision frequencies $\nu = 10^{-5}\omega_p, 5 \cdot 10^{-2}\omega_p, 10^{-1}\omega_p$ 1/sec. Film thickness $d = 10$ nm, coefficient of specular reflection $p = 1$ (case of specular reflection).

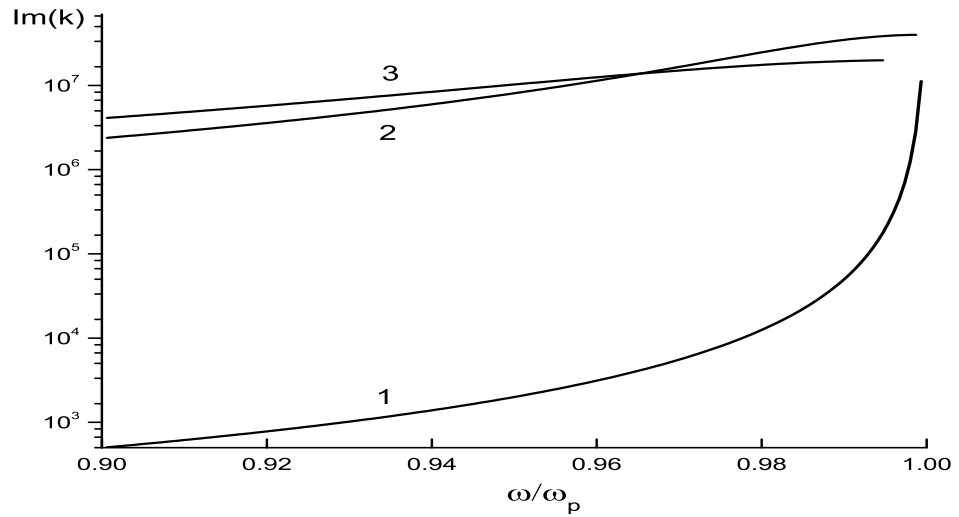


Figure 7. Imaginary part of wave number, curves 1, 2, 3 correspond to values of electron collision frequencies $\nu = 10^{-5}\omega_p, 5 \cdot 10^{-2}\omega_p, 10^{-1}\omega_p$ 1/sec. Film thickness $d = 10$ nm, coefficient of specular reflection $p = 1$ (case of specular reflection).

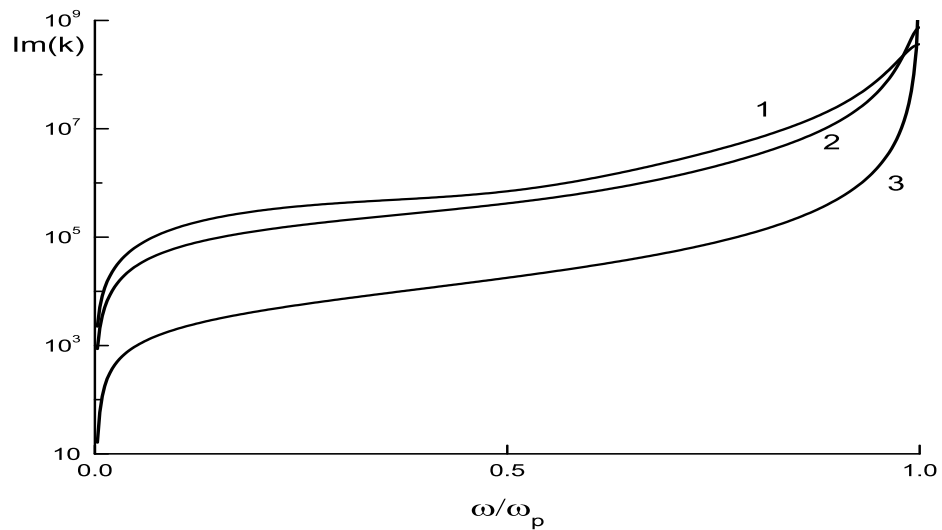


Figure 8. Imaginary part of the wave number $\text{Im } k(\omega)$. Curves 1, 2, 3 correspond to values of coefficient of specular reflection $p = 0, 0.5, 1$. Electron collision frequencies is equal $\nu = 10^{-3}\omega_p$ 1/sec. Film thickness $d = 1$ nm.