

Structure of p-shell hypernuclei

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Abstract. Shell-model calculations that include both Λ and Σ configurations with p-shell cores are used to interpret γ -ray transitions in ${}^7_{\Lambda}\text{Li}$, ${}^9_{\Lambda}\text{Be}$, ${}^{10}_{\Lambda}\text{B}$, ${}^{11}_{\Lambda}\text{B}$, ${}^{12}_{\Lambda}\text{C}$, ${}^{15}_{\Lambda}\text{N}$, and ${}^{16}_{\Lambda}\text{O}$ observed with the Hyperball array of Ge detectors. It is shown that the data puts strong constraints on the spin dependence of the ΛN effective interaction and that the Λ - Σ coupling plays an important role.

1. Introduction

Beginning in the 1970's, experiments at CERN, BNL, and KEK using the (K_{stop}^-, π^-) and in-flight (K^-, π^-) and (π^+, K^+) reactions (see table 2 of [1]) established that the Λ single-particle energies in hypernuclei up to ${}^{208}_{\Lambda}\text{Pb}$ form a textbook example of single-particle behavior, describable by a Woods-Saxon well of depth 28 – 30 MeV [1, 2] (consistent with earlier analyses of B_{Λ} values from emulsion experiments [3]).

The best energy resolution achieved in such experiments was 1.45 MeV using a thin carbon target [4]. Recently, ($e, e'K^+$) experiments in Hall A and Hall C at JLab have achieved resolutions of ~ 700 keV [5] and ~ 400 keV [6], respectively. In general, this is still insufficient to measure the doublet splittings that result when a Λ in an s orbit couples to a state of a p-shell nucleus with non-zero spin.

Because the Λ is in an s orbit, only a spatial monopole interaction operates between the nucleon and the Λ in such doublets. This means that there is little configuration mixing between members of different doublets and that the doublet splittings depend almost entirely on the spin-spin, Λ spin-orbit, and tensor components of the effective ΛN interaction, together with a contribution from Λ - Σ coupling. Given that data on the free YN ($\Lambda N + \Sigma N$) interaction are very sparse and essentially constrain only spin-averaged s-wave scattering for ΛN , the s_{Λ} doublet spacings are an important source of information on the YN interaction. This requires hypernuclear γ -ray spectroscopy.

The earliest measurements were made with NaI detectors [7, 8]. The excited 1^+ states of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$ were found to be at 1.04(4) MeV and 1.15(4) MeV, respectively [7]. With just a ΛN interaction, this implies that the spin-singlet central interaction is more attractive than the triplet, as is also necessary to obtain a $1/2^+$ hypertriton. Now, it is recognized that Λ - Σ coupling contributes significantly to the $1^+ - 0^+$ spacing mainly by increasing the binding energy of the 0^+ ground state; this also permits a consistent description of the binding energies of the s-shell hypernuclei [9, 10, 11, 12]. The observation of γ -rays in ${}^7_{\Lambda}\text{Li}$ and ${}^9_{\Lambda}\text{Be}$ [8] enabled progress to be made in the use of shell-model calculations to extract information on the spin-dependence of the ΛN interaction for p-shell hypernuclei [13] after the pioneering efforts of Gal, Soper, and

Dalitz [14] in which the ground-state B_Λ values alone proved insufficient to give fits with any degree of uniqueness.

However, the superior (\sim keV) resolution of Ge detectors turns out to be essential. Following a pioneering experiment at BNL [15], the capabilities of a large-acceptance Ge detector array (Hyperball) have been exploited in a series of experiments on p-shell targets carried out at KEK and BNL between 1998 and 2005 using the $(\pi^+, K^+\gamma)$ and $(K^-, \pi^-\gamma)$ reactions, respectively (see table 3 of [1]). As well as γ -ray transitions between bound states of the primary hypernucleus, γ -ray transitions are often seen from daughter hypernuclei formed by particle emission (most often a proton) from unbound states of the primary hypernucleus.

A total of 22 γ -rays, including nine doublet spacings, have now been observed with the Hyperball or Hyperball-2 Ge-detector arrays [1, 16] in six p-shell hypernuclei, namely ${}^7_\Lambda\text{Li}$, ${}^9_\Lambda\text{Be}$, ${}^{11}_\Lambda\text{B}$, ${}^{12}_\Lambda\text{C}$, ${}^{15}_\Lambda\text{N}$, and ${}^{16}_\Lambda\text{O}$. The newest information concerns three γ -ray transitions each in ${}^{12}_\Lambda\text{C}$ and ${}^{11}_\Lambda\text{B}$ from KEK E566 [17, 18] that used the $(\pi^+, K^+\gamma)$ reaction on a ${}^{12}\text{C}$ target and the Hyperball-2 detector. The placements of most these γ rays are summarized in figure 1 (except for some transitions involving higher levels of ${}^{11}_\Lambda\text{B}$).

After a brief description of the shell-model calculations and the parametrization of the YN interactions, a ΛN parameter set that fits ${}^7_\Lambda\text{Li}$ and another that fits the heavier p-shell hypernuclei are given in section 2. The fitting procedure is described in section 3. The details of these fits have been covered extensively in recent publications [19, 20, 21, 22]. Section 4 gives matrix elements derived from free YN interactions and conclusions are presented in section 5.

2. Shell-model calculations

Shell-model calculations for p-shell hypernuclei start with the Hamiltonian

$$H = H_N + H_Y + V_{NY} , \quad (1)$$

where H_N is an empirical Hamiltonian for the p-shell core, the single-particle H_Y supplies the ~ 80 MeV mass difference between Λ and Σ , and V_{NY} is the YN interaction. The shell-model basis states are chosen to be of the form $|(p^n \alpha_c J_c T_c, j_Y t_Y) J T\rangle$, where the hyperon is coupled in angular momentum and isospin to eigenstates of the p-shell Hamiltonian for the core, with up to three values of T_c contributing for Σ -hypernuclear states. This is known as a weak-coupling basis and, indeed, the mixing of basis states in the hypernuclear eigenstates is generally very small. In this basis, the core energies are taken from experiment where possible and from the p-shell calculation otherwise.

To perform shell-model calculations, the YN interaction is written as [23]

$$V = \sum_{\alpha} C(\alpha) \left[\left[a_{j_N}^+ \tilde{a}_{j'_N} \right]^{J_\alpha T_\alpha} \left[a_{j_Y}^+ \tilde{a}_{j'_Y} \right]^{J_\alpha T_\alpha} \right]^{00} , \quad (2)$$

where $C(\alpha)$ represents linear combinations of the two-body matrix elements, α stands for all the quantum numbers, and the tilde denotes properly phased annihilation operators. The basic input from the p-shell calculation is then a set of one-body density-matrix elements (OBDME) between all pairs of nuclear core states that are to be included in the hypernuclear shell-model calculation. From the isospins of the hyperons, it is clear that only isoscalar OBDME are needed for coupling Λ configurations while isovector OBDME are needed for coupling Λ configurations to Σ configurations. For the p-shell wave functions of the core, early hypernuclear shell-model calculations of the γ -ray era [13, 24] used one or more of the three Cohen and Kurath interactions [25]. More recently, interactions fitted to p-shell energy-level data with the strength of the tensor interaction fixed to reproduce the cancellation in the ${}^{14}\text{C}$ β decay (${}^{14}\text{N}$ ground-state wave function) have been used [19].

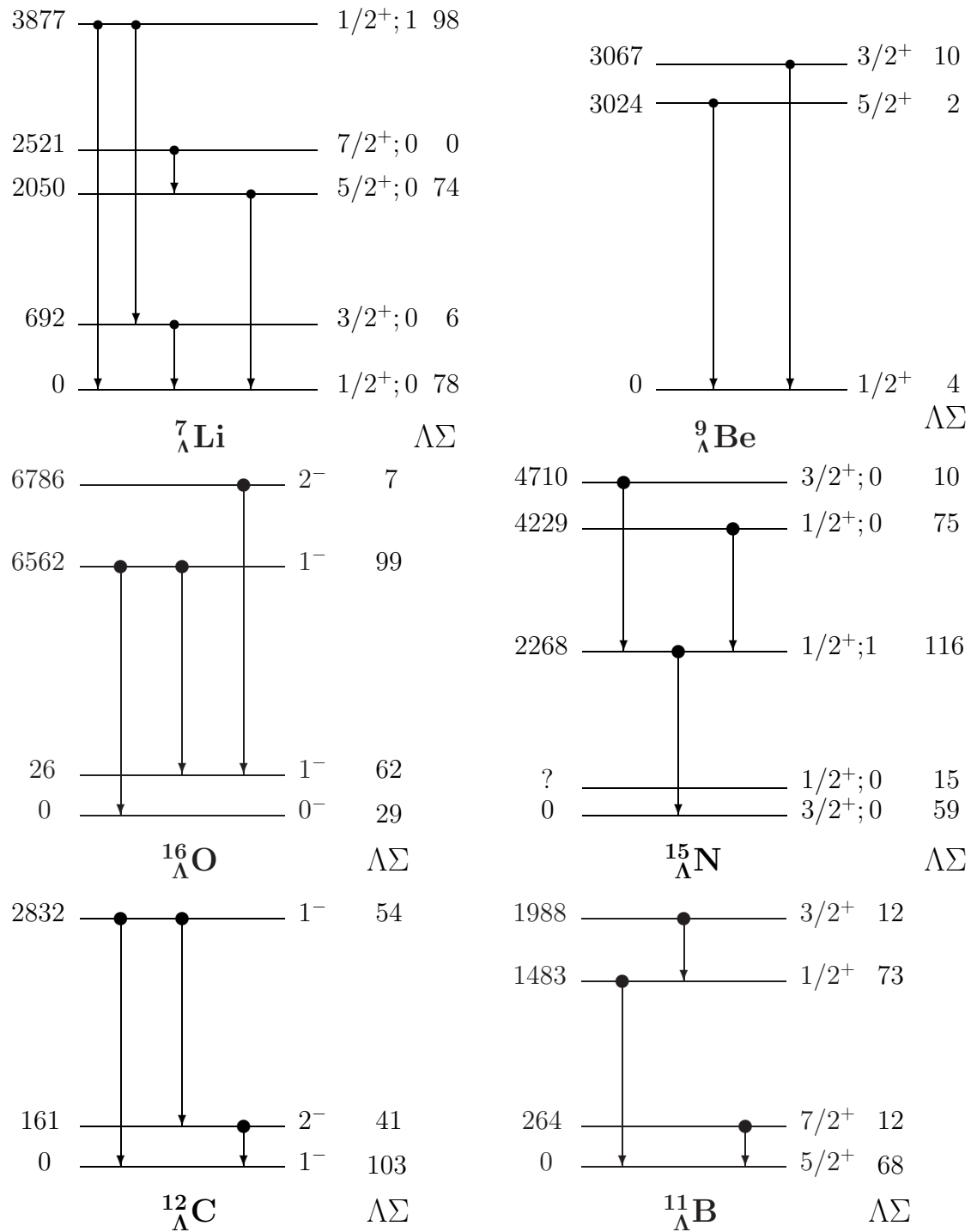


Figure 1. The spectra of ${}^7_{\Lambda}\text{Li}$, ${}^9_{\Lambda}\text{Be}$, ${}^{16}_{\Lambda}\text{O}$, and ${}^{15}_{\Lambda}\text{N}$, ${}^{12}_{\Lambda}\text{C}$, and ${}^{11}_{\Lambda}\text{B}$ determined from experiments KEK E419, E518, E566, and BNL E930 with the Hyperball detector. All energies are in keV. The arrows denote observed γ -ray transitions. For each state the calculated energy shifts due to Λ - Σ coupling are given.

The ΛN effective interaction can be written [14]

$$V_{\Lambda N}(r) = V_0(r) + V_\sigma(r) \vec{s}_N \cdot \vec{s}_\Lambda + V_\Lambda(r) \vec{l}_{N\Lambda} \cdot \vec{s}_\Lambda + V_N(r) \vec{l}_{N\Lambda} \cdot \vec{s}_N + V_T(r) S_{12}, \quad (3)$$

where $S_{12} = 3(\vec{\sigma}_N \cdot \vec{r})(\vec{\sigma}_\Lambda \cdot \vec{r}) - \vec{\sigma}_N \cdot \vec{\sigma}_\Lambda$. The five $p_N s_\Lambda$ two-body matrix elements depend on the radial integrals associated with each component in equation 3. They are denoted by the parameters \bar{V} , Δ , S_Λ , S_N and T [14]. By convention [14], S_Λ and S_N are actually the coefficients of $\vec{l}_N \cdot \vec{s}_\Lambda$ and $\vec{l}_N \cdot \vec{s}_N$. Then, the operators associated with Δ and S_Λ are $\vec{S}_N \cdot \vec{s}_\Lambda$ and $\vec{L}_N \cdot \vec{s}_\Lambda$. In an LS basis for the core, the matrix elements of $\vec{S}_N \cdot \vec{s}_\Lambda$ are diagonal (similarly for $\vec{L}_N \cdot \vec{s}_\Lambda = (\vec{J}_N - \vec{S}_N) \cdot \vec{s}_\Lambda$) and therefore depend only on the intensities of the different L_c and S_c in the core wave functions. Because supermultiplet symmetry $[f_c]K_c L_c S_c J_c T_c$ is a rather good symmetry for p-shell core states, only a few values of L_c and S_c are important leading to a simple understanding of the contributions from Δ and S_Λ . \bar{V} contributes only to the overall binding energy; S_N does not contribute to doublet splittings in the weak-coupling limit but augments the nuclear spin-orbit interaction and contributes to the spacings between states based on different core states; in general, there are not simple expressions for the coefficients of T .

The parametrization of equation 3 applies to the direct ΛN interaction, the ΛN - ΣN coupling interaction, and the direct ΣN interaction for both isospin 1/2 and 3/2. A set of parameters that fits the energy spacings between levels of ${}^7_\Lambda\text{Li}$ and ${}^9_\Lambda\text{Be}$ is (parameters in MeV)

$$\Delta = 0.430 \quad S_\Lambda = -0.015 \quad S_N = -0.390 \quad T = 0.030, \quad (4)$$

while a set that fits the heavier p-shell hypernuclei is

$$\Delta = 0.330 \quad S_\Lambda = -0.015 \quad S_N = -0.350 \quad T = 0.0239. \quad (5)$$

The corresponding matrix elements for the Λ - Σ coupling interaction, based on the G-matrix calculations of [9] for the $nsc97e, f$ interactions [26], are [20, 19]

$$\bar{V}' = 1.45 \quad \Delta' = 3.04 \quad S'_\Lambda = S'_N = -0.09 \quad T' = 0.16. \quad (6)$$

These parameters are kept fixed in the present calculations.

From equation 6 it is clear that the central terms, \bar{V}' and Δ' , of Λ - Σ coupling interaction are important. Formally, one could include an overall factor $t_N \cdot t_{\Lambda\Sigma}$ in the analog of equation 3 that defines the interaction, where $t_{\Lambda\Sigma}$ is the operator that converts a Λ into a Σ . Then, the core operator associated with \bar{V}' is $T_N = \sum_i t_{N_i}$. This leads to a non-zero matrix element only between Λ and Σ states that have the same core, with the value

$$\langle (J_c T, s_\Sigma) J T | V'_{\Lambda\Sigma} | (J_c T, s_\Lambda) J T \rangle = \sqrt{4/3} \sqrt{T(T+1)} \bar{V}', \quad (7)$$

in analogy to Fermi β decay of the core nucleus. Similarly, the spin-spin term involves $\sum_i s_{N_i} t_{N_i}$ for the core and connects core states that have large Gamow-Teller matrix elements between them. These are states with mainly the same $[f_c]K_c L_c$ because the Gamow-Teller operator can't change any spatial quantum numbers. For a $T=0$ core nucleus, only Δ' contributes.

Perturbatively, the energy shift that Λ -hypernuclear state gets from mixing with a Σ -hypernuclear state goes as the mixing matrix element squared over an energy denominator that is of the order of 80 MeV. Equation 7 shows that the contribution from \bar{V}' increases quadratically

Table 1. Doublet spacings in p-shell hypernuclei. Entries in the top (bottom) half of the table are calculated using the parameters in equation 4 (equation 5). The individual contributions do not sum to exactly ΔE^{th} , which comes from the diagonalization, because small contributions from the energies of admixed core states are not included. The coefficients of Δ , S_Λ , S_N , and T can be obtained by dividing the contributions in the table by the values of the parameters in equation 4 or equation 5. Alternatively, they are given in Refs. [19, 20, 21, 22].

	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
${}^7_\Lambda\text{Li}$	$3/2^+$	$1/2^+$	72	628	-1	-4	-9	693	692
${}^7_\Lambda\text{Li}$	$7/2^+$	$5/2^+$	74	557	-32	-8	-71	494	471
${}^8_\Lambda\text{Li}$	2^-	1^-	151	396	-14	-16	-24	450	(442)
${}^9_\Lambda\text{Li}$	$5/2^+$	$3/2^+$	116	530	-17	-18	-1	589	
${}^9_\Lambda\text{Li}$	$3/2_2^+$	$1/2^+$	-80	231	-13	-13	-93	-9	
${}^9_\Lambda\text{Be}$	$3/2^+$	$5/2^+$	-8	-14	37	0	28	44	43
${}^{10}_\Lambda\text{B}$	2^-	1^-	-15	188	-21	-3	-26	120	< 100
${}^{11}_\Lambda\text{B}$	$7/2^+$	$5/2^+$	56	339	-37	-10	-80	267	264
${}^{11}_\Lambda\text{B}$	$3/2^+$	$1/2^+$	61	424	-3	-44	-10	475	505
${}^{12}_\Lambda\text{C}$	2^-	1^-	61	175	-12	-13	-42	153	161
${}^{15}_\Lambda\text{N}$	$1/2_1^+$	$3/2_1^+$	44	244	34	-8	-214	99	
${}^{15}_\Lambda\text{N}$	$3/2_2^+$	$1/2_2^+$	65	451	-2	-16	-10	507	481
${}^{16}_\Lambda\text{O}$	1^-	0^-	-33	-123	-20	1	188	23	26
${}^{16}_\Lambda\text{O}$	2^-	1_2^-	92	207	-21	1	-41	248	224

with the isospin for neutron-rich hypernuclei. The total Gamow-Teller strength, proportional to $N - Z$, increases linearly with isospin. However, not all the Gamow-Teller strength is operative for a given J . An important feature is the magnitude and sign of the contribution from Δ' to the “diagonal” matrix element in equation 7. This is illustrated by the off-diagonal matrix elements $v(J)$ in a simple s^3s_Λ plus s^3s_Σ model [9, 19] for the 0^+ and 1^+ states of ${}^4_\Lambda\text{He}$ (or ${}^4_\Lambda\text{H}$) where

$$v(0) = \overline{V}_s' + 3/4\Delta'_s, \quad v(1) = \overline{V}_s' - 1/4\Delta'_s. \quad (8)$$

The pure s-state matrix elements take roughly double the p-shell values in equation 7 leading to 7.46 MeV for $v(0)$ and a much smaller value for $v(1)$. The resultant energy shift is ~ 700 keV for the ${}^4_\Lambda\text{He}$ ground state. Thus, the ΛN spin-spin interaction and Λ - Σ coupling make comparable contributions to the $1^+ - 0^+$ doublet splitting. Few-body calculations have confirmed this basic picture [10, 11, 12].

3. Analysis of the p-shell hypernuclei

Table 1 gives the breakdown of the contributions from Λ - Σ coupling and the ΛN interaction parameters to all 9 of the measured doublet spacings and several more doublets spacings of interest in ${}^8_\Lambda\text{Li}$, ${}^9_\Lambda\text{Li}$, ${}^{10}_\Lambda\text{B}$, and ${}^{15}_\Lambda\text{N}$. The strategy for fixing the values of the parameters goes as follows.

The ${}^9_\Lambda\text{Be}$ doublet spacing demands a small value for S_Λ . This was already clear from the limit of 100 keV placed on the spacing using NaI detectors [8, 13]. This limit relies on theory only to the extent that the doublet members are expected to be populated almost equally in the (K^-, π^-) reaction [27]. The unbound 2^+ excited state of ${}^8\text{Be}$ (bound in the presence of the Λ) has dominantly $L=2$ and $S=0$, in which case the $3/2^+$ lies above the $5/2^+$ state by $-5/2S_\Lambda$.

The small $S=1$ admixtures in the 2^+ wave function, necessary to account for the Gamow-Teller decays of ${}^8\text{Li}$ and ${}^8\text{B}$, lead to small contributions from Δ and T to the doublet spacing. The small Gamow-Teller matrix elements also mean that the contribution from Λ - Σ coupling (Δ') is small and it so happens that the contributions other than from S_Λ more or less cancel. The parameter set chosen puts the $3/2^+$ state above the $5/2^+$ state but the order is not determined in the original experiment [28]. However, in the 2001 run of BNL E930 on a ${}^{10}\text{B}$ target, only the upper level is seen strongly following proton emission from ${}^{10}_\Lambda\text{B}$ [29]. It can then be deduced that the $3/2^+$ state is the upper member of the doublet [19].

Four of the five γ rays in ${}^7_\Lambda\text{Li}$ were observed in the first Hyperball experiment [30] while the excited-state doublet spacing was observed following ${}^3\text{He}$ emission from $0s$ -hole states in ${}^{10}_\Lambda\text{B}$ [31]. The ground state of ${}^6\text{Li}$ is mainly $L=0$ and $S=1$ [19] which means that the spins ground-state doublet members in ${}^7_\Lambda\text{Li}$ are all due to intrinsic spin and the doublet spacing is given by $3/2 \Delta$ plus the contribution from Λ - Σ coupling (due to Δ'). The latter accounts for $\sim 10\%$ of the spacing. The excited 3^+ state of ${}^6\text{Li}$ is purely $L=2, S=1$ leading, in this limit, to a doublet spacing of [27]

$$\Delta E = 7/6 \Delta + 7/3 S_\Lambda - 14/5 T . \quad (9)$$

Again, the spin-spin interaction dominates but the spacing is reduced by contributions from S_Λ and T . Initially, T was taken to be small based on the available YN interactions [32]. The first indication of a substantial negative value for S_N came from the excitation energy of the $5/2^+$ state [8, 32, 24] which, being based on the lowest member of an $L=2, S=1$ triplet, is lowered by an enhanced nuclear spin-orbit interaction (see table 2); S_N also makes an important contribution to the excitation energy of the $1/2^+; 1$ state.

The $A=7$ hypernuclei have also been extensively studied using three-body [33, 34] and four-body [35, 36] models. These models lack a tensor interaction and explicit Λ - Σ coupling but can treat the radial structure of these hypernuclei. Indeed, a reduced $B(E2)$ for the $5/2^+ \rightarrow 1/2^+$ transition relative to the known $3^+ \rightarrow 1^+$ core transition in ${}^6\text{Li}$ was predicted due to a contraction brought about by the extra binding energy in the presence of a Λ , as subsequently found experimentally [37].

The determination of T rests on the measurement of the ground-state doublet spacing in ${}^{16}_\Lambda\text{O}$ [38, 16]. Because the $1^-, 0^-$ doublet spacing, given by

$$\Delta E = -1/3 \Delta + 4/3 S_\Lambda + 8 T , \quad (10)$$

could be small, the experiment was set up to measure the γ -ray energies from the 1_2^- state to the members of the ground-state doublet. A weak transition, tentatively assigned as from the 2^- level, was also observed [16], thus providing the $2^-, 1^-$ excited-state doublet spacing.

In the same experiment [16], three γ -ray transitions in ${}^{15}_\Lambda\text{N}$ were observed following proton emission from unbound states of ${}^{16}_\Lambda\text{O}$. In the simplest model of $p_{1/2}^- s_\Lambda$, the ground-state doublet spacing should be just $3/2$ that of ${}^{16}_\Lambda\text{O}$ [27, 32] with the $1/2^+$ state lowest and should be measurable from the decays of the 2268-keV $1/2^+; 1$ level (see figure 1). This would provide a check on the value of T from ${}^{16}_\Lambda\text{O}$. However, only one very sharp γ ray corresponding to a long lifetime is observed. In addition, the lifetime of the 2268-keV level is measured to be 15 times longer than that of the $0^+; 1$ core state [16]. The core transition is a weak, mainly orbital, M1 transition because the spin matrix element is almost zero (cf. ${}^{14}\text{C}$ β^- decay). It turns out that small $1_2^+; 0 \times s_\Lambda$ admixtures introduce a strong spin M1 matrix element that produces strong cancellations, the more so for the decay to the $1/2^+$ member of the ground-state doublet [19, 16]. The ground-state is predicted to be $3/2^+$, opposite to that expected from the simple model mentioned above, and this is confirmed by recent results on the mesonic weak

Table 2. Non-doublet excitation energies of states in p-shell hypernuclei. The entry for ${}^7_{\Lambda}\text{Li}$ is calculated using the parameters in equation 4. The remainder are calculated using equation 5. See table 1 for more details. ΔE_c is the excitation energy of the core state. Taking the centroid of doublets would eliminate most of the dependence on Δ , S_{Λ} , and T . All energies are in keV.

	$J^{\pi}; T$	ΔE_c	$\Lambda\Sigma$	Δ	S_{Λ}	S_N	T	E_x^{th}	E_x^{exp}
${}^7_{\Lambda}\text{Li}$	$5/2^+$	2186	4	77	17	-288	33	2047	2050
${}^7_{\Lambda}\text{Li}$	$1/2^+; 1$	3565	-23	418	0	-82	-3	3883	3877
${}^{11}_{\Lambda}\text{B}$	$1/2^+; 0$	718	5	-88	-19	391	-38	968	1483
${}^{12}_{\Lambda}\text{C}$	$1^-; 1/2$	2000	49	117	-17	309	20	2430	2832
${}^{13}_{\Lambda}\text{C}$	$3/2^+; 0$	4439	1	-11	22	203	-22	4630	4880
${}^{15}_{\Lambda}\text{N}$	$1/2^+; 1$	2313	-57	86	11	-6	-71	2274	2268
${}^{15}_{\Lambda}\text{N}$	$1/2_2^+; 0$	3948	-16	-208	13	473	-67	4120	4229
${}^{16}_{\Lambda}\text{O}$	$1_2^-; 1/2$	6176	-70	-207	-2	524	170	6582	6562

decay of ${}^{15}_{\Lambda}\text{N}$ [39, 40]. The other two observed γ rays determine the spacing of the excited-state doublet. Here the core state is largely $L = 0$, $S = 1$ and, as for the ground-state doublet in ${}^7_{\Lambda}\text{Li}$, the spacing is driven by Δ and is well fitted by the value in equation 5. The excited-state doublet spacing in ${}^{11}_{\Lambda}\text{B}$ provides a similar example.

The data on ${}^{12}_{\Lambda}\text{C}$ and ${}^{11}_{\Lambda}\text{B}$ in figure 1 come from KEK E566 [17, 18], the ${}^{11}_{\Lambda}\text{B}$ γ rays following proton emission from unbound states of ${}^{12}_{\Lambda}\text{C}$ (a total of six γ rays are known in ${}^{11}_{\Lambda}\text{B}$ [41, 1]).

As can be seen from table 1, there is a consistent description of the doublet spacings once a larger value of Δ is taken for ${}^7_{\Lambda}\text{Li}$. The ground-state doublet of ${}^8_{\Lambda}\text{Li}$ is included because there is a candidate γ -ray [15]. The ground-state doublet of ${}^9_{\Lambda}\text{Li}$ could be measured using the ${}^9\text{Be}(e, e'K^+){}^9_{\Lambda}\text{Li}$ reaction at Jefferson Laboratory (the spacing is 470 keV with the other parameter set). The excited-state doublet of ${}^9_{\Lambda}\text{Li}$ is included to show that the contributions from Δ and Λ - Σ coupling are not always of the same sign.

The remaining point with regard to doublet spacings concerns the ground-state doublet spacings of ${}^{12}_{\Lambda}\text{C}$ and ${}^{10}_{\Lambda}\text{B}$. The limit on the ${}^{10}_{\Lambda}\text{B}$ spacing [15] was the reason that Fetisov et al. chose a small value of Δ [24]. With the inclusion of Λ - Σ coupling, it can be seen from table 1 that Λ - Σ coupling reduces the spacing in ${}^{10}_{\Lambda}\text{B}$ and increases it in ${}^{12}_{\Lambda}\text{C}$ [19, 22]. The recent measurement of the ground-state doublet spacing in ${}^{12}_{\Lambda}\text{C}$ provides an important check on this effect. It is possible to reduce the ${}^{10}_{\Lambda}\text{B}$ spacing further by adjustments to the Λ - Σ coupling parameters in equation 6 [22]. It should also be borne in mind that the protons are relatively loosely bound in ${}^{10}_{\Lambda}\text{B}$ (${}^9\text{B}$ is unbound to proton emission) but the effect of this is hard to estimate because the p-shell parentage is widely spread.

Table 2 shows the breakdown of contributions to non-doublet excitation energies for the hypernuclei in figure 1. This shows the influence of S_N on these spacings; by taking the centroid of doublets one could remove most of the contributions from Δ , S_{Λ} , and T but the contribution from S_N would not be much affected, being essentially the same for doublet members (table 1). As can be seen S_N always contributes in the right direction to improve agreement with experiment, but leads to an underestimate of the excitation energies for ${}^{11}_{\Lambda}\text{B}$, ${}^{12}_{\Lambda}\text{C}$, and ${}^{13}_{\Lambda}\text{C}$ near the middle of the shell (there is sensitivity to the core wave functions). Possible reasons for this discrepancy will be addressed in section 5.

Table 3. Parameter values for p-shell and s-shell hypernuclei calculated using Woods-Saxon wave functions and Gaussian or Yukawa representations of ΛN - ΛN G-matrix elements.

		\bar{V}	Δ	p-shell			s-shell	
				S_Λ	S_N	T	\bar{V}_s	Δ_s
fit-djm	${}^7_\Lambda\text{Li}$	-1.142	0.438	-0.008	-0.414	0.031	-1.387	0.497
fit-djm	${}^{16}_\Lambda\text{O}$	-1.161	0.441	-0.007	-0.401	0.030		
nsc97f	${}^7_\Lambda\text{Li}$	-1.086	0.421	-0.149	-0.238	0.055	-1.725	0.775
esc04a	${}^7_\Lambda\text{Li}$	-1.287	0.381	-0.108	-0.236	0.013	-1.577	0.850
esc08a	${}^7_\Lambda\text{Li}$	-1.221	0.146	-0.074	-0.241	0.055	-1.796	0.650

4. Matrix elements from baryon-baryon interactions

In the preceding section, the ΛN matrix elements have been treated as parameters independent of mass number, except (essentially) for the larger value of Δ for ${}^7_\Lambda\text{Li}$. This is reasonable because the stable p-shell nuclei exhibit almost constant rms charge radii [19]. To illustrate this point for hypernuclei, we show in table 3 the values obtained for the parameters when they are calculated from fixed radial forms of the potentials using Woods-Saxon wave functions. Of the nsc97 interactions [26], only nsc97f gives a value for Δ that comes close to the empirical value. The radial representation of nsc97f then forms a starting point for scaling the strengths in the various central, spin-orbit, antisymmetric, and tensor channels to obtain a fit to either of the empirically determined parameter sets; fit-djm is the result for the ${}^7_\Lambda\text{Li}$ set. Most recent YN models (such as esc04 [42], esc08 [43], or the new Jülich models [44]), use some constraint to obtain a more attractive singlet s-wave interaction than triplet to bind the hypertriton, and give a reasonable $A = 4$ $0^+/1^+$ doublet spacing. The Yukawa representation of the esc04a potential comes from the work of Halderson [45] (similarly esc08a).

The fit-djm results for ${}^7_\Lambda\text{Li}$ and ${}^{16}_\Lambda\text{O}$ are calculated from the same interaction for Woods-Saxon radii that scale as $A^{1/3}$ and depths that are fitted to the experimental binding energies. As can be seen from the first two lines of table 3, the calculated parameters stay remarkably constant (the rms radii of the p nucleon orbits stay very nearly constant at around 2.9 fm while the rms radius of the s_Λ orbit goes from 2.34 fm for ${}^{16}_\Lambda\text{O}$ to 2.60 fm for ${}^7_\Lambda\text{Li}$). The cancellation between the symmetric and antisymmetric spin-orbit interactions for nsc97f, esc04a, and esc08a is in the right direction, but not large enough, to reproduce the very small empirical value of S_Λ .

In the p shell, Δ receives contributions from the spin-dependence in both relative s states and p states [19] (cf. [45]). These the p-wave contributions are repulsive for both spin channels in nsc97f, repulsive for singlet in esc04a and esc08a, and attractive for both in fit-djm. Despite the smaller size of p-wave matrix elements relative to s-wave matrix elements, the contribution to Δ can be substantial when the singlet and triplet p-wave interactions are of opposite sign, as they are for esc04a and esc08a. This results in an unsatisfactorily small value of Δ for esc08a. An effect of this phenomenon is also seen in the substantial variation in the s-shell parameters (similarly calculated) in table 3 where the contributions from odd-state central interactions are absent. A few-body calculation of ${}^4_\Lambda\text{H}$ and ${}^4_\Lambda\text{He}$ versus ${}^7_\Lambda\text{Li}$ would then test the even/odd character of the central interaction in addition to the role of Λ - Σ coupling [11].

Finally, table 4 shows calculated Λ - Σ coupling parameter values for several of the Nijmegen baryon-baryon interactions. These are dominantly effective central interactions that arise from the strong tensor interaction of Λ - Σ coupling acting in second order. The esc04 interactions seem to have an unphysical radial behavior [45].

Table 4. p_{NSY} Λ - Σ coupling parameters from several of the Nijmegen baryon-baryon potentials.

Source	Interaction	\bar{V}'	Δ'	S'_Λ	S'_N	T'
Akaishi (s-shell)	NSC97e/f	1.45	3.04	-0.09	-0.09	0.16
Yamamoto	NSC97f	0.96	3.62	-0.07	-0.07	0.31
Halderson	NSC97e	0.75	3.51	-0.45	-0.24	0.31
Halderson	NSC97f	1.10	3.73	-0.45	-0.23	0.30
Halderson	ESC04a	-2.30	-2.59	-0.17	-0.17	0.23
Halderson	ESC08a	1.05	4.71	-0.07	0.02	0.32

5. Discussion

Section 3 and table 1 show that the doublet spacings in p-shell hypernuclei are rather well accounted for by a consistent set of ΛN interaction parameters Δ , S_Λ , and T together with the contributions from Λ - Σ coupling. The main problem with consistency is that a larger value of the spin-spin matrix element Δ is required near the beginning of the shell, most certainly for ${}^7_\Lambda\text{Li}$. The discussion in section 4 shows that this is unlikely to be simply an effect of nuclear size. Another possibility is that restricting the core wave functions to p-shell configurations is inadequate. When the core bases are expanded by adding higher configurations, the most important admixtures at the beginning of the p shell involve nucleons excited from the s shell to the p shell, For the specific case of ${}^6\text{Li}$, promoting an np pair from the s shell to the p shell forms a particularly stable α -like $2n2p$ system (${}^8\text{Be}$) in the p shell in analogy to the classic low-lying $4p2h$ states in the $A=18$ nuclei. This leaves an active np pair in the s shell for which the $s_N s_\Lambda$ matrix element is larger than the $p_N s_\Lambda$ (central) matrix element by roughly a factor of two. In contrast, beyond ${}^8\text{Be}$ the low-lying excitations involve nucleons excited from the p shell to the sd shell. In ${}^{10}\text{B}$ for example, the lowest $(sd)^2$ levels are the 5.18-MeV $1^+; 0$ level and the 7.56-MeV $0^+; 1$ level. In this case, the $(sd)_N s_\Lambda$ matrix elements are smaller than those for $p_N s_\Lambda$.

The other major discrepancy found in the present shell-model treatment is that the nuclear-spin-dependent spin-orbit term S_N does not provide a large enough contribution for the mid-shell hypernuclei (table 2). In the core nuclei, antisymmetric spin-orbit interactions play an important role in the latter half of the p-shell, mocking up the effect of NNN interactions in producing a 3^+ ground state for ${}^{10}\text{B}$ and enough “spin-orbit” splitting at $A=15$. These can be obtained by averaging the NNN interaction over the closed shell of $0s$ nucleons. For hypernuclei, the double one-pion exchange ΛNN interaction [14] is independent of the Λ spin and gives, when averaged over the s_Λ wave function,

$$V_{NN}^{eff} = \sum_{klm} Q_{lm}^k(r_1, r_2) [\sigma_1, \sigma_2]^k \cdot [C_l(\hat{r}_1), C_m(\hat{r}_2)]^k \tau_1 \cdot \tau_2. \quad (11)$$

The Q_{00}^0 and Q_{22}^0 terms give repulsive contributions to B_Λ that depend quadratically on the number of p-shell nucleons in the core while Q_{22}^1 represents an anti-symmetric spin-orbit interaction that behaves rather like S_N [14], and could act synergistically with the vector interactions in the core. This would help alleviate the general problem of failing to get enough spacing between states based on different core levels for hypernuclei near the middle of the p shell despite the fact that S_N works in the right direction.

Theoretically, then, the next step is to expand the basis for the nuclear core states and to explicitly include ΛNN interactions. Experimentally, progress will come from experiments using

the new Hyperball-J detector at J-PARC [18].

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