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In Pursuit of New Physics with $B_s^0 \to K^+K^-$

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Abstract

The $B_s^0 \to K^+K^-$ decay is the U-spin partner of $B_d^0 \to \pi^+\pi^-$ and allows a determination of the angle γ of the unitarity triangle of the Cabibbo–Kobayashi–Maskawa matrix. Using updated information on the branching ratio and the relevant hadronic form factors, we discuss the picture that emerges for γ from the currently available data, which is in good agreement with the fits of the unitarity triangle. We point out that the $B_s^0 \to K^+K^-$ decay also offers interesting probes to search for new physics in B_s^0–$\bar{B}_s^0$ mixing, thereby complementing the well-known $B_s^0 \to J/\psi\phi$ analyses. The relevant observables are the effective lifetime of this channel and its mixing-induced CP asymmetry. We calculate correlations between these quantities and the CP-violating B_s^0–$\bar{B}_s^0$ mixing phase, which serve as target regions for improved measurements at the Tevatron and the early data taking at the LHCb experiment. Finally, we discuss the expected situation for the optimal determination of γ at LHCb, exploiting precise measurements of the CP violation in $B_s^0 \to K^+K^-$.

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1 Introduction

The decay $B_s^0 \to K^+K^-$, which is governed by QCD penguins and has a doubly Cabibbo-suppressed tree contribution in the Standard Model (SM), has a very interesting physics potential for B-decay experiments at hadron colliders. It is related to the $B_d^0 \to \pi^+\pi^-$ channel through the U-spin flavour symmetry of strong interactions, which allows a determination of the angle γ of the unitarity triangle of the Cabibbo–Kobayashi–Maskawa (CKM) matrix and certain hadronic parameters [1]. The advantage of this U-spin strategy with respect to the conventional $SU(3)$ flavour-symmetry strategies is twofold:

- no additional dynamical assumptions have to be made (such as the neglect of particular topologies), which could be spoiled by large rescattering effects;
- electroweak penguins, which are not invariant under the isospin symmetry because of the different up- and down-quark charges, are automatically included.

In Ref. [2], a detailed discussion of the status of the extraction of γ coming from the first measurement of the $B_s^0 \to K^+K^-$ branching ratio at the Tevatron was given, addressing also tests of the U-spin symmetry.

In the current paper, we have a fresh look at the determination of γ both in view of updated information on the $B_s^0 \to K^+K^-$ branching ratio and in view of updated information on the relevant U-spin-breaking form-factor ratio. The main focus of this paper, however, is to point out the usefulness of measurements of the $B_s^0 \to K^+K^-$ channel's effective lifetime $\tau_{K^+K^-}$ and mixing-induced CP asymmetry for probing New Physics (NP) in B_s^0–$\bar{B}_s^0$ mixing. This analysis provides target ranges for these observables for improved measurements at the Tevatron and the LHCb experiment at CERN's Large Hadron Collider (LHC) and complements nicely analyses of CP-violating effects in the angular distributions of the decay products of $B_s^0 \to J/\psi[\to \mu^+\mu^-]\phi[\to K^+K^-]$ [3, 4]. With respect to an analysis of various $B_s^0 \to K^+K^-$ correlations performed almost a decade ago [5], we now obtain a significantly sharper picture.

The CDF and DØ collaborations have used $B_s^0 \to J/\psi\phi$ measurements to obtain the first direct results for the CP-violating B_s^0–$\bar{B}_s^0$ mixing phase ϕ_s. In the SM this quantity is fixed in terms of the Wolfenstein parameters [6] by $-2\lambda^2\eta \sim -2°$ but could be enhanced significantly through NP effects. Unfortunately, the most recent data are still far from being conclusive: while the CDF collaboration finds $\phi_s \in [-59.6°, -2.29°] \sim -30° \vee [-177.6°, -123.8°] \sim -150°$ (68% C.L.) [7], DØ quotes a best fit value around $\phi_s \sim -45°$ [8], taking also information form the dimuon charge asymmetry and the measured $B_s \to D_s^{(*)+}D_s^{(*)-}$ branching ratio into account. As we will show in the current paper, the $B_s^0 \to K^+K^-$ channel offers an excellent alternative probe for such effects, thereby nicely complementing the $B_s^0 \to J/\psi\phi$ analysis and allowing us also to resolve a twofold discrete ambiguity for the extracted value of ϕ_s.

The outline of this paper is as follows: in Section 2, we discuss the picture for γ arising from the current data. In Section 3, we calculate the effective lifetime $\tau_{K^+K^-}$ and study its correlation with the CP-violating B_s^0–$\bar{B}_s^0$ mixing phase. A similar analysis is performed for the mixing-induced CP asymmetry of $B_s^0 \to K^+K^-$ in Section 4. In Section 5, we discuss and illustrate the optimal determination of γ using the CP asymmetries of $B_s^0 \to K^+K^-$. Finally, we summarize our conclusions in Section 6.

2 Determination of γ

In the SM, the $B_s^0 \to K^+K^-$ and $B_d^0 \to \pi^+\pi^-$ decay amplitudes can be written as follows [1]:

$$A(B_s^0 \to K^+K^-) = e^{i\gamma}\lambda\,\mathcal{C}'\left[1+\frac{1}{\epsilon}d'e^{i\theta'}e^{-i\gamma}\right] \tag{1}$$

$$A(B_d^0 \to \pi^+\pi^-) = e^{i\gamma}\left(1-\frac{\lambda^2}{2}\right)\mathcal{C}\left[1-d\,e^{i\theta}e^{-i\gamma}\right], \tag{2}$$

where $\lambda \equiv |V_{us}| = 0.22543 \pm 0.00077$ is the Wolfenstein parameter [9], $\epsilon \equiv \lambda^2/(1-\lambda^2)$, $\mathcal{C}$ and $\mathcal{C}'$ are CP-conserving strong amplitudes that are governed by the tree contributions, while the CP-consering hadronic parameters $de^{i\theta}$ and $d'e^{i\theta'}$ measure – loosely speaking – the ratio of penguin to tree amplitudes.

If we apply the U-spin symmetry, we obtain the relations [1]

$$d' = d, \quad \theta' = \theta. \tag{3}$$

As was pointed out in Ref. [1], these relations are not affected by factorizable U-spin-breaking corrections, i.e. the relevant form factors and decay constants cancel. This feature holds also for chirally enhanced contributions to the transition amplitudes.

On the other hand, the U-spin symmetry also implies $|\mathcal{C}'/\mathcal{C}| = 1$. Here, however, the corresponding decay constants and form factors do not cancel, so that we obtain the following result in the factorization approximation:

$$\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|_{\rm fact} = \frac{f_K}{f_\pi}\frac{F_{B_sK}(M_K^2;0^+)}{F_{B_d\pi}(M_\pi^2;0^+)}\left(\frac{M_{B_s}^2-M_K^2}{M_{B_d}^2-M_\pi^2}\right). \tag{4}$$

Using the updated QCD light-cone sum rule calculation of Ref. [10] yields

$$\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|_{\rm fact}^{\rm QCDSR} = 1.41^{+0.20}_{-0.11}, \tag{5}$$

which is consistent within the errors with the numerical value of $1.52^{+0.18}_{-0.14}$ obtained previously by the authors of Ref. [11] that was used in Ref. [2].

The $B_s^0 \to K^+K^-$ decay is now well established and the Heavy Flavour Averaging Group (HFAG) gives the following average for its branching ratio [12]:

$$\mathrm{BR}(B_s^0 \to K^+K^-) = (26.5 \pm 4.4) \times 10^{-6}, \tag{6}$$

which is a combination of measurements by CDF [13] at the Tevatron and by Belle [14] at KEKB runs at the $\Upsilon(5S)$ resonance. A major source of uncertainty for the branching ratio measurement of any B_s decay is the ratio f_s/f_d of fragmentation functions f_q describing the probability that a b quark will hadronize as a $\bar{B}_q$ meson. Using a newly proposed strategy [15], this ratio can be measured at LHCb with an expected uncertainty that is two times smaller than that of current compilations in the literature.

For the extraction of γ, it is useful to introduce the following ratio of CP-averaged branching ratios:

$$K = \frac{1}{\epsilon}\left|\frac{\mathcal{C}}{\mathcal{C}'}\right|^2\left[\frac{M_{B_s}}{M_{B_d}}\frac{\Phi(M_\pi/M_{B_d},M_\pi/M_{B_d})}{\Phi(M_K/M_{B_s},M_K/M_{B_s})}\frac{\tau_{B_d}}{\tau_{B_s}}\right]\left[\frac{\mathrm{BR}(B_s\to K^+K^-)}{\mathrm{BR}(B_d\to\pi^+\pi^-)}\right]$$
$$= \frac{1}{\epsilon^2}\left[\frac{\epsilon^2+2\epsilon d'\cos\theta'\cos\gamma+d'^2}{1-2d\cos\theta\cos\gamma+d^2}\right] \overset{\mathrm{exp}}{=} 51.8^{+12.7}_{-15.0}, \tag{7}$$

where we have used (1) and (2). The numerical value results from a combination of (4) and (6) with the HFAG average $\mathrm{BR}(B_d\to\pi^+\pi^-)=(5.16\pm0.22)\times10^{-6}$, where we have added all errors in quadrature.

The $B_s^0\to K^+K^-$ and $B_d^0\to\pi^+\pi^-$ decays are into CP-even eigenstates and offer the following time-dependent CP asymmetries:

$$\frac{\Gamma(B_q^0(t)\to f)-\Gamma(\bar{B}_q^0(t)\to f)}{\Gamma(B_q^0(t)\to f)+\Gamma(\bar{B}_q^0(t)\to f)}$$
$$= \left[\frac{\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_q\to f)\,\cos(\Delta M_q t)+\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_q\to f)\,\sin(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2)+\mathcal{A}_{\Delta\Gamma}(B_q\to f)\,\sinh(\Delta\Gamma_q t/2)}\right], \tag{8}$$

where $\Delta M_q\equiv M_{\mathrm{H}}^{(q)}-M_{\mathrm{L}}^{(q)}$ and $\Delta\Gamma_q\equiv\Gamma_{\mathrm{L}}^{(q)}-\Gamma_{\mathrm{H}}^{(q)}$ are the mass and width differences of the "heavy" and "light" B_q mass eigenstates, respectively. The width difference is negligible in the B_d system but is expected at the 15% level in the B_s case within the SM, as we will discuss in more detail in Section 3. Note that for the sign definition of $\Delta\Gamma_q$ given above, the Standard-Model value $\Delta\Gamma_s^{\mathrm{SM}}$ is positive.

In the case of the $B_s^0\to K^+K^-$ channel, we unfortunately do not yet have a measurement of the CP asymmetry in (8) available. On the other hand, measurements of the CP-violating observables of the $B_d^0\to\pi^+\pi^-$ channel have been performed at the B factories. By using (2), we can derive the expressions

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d\to\pi^+\pi^-) = -\left[\frac{2\,d\sin\theta\sin\gamma}{1-2\,d\cos\theta\cos\gamma+d^2}\right],$$
$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d\to\pi^+\pi^-) = +\left[\frac{\sin(\phi_d+2\gamma)-2\,d\,\cos\theta\,\sin(\phi_d+\gamma)+d^2\sin\phi_d}{1-2\,d\cos\theta\cos\gamma+d^2}\right], \tag{9}$$

where ϕ_d denotes the CP-violating B_d^0–$\bar{B}_d^0$ mixing phase which is measured by means of CP violation in $B_d\to J/\psi K_{\mathrm{S,L}}$ type decays. Taking also hadronic corrections from doubly Cabibbo-suppressed penguin contributions through $B_d^0\to J/\psi\pi^0$ data into account gives [16]

$$\phi_d=(42.4^{+3.4}_{-1.7})^\circ. \tag{10}$$

The current experimental status of the CP violation in $B_d^0\to\pi^+\pi^-$ can be summarized as

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d\to\pi^+\pi^-) = \begin{cases} 0.68\pm0.10\pm0.03 \text{ (BaBar [17])} \\ 0.61\pm0.10\pm0.04 \text{ (Belle [18])}, \end{cases} \tag{11}$$

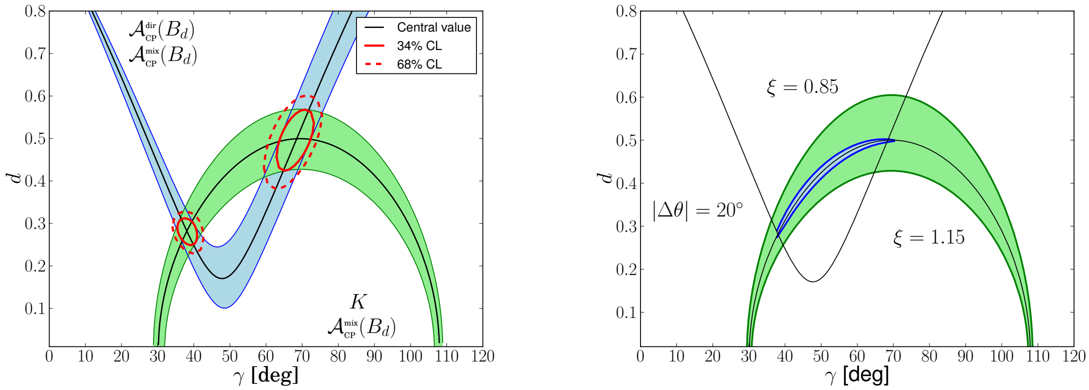

Figure 1: The contours in the γ–d plane fixed through the CP-violating $B_d^0 \to \pi^+\pi^-$ observables and K. Left panel: $1\,\sigma$ error bands and the 68% C.L. regions, right panel: illustration of U-spin-breaking effects.

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = \begin{cases} -0.25 \pm 0.08 \pm 0.02 \text{ (BaBar [17])} \\ -0.55 \pm 0.08 \pm 0.05 \text{ (Belle [18])}. \end{cases} \tag{12}$$

While there is a good agreement between BaBar and Belle for the measurement of the mixing-induced CP asymmetry, yielding the average [12]

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-) = 0.65 \pm 0.07, \tag{13}$$

we are unfortunately still facing a discrepancy between the BaBar and Belle results for the direct CP asymmetry, which, in our opinion, makes it problematic to combine them in an average. This feature is reflected by the averages appearing in the literature: HFAG gives -0.38 ± 0.06, whereas the Particle Data Group (PDG) quote the same central value with a larger error of 0.17 [19].

In view of this unsatisfactory situation, we would like to avoid these averages. An alternative is offered by the direct CP violation in $B_d^0 \to \pi^- K^+$, where all measurements are consistent with one another, yielding the average $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm) = 0.098^{+0.011}_{-0.012}$ (for a sign convention consistent with (8)) [12]. The $B_d^0 \to \pi^- K^+$ and $B_s^0 \to K^+K^-$ channels differ only in their spectator quarks. We can neglect exchange and penguin annihilation topologies, as these are expected to be tiny and can further be constrained by the absence of anomalous behaviour in $B_d^0 \to K^+K^-$ and $B_s^0 \to \pi^+\pi^-$ data [2]. Consequently, $SU(3)$ flavour symmetry implies the following relation:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = \\ -\left(\frac{f_\pi}{f_K}\right)^2 \left[\frac{{\rm BR}(B_d \to \pi^\mp K^\pm)}{{\rm BR}(B_d \to \pi^+\pi^-)}\right] \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm) = -0.26 \pm 0.03, \tag{14}$$

which is in excellent agreement with the BaBar result (12) (see also Ref. [20]).

In the proceeding numerical analysis, we shall use $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = -0.26 \pm 0.10$, i.e. we increase the error of (14) generously to allow for possible $SU(3)$-breaking corrections. It will turn out that this CP asymmetry plays a minor role for the error budget of the extraction of γ.

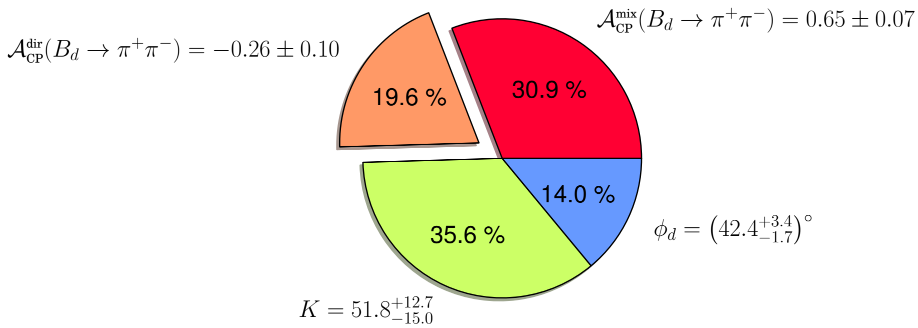

Figure 2: The error budget for the extracted value of γ in (17) that is associated with the used input data.

In order to determine γ, we can convert the direct and mixing-induced CP asymmetry of the $B_d^0 \to \pi^+\pi^-$ channel into a theoretically clean contour in the γ–d plane; the corresponding formulae are given in Ref. [1]. Furthermore, using the U-spin relation (3) in (7), we can determine a second contour. The intersection of both contours then allows us to determine γ and d, so that we can also extract the strong phase θ. In Fig. 1, we show the situation arising for the current data. The plot on the left-hand side shows the $1\,\sigma$ error bands and the 34% and 68% confidence regions arising from a χ^2 fit, whereas the plot on the right-hand side illustrates the impact of U-spin-breaking corrections to (3), which we have parameterized as

$$\xi \equiv d'/d = 1 \pm 0.15, \tag{15}$$

$$\Delta\theta \equiv \theta' - \theta = \pm 20^\circ. \tag{16}$$

As discussed in Ref. [2], the discrete ambiguity can be resolved, yielding

$$\gamma = \left(68.3^{+4.9}_{-5.7}\big|_{\rm input}{}^{+5.0}_{-3.7}\big|_{\xi}{}^{+0.1}_{-0.2}\big|_{\Delta\theta}\right)^\circ \tag{17}$$

and

$$d = 0.499^{+0.069}_{-0.076}\big|_{\rm input}{}^{+0.101}_{-0.074}\big|_{\xi}{}^{+0.002}_{-0.005}\big|_{\Delta\theta}, \quad \theta = \left(153.7^{+10.8}_{-13.6}\big|_{\rm input}{}^{+3.8}_{-3.9}\big|_{\xi}{}^{+0.1}_{-0.2}\big|_{\Delta\theta}\right)^\circ, \tag{18}$$

where the input errors are the 68% confidence intervals of the χ^2 fit. In Fig. 2, we show the error budget for γ coming from the individual input quantities. We observe that K and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ have a similar impact on the error, while $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ plays a significantly less important role. This is a nice feature in view of the unsatisfactory experimental situation corresponding to the direct measurement of this observable discussed above. Using the central value from the HFAG and PDG averages of the BaBar and Belle data, $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = -0.38$, yields a central value of $\gamma = 65^\circ$, which is fully consistent with (17). Interestingly, also the error of ϕ_d given in (10) has a small but non-negligible impact on the overall error. In Ref. [21], a strategy to include penguin effects in the determinations of ϕ_d using $B_s^0 \to J/\psi K_{\rm S}$ was discussed, allowing us to match the experimental precision for the measurement of that phase at LHCb.

The extracted value given in (17) is in excellent agreement with the fits of the unitarity triangle, yielding

$$\gamma = \begin{cases} (67.2^{+3.9}_{-3.9})^\circ & \text{(CKMfitter [9])} \\ (69.6 \pm 3.1)^\circ & \text{(UTfit [22]).} \end{cases} \tag{19}$$

In view of this feature, large NP effects at the amplitude level are already excluded by the current data. We shall therefore assume the SM expressions in (1) and (2) for the following discussion.

The determination of γ can be significantly improved once a measurement of CP-violating observables of the $B_s^0 \to K^+K^-$ channel are available. We will discuss this in more detail in Section 5. Let us next have a closer look at the effective $B_s^0 \to K^+K^-$ lifetime, which is particularly interesting for improved measurements by CDF at the Tevatron and the early data taking at LHCb.

3 The Effective $B_s^0 \to K^+K^-$ Lifetime

The "untagged" rate of initially, i.e. at time $t = 0$, present B_s^0 or $\bar{B}_s^0$ decays into the K^+K^- final state can be written as follows [23]:

$$\begin{aligned} \langle \Gamma(B_s(t) \to K^+K^-) \rangle &\equiv \Gamma(B_s^0(t) \to K^+K^-) + \Gamma(\bar{B}_s^0(t) \to K^+K^-) \\ &= R_{\mathrm{H}}(B_s \to K^+K^-) e^{-\Gamma_{\mathrm{H}}^{(s)} t} + R_{\mathrm{L}}(B_s \to K^+K^-) e^{-\Gamma_{\mathrm{L}}^{(s)} t} \\ &\propto e^{-\Gamma_s t} \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + \mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-) \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \right], \end{aligned} \tag{20}$$

where

$$\Gamma_s \equiv \frac{\Gamma_{\mathrm{H}}^{(s)} + \Gamma_{\mathrm{L}}^{(s)}}{2} = \tau_{B_s}^{-1}, \tag{21}$$

with τ_{B_s} denoting the B_s^0 lifetime. The observable $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$, which enters also the time-dependent CP asymmetry (8), is given by

$$\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-) = \frac{R_{\mathrm{H}}(B_s \to K^+K^-) - R_{\mathrm{L}}(B_s \to K^+K^-)}{R_{\mathrm{H}}(B_s \to K^+K^-) + R_{\mathrm{L}}(B_s \to K^+K^-)}. \tag{22}$$

Using the parametrization in (1), we obtain

$$\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-) = -\left[\frac{d'^2 \cos\phi_s + 2\epsilon d' \cos\theta' \cos(\phi_s + \gamma) + \epsilon^2 \cos(\phi_s + 2\gamma)}{d'^2 + 2\epsilon d' \cos\theta' \cos\gamma + \epsilon^2} \right]. \tag{23}$$

A particularly nice and simple observable that is offered by the $B_s^0 \to K^+K^-$ decay is its effective lifetime, which is defined as

$$\tau_{K^+K^-} \equiv \frac{\int_0^\infty t \, \langle \Gamma(B_s(t) \to K^+K^-) \rangle \, dt}{\int_0^\infty \langle \Gamma(B_s(t) \to K^+K^-) \rangle \, dt}. \tag{24}$$

This quantity is also the resulting lifetime if the untagged rate with the two exponentials in (20) is fitted to a single exponential [4]. Using the two-exponential form in (20) with $R_{\mathrm{H,L}} \equiv R_{\mathrm{H,L}}(B_s \to K^+K^-)$ yields

$$\tau_{K^+K^-} = \frac{R_{\mathrm{L}}/\Gamma_{\mathrm{L}}^{(s)2} + R_{\mathrm{H}}/\Gamma_{\mathrm{H}}^{(s)2}}{R_{\mathrm{L}}/\Gamma_{\mathrm{L}}^{(s)} + R_{\mathrm{H}}/\Gamma_{\mathrm{H}}^{(s)}}, \tag{25}$$

which can be written in terms of

$$y_s \equiv \frac{\Delta\Gamma_s}{2\Gamma_s}, \tag{26}$$

the observable $\mathcal{A}_{\Delta\Gamma} \equiv \mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$, and the B_s^0 lifetime as

$$\frac{\tau_{K^+K^-}}{\tau_{B_s}} = \frac{1}{1-y_s^2}\left[\frac{1+2\mathcal{A}_{\Delta\Gamma}y_s + y_s^2}{1+\mathcal{A}_{\Delta\Gamma}y_s}\right] = 1 + \mathcal{A}_{\Delta\Gamma}y_s + \left(2-\mathcal{A}_{\Delta\Gamma}^2\right)y_s^2 + \mathcal{O}\left(y_s^3\right). \tag{27}$$

First studies along these lines were performed by the CDF collaboration in 2006 [24], yielding a value of

$$\tau_{K^+K^-} = (1.53 \pm 0.18 \pm 0.02)\,\mathrm{ps}. \tag{28}$$

Unfortunately, this analysis has, to the best of our knowledge, not been updated, which we strongly encourage in view of the results discussed below.

In the following analysis, we assume that γ is known, with a value of $\gamma = (68 \pm 7)^\circ$ in agreement with (17) and the fits of the UT in (19). Using then K as given in (7) with the U-spin relation (3) and the direct CP asymmetry

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to K^+K^-) = \frac{2\epsilon d'\sin\theta'\sin\gamma}{d'^2 + 2\epsilon d'\cos\theta'\cos\gamma + \epsilon^2}, \tag{29}$$

we can determine d' and $\cos\theta'$, allowing us to calculate $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$ for a given value of the B_s^0–$\bar{B}_s^0$ mixing phase ϕ_s. Parametrizing the U-spin-breaking effects for $d^{(\prime)}$ through (15), the corresponding formulae from which d' and $\cos\theta'$ can be extracted are

$$2d'\cos\theta' = \frac{-d'^2 + \epsilon^2\left[K(1+d'^2\xi^{-2}) - 1\right]}{\epsilon\cos\gamma\,(1+\epsilon\xi^{-1}K)}, \tag{30}$$

$$d'^2 = \epsilon^2\left[\frac{b \pm \sqrt{b^2 - ac}}{a}\right], \tag{31}$$

with

$$a = \epsilon^2\xi^{-2}\left(1+\epsilon\xi^{-1}\right)^2(\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}})^2K^2\cot^2\gamma + \left(1-\epsilon^2\xi^{-2}K\right)^2, \tag{32}$$

$$b = -\epsilon\xi^{-1}\left(1+\epsilon\xi^{-1}\right)^2(\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}})^2K^2\cot^2\gamma + 2\cos^2\gamma\left(1+\epsilon\xi^{-1}K\right)^2 \tag{33}$$

$$+ (K-1)(1-\epsilon^2\xi^{-2}K), \tag{34}$$

$$c = \left(1+\epsilon\xi^{-1}\right)^2(\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}})^2K^2\cot^2\gamma + (K-1)^2, \tag{35}$$

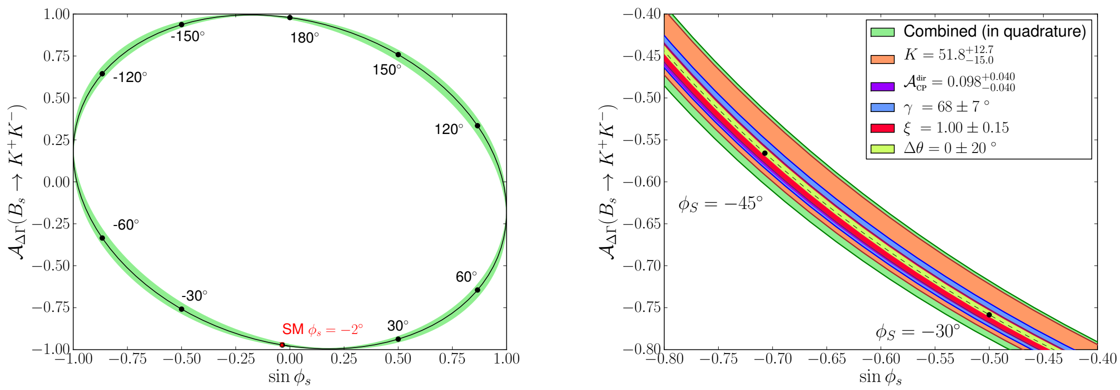

Figure 3: Left panel: correlation between $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$ and $\sin\phi_s$. Right panel: errors associated with the input observables/parameters, zoomed in for a NP region of $\phi_s \in [-25°, -50°]$ and overlayed on top of one another. The legend lists the error contributions from largest to smallest.

where $\mathcal{A}_{\rm CP}^{\rm dir} \equiv \mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$. The U-spin-breaking effects of $\theta^{(\prime)}$, as given in (16), are difficult to include in the above analytic expressions but are straightforward to calculate numerically.

As we have noted above, the CP-violating observables of the $B_s^0 \to K^+K^-$ channel have not yet been measured. However, as the $B_d^0 \to \pi^-K^+$ and $B_s^0 \to K^+K^-$ channels differ only in their spectator quarks (see the discussion of (14)), we expect

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) \approx \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm) = 0.098^{+0.011}_{-0.012}. \tag{36}$$

In order to take possible corrections into account, we increase the error generously and use

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) = 0.098 \pm 0.04 \tag{37}$$

in the following numerical analysis.

Given all of these inputs, including the U-spin breaking effects (15) and (16), we arrive at the following results for the hadronic parameters:

$$d' = 0.499^{+0.085}_{-0.101}, \quad \cos\theta' = -0.886^{+0.131}_{-0.089}, \tag{38}$$

where all errors were added in quadrature.

The SM prediction for the B_s^0–$\bar{B}_s^0$ mixing phase is $\phi_s = -(2.1 \pm 0.1)°$ [9,22], where the error has an essentially negligible impact on the following analysis. The corresponding prediction for the "untagged" observable is

$$\begin{aligned}\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)\Big|_{\rm SM} &= -0.97178^{+0.0133}_{-0.0064}\Big|_K{}^{+0.0047}_{-0.0046}\Big|_\gamma{}^{+0.00005}_{-0.00003}\Big|_{\mathcal{A}_{\rm CP}^{\rm dir}}{}^{+0.0022}_{-0.0031}\Big|_\xi{}^{+0.0016}_{-0.0008}\Big|_{\Delta\theta} \\ &= -0.972^{+0.014}_{-0.009}, \end{aligned} \tag{39}$$

where all errors have again been combined in quadrature. Particularly interesting is the small influence of the U-spin breaking errors and $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$ on the total error.

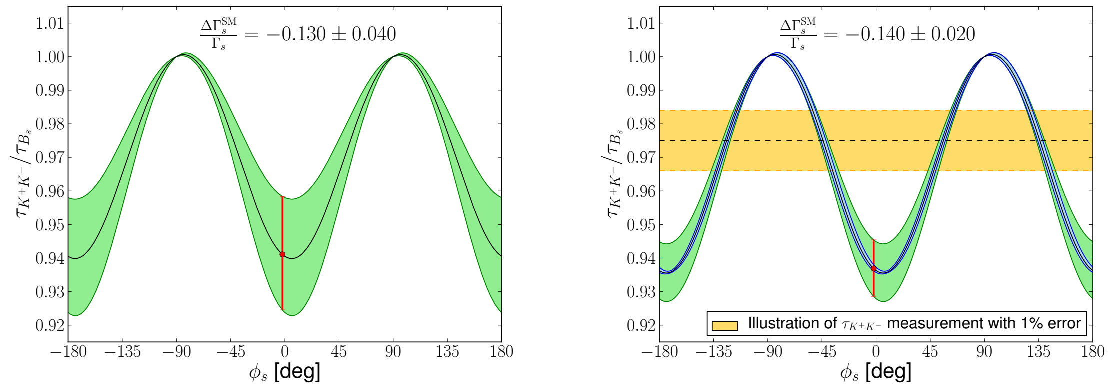

Figure 4: The dependence of $\tau_{K^+K^-}/\tau_{B_s}$ on the B_s^0–$\bar{B}_s^0$ mixing phase. Left panel: uses $\Delta\Gamma_s^{\rm SM}/\Gamma_s$ with the larger error in (41), as in (42). Right panel: illustration of the impact of a measurement of $\tau_{K^+K^-}/\tau_{B_s}$ with 1% uncertainty (horizontal band) for $\Delta\Gamma_s^{\rm SM}/\Gamma_s$ with the smaller error in (42). The narrow band on the curve corresponds to the errors of the input quantities entering our prediction of $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$.

In Fig. 3 we treat ϕ_s as a free parameter and show the correlation between $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$ and $\sin\phi_s$, as well as errors related to the input quantities overlayed on top of one another and centred on the central value. It is remarkable that $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$ is very robust with respect to the input errors for the whole range of ϕ_s.

The remaining ingredient for a determination of the effective lifetime is the width difference $\Delta\Gamma_s$. In the presence of CP-violating NP contributions to B_s^0–$\bar{B}_s^0$ mixing, it takes the following form [25]:

$$\Delta\Gamma_s = \Delta\Gamma_s^{\rm SM}\cos\phi_s, \tag{40}$$

where $\Delta\Gamma_s^{\rm SM}$ is the width difference of the B_s-meson system in the SM. The most recent updates for the theoretical results for this quantity are

$$\frac{\Delta\Gamma_s^{\rm SM}}{\Gamma_s} = \begin{cases} 0.13 \pm 0.04 \ [26], \\ 0.14 \pm 0.02 \ [27]. \end{cases} \tag{41}$$

Using the expression in (27) with (39) and the value with the larger error in (41), we obtain the following SM prediction of the effective lifetime ratio:

$$\left.\frac{\tau_{K^+K^-}}{\tau_{B_s}}\right|_{\rm SM} = 0.9411^{+0.0011}_{-0.0006}\Big|_{\mathcal{A}_{\Delta\Gamma}} {}^{+0.0173}_{-0.0165}\Big|_{\Delta\Gamma_s/\Gamma_s} = 0.941^{+0.017}_{-0.017}, \tag{42}$$

where the errors have been added in quadrature. Combining this with the measurement $\tau_{B_s} = (1.477 \pm 0.022)\,$ps [12] gives a SM prediction for the lifetime of

$$\tau_{K^+K^-}|_{\rm SM} = (1.390 \pm 0.032)\,{\rm ps}, \tag{43}$$

which is fully consistent with the CDF result in (28) although the errors are too large to draw any further conclusions at this point. LHCb is expected to achieve a precision of 2% (or better) with the data sample foreseen to be accumulated in 2011, corresponding to 1fb^{-1} at the LHC centre-of-mass energy of 7 TeV [28].

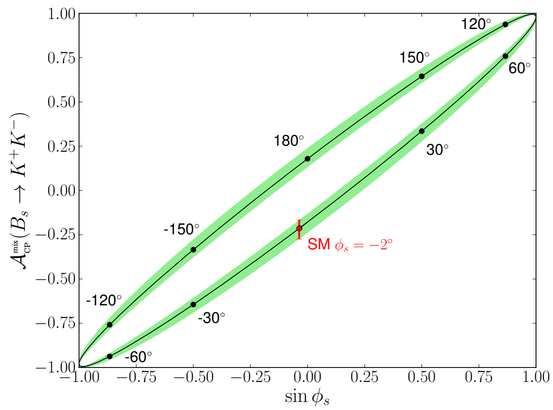

Figure 5: The correlation between $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ and $\sin\phi_s$. The error band takes the uncertainties due to the input parameters and observables into account, as well as possible U-spin-breaking corrections. The numbers label the values of ϕ_s.

In Fig. 4, we show the dependence of $\tau_{K^+K^-}/\tau_{B_s}$ on the mixing phase ϕ_s and illustrate the impact of a measurement of this lifetime ratio with a precision of 1%. Here we assume a value of $\phi_s = -45°$, which is the best fit value of the current DØ data (see Section 1). The figure clearly shows that a measurement of the effective $B_s^0 \to K^+K^-$ lifetime will offer an interesting alternative tool for finding evidence of a sizeable NP value for ϕ_s, thereby complementing the "conventional" analyses. The error band of the theoretical prediction is essentially fully dominated by that of $\Delta\Gamma_s^{\rm SM}/\Gamma_s$. The discrete ambiguity for ϕ_s emerging from the lifetime analysis can be resolved with the help of the mixing-induced CP violation in $B_s^0 \to K^+K^-$, as we will see in the next section.

As an alternative for using the theoretical SM value in (27), we may also use the experimental value of $\Delta\Gamma_s/\Gamma_s$, which depends implicitly on ϕ_s through (40). By the end of 2011, LHCb is expected to reach a sensitivity of about 0.03 for this measurement from $B_s^0 \to J/\psi\phi$ [29]. For a compilation of current experimental information of $\Delta\Gamma_s/\Gamma_s$, assuming sometimes the SM, the reader is referred to Ref. [12].

4 Mixing-Induced CP Violation in $B_s^0 \to K^+K^-$

The final observable that is offered by the $B_s^0 \to K^+K^-$ channel is its mixing-induced CP asymmetry, which takes the following form [1]:

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-) = +\left[\frac{d'^2 \sin\phi_s + 2\epsilon d' \cos\theta' \sin(\phi_s + \gamma) + \epsilon^2 \sin(\phi_s + 2\gamma)}{d'^2 + 2\epsilon d' \cos\theta' \cos\gamma + \epsilon^2}\right]. \quad (44)$$

The structure of this expression is very similar to (23), i.e. the strong phase enters only as $\cos\theta'$. Consequently, we can use the formulae given in the previous section to perform an analysis of $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ that is analogous to that of $\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-)$. For

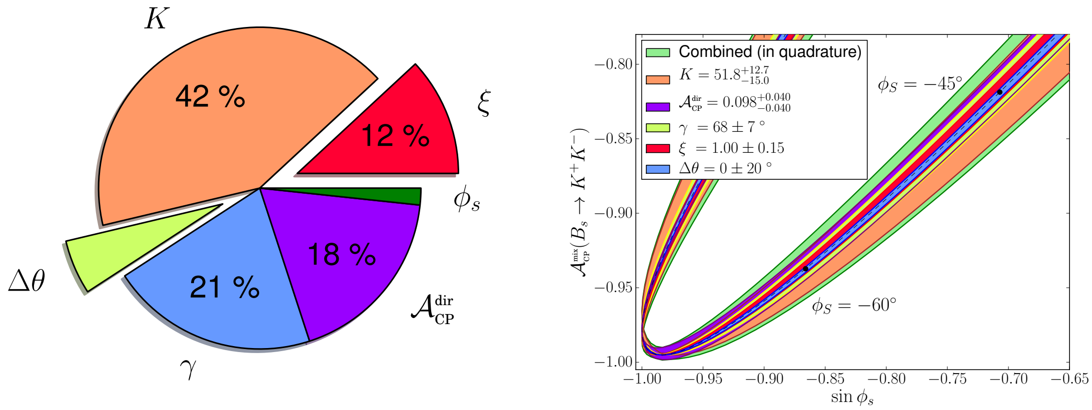

Figure 6: The error budget for $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$. Left panel: pie chart of the relative contribution of each input error for the SM case of $\phi_s^{\rm SM}$. Right panel: errors overlayed on top of one another for the correlation in Fig. 5. The legend lists the error contributions from largest to smallest.

the SM we obtain the prediction

$$\begin{aligned}\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)\Big|_{\rm SM} &= -0.215^{+0.031}_{-0.053}\Big|_K {}^{+0.022}_{-0.020}\Big|_\gamma {}^{+0.023}_{-0.014}\Big|_{\mathcal{A}_{\rm CP}^{\rm dir}} {}^{+0.014}_{-0.010}\Big|_\xi {}^{+0.004}_{-0.007}\Big|_{\Delta\theta} \\ &= -0.215^{+0.047}_{-0.060}, \end{aligned} \tag{45}$$

where the errors have been combined in quadrature. In Fig. 5, we show the dependence of $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ on $\sin\phi_s$, with a range of ϕ_s points marked explicitly. The latter quantity is conventionally measured through the CP-violating effects in the $B_s^0 \to J/\psi\phi$ angular distribution, as discussed above. The error bars on the SM point correspond to those given above. This plot illustrates two interesting features:

- $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ offers a powerful tool to search for footprints of a sizable NP phase ϕ_s, and deviates already significantly from the SM value for moderate values of this phase (such as $\phi_s \sim -30°$).

- $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ allows us to resolve the twofold ambiguity for the value of ϕ_s resulting from the analyses of $B_s^0 \to J/\psi\phi$. In particular, we can then also distinguish between the SM case with $\phi_s \sim 0°$ and a NP scenario with $\phi_s \sim 180°$, both leading to small CP violation in $B_s^0 \to J/\psi\phi$.

The correlation in Fig. 5 was first discussed in Ref. [2] (its counterpart for $B_s^0 \to J/\psi K_{\rm S}$ was recently studied in Ref. [21]). Here we go beyond that analysis by making a detailed analysis of the corresponding errors and using γ as an input. As in the previous section, we observe that the calculation is remarkably stable with respect to possible U-spin-breaking corrections and input errors. In Fig. 6, we show the error budget corresponding to the various input parameters and observables: for the SM case, we give a pie chart of the relative contribution of each error, and for a NP region, zoomed in on the range

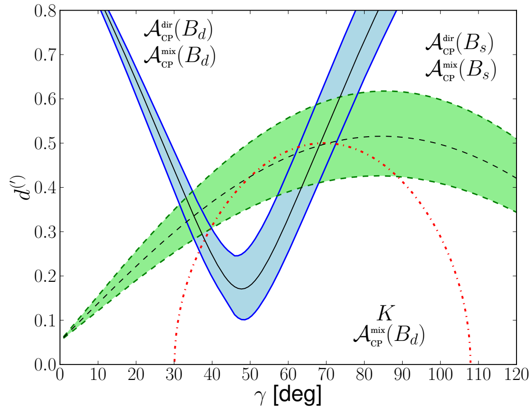

Figure 7: Illustration of the optimal determination of γ from the CP-violating observables of the $B_s \to K^+K^-$, $B_d \to \pi^+\pi^-$ system for the SM case. The band of the $B_s \to K^+K^-$ contour represents the 1σ errors in (46) and (47).

$\phi_s \in [-40^\circ, -120^\circ]$, we show, as in the right panel of Fig. 3, the errors overlayed on top of one another and centred on the central value.

The experimental sensitivities of the CP-violating $B_s^0 \to K^+K^-$ observables at LHCb were studied in Ref. [30]. Already with an integrated luminosity of $200\,\mathrm{pb}^{-1}$ at the 7 TeV run of the LHC, a statistical sensitivity for $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ of 0.11 can be obtained at this experiment [31]. Consequently, already for the first LHCb measurement of CP violation in $B_s^0 \to K^+K^-$ it will be exciting to confront the correlation in Fig. 5 with real data.

5 Optimal Determination of γ

The analyses discussed in the previous two sections provide target regions for improved measurements at the Tevatron and for the early data taking at LHCb. In the long run, the major application of the $B_s^0 \to K^+K^-$ decay will be the determination of γ. The key improvement with respect to the discussion in Section 2 will be made possible by the measurement of both $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ [1] (the expected LHCb statistical sensitivity for $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$ is 0.15 for already $200\,\mathrm{pb}^{-1}$ [31]). We can then convert the corresponding values into a contour in the γ–d' plane that is theoretically clean, in analogy to the γ–d contour following from the CP-violating observables of the $B_d^0 \to \pi^+\pi^-$ channel shown in Fig. 1. Using then the U-spin relation $d' = d$, we can determine γ and d from the intersection of the contours, as well as θ and θ', allowing an internal consistency check of the U-spin assumption. In particular, the quantity K does not enter this determination of γ. It can then instead be used to extract $|\mathcal{C}'/\mathcal{C}|$ for comparison with theoretical analyses of this ratio.

In Fig. 7, we illustrate the corresponding contour in the γ–d' plane that is determined through the CP asymmetries of the $B_s^0 \to K^+K^-$ channel. To this end, we refer to the γ analysis of Section 2. Using the values of γ, d and θ in (17) and (18), and neglecting the corresponding U-spin-breaking errors, we obtain

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) = 0.094^{+0.044}_{-0.039}, \tag{46}$$

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)|_{\rm SM} = -0.218^{+0.037}_{-0.036}. \tag{47}$$

These numbers are fully consistent with (37) and (45), which rely on different inputs, thereby further supporting our numerical analysis. The band in Fig. 7 referring to the CP-violating $B_s^0 \to K^+K^-$ observables corresponds to the central values in (46) and (47) and their 1σ ranges. The $B_d^0 \to \pi^+\pi^-$ contour is the same as in the left panel of Fig. 1, and, in order to guide the eye, we have also included the central value of the contour fixed through K and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$. It is interesting to observe that the $B_s^0 \to K^+K^-$ and $B_d^0 \to \pi^+\pi^-$ contours are intersecting with a large angle, thereby leading again to a situation that is very robust with respect to possible U-spin-breaking corrections to $d' = d$. It will be interesting to confront the contours in Fig. 7 with future LHCb data.

6 Conclusions

We have performed an analysis of the U-spin-related $B_s^0 \to K^+K^-$, $B_d^0 \to \pi^+\pi^-$ system in view of updated experimental and theoretical information and the first measurements from the LHCb experiment that are expected to arrive soon. We obtain a value of $\gamma = (68.3^{+4.9}_{-5.7}|_{\rm input}{}^{+5.0}_{-3.7}|_{\xi}{}^{+0.1}_{-0.2}|_{\Delta\theta})^\circ$, which is very competitive with other direct determinations of this angle. Moreover, our result is in excellent agreement with the fits for the unitarity triangle, thereby excluding large NP effects at the decay amplitude level.

A particularly interesting first observable of the $B_s^0 \to K^+K^-$ decay to be measured at LHCb will be its effective lifetime $\tau_{K^+K^-}$. We have calculated this observable as a function of the B_s^0–$\bar{B}_s^0$ mixing phase and arrive at a picture that is remarkably stable both with respect to the current errors of the relevant input quantities and with respect to possible U-spin-breaking effects. Moreover, we have shown that $\tau_{K^+K^-}$ offers an interesting alternative tool to get evidence for a sizeable NP value of ϕ_s, thereby complementing the analyses of CP violation in $B_s^0 \to J/\psi\phi$.

The next newly measured observable to enter the stage should be the mixing-induced CP asymmetry of $B_s^0 \to K^+K^-$, which is correlated with $\sin\phi_s$ in an interesting way. This correlation is again remarkably stable with respect to the errors of the input quantities and possible U-spin-breaking corrections. Even for moderate values of ϕ_s, $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ deviates significantly from its SM value and therefore provides another sensitive probe for CP-violating NP effects in B_s^0–$\bar{B}_s^0$ mixing. The measurement of $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ will then also allow us to determine ϕ_s unambiguously and, in particular, to distinguish the SM case with $\phi_s \sim 0^\circ$ from a NP scenario with $\phi_s \sim 180^\circ$, both leading to small CP violation in $B_s^0 \to J/\psi\phi$.

Finally, once the direct CP asymmetry of $B_s^0 \to K^+K^-$ is also accurately measured, we can optimize the extraction of the angle γ of the unitarity triangle, which will be the major application of the $B_s^0 \to K^+K^-$, $B_d^0 \to \pi^+\pi^-$ system at LHCb in the long run. Moreover, we can then also perform internal checks of the U-spin symmetry assumption.

The picture following from the current data points towards a stable and favourable situation with respect to possible U-spin-breaking effects.

We look forward to confronting the "target regions" in observable space calculated in this paper with real experimental data!

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