

Discrimination of Supersymmetric Grand Unified Models in Gaugino Mediation

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Abstract

We consider supersymmetric grand unified theory (GUT) with the gaugino mediated supersymmetry breaking and investigate a possibility to discriminate different GUT models in terms of predicted sparticle mass spectra. Taking two example GUT models, the minimal $SU(5)$ and a simple $SO(10)$ models, and imposing a variety of theoretical and experimental constraints, we calculate sparticle masses. Fixing parameters of each model so as to result in the same mass of neutralino as the lightest supersymmetric particle (LSP), giving the observed dark matter relic density, we find sizable mass differences in the left-handed slepton and right-handed down-type squark sectors in two models, which can be a probe to discriminate the GUT models realized at the GUT scale far beyond the reach of collider experiments.

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1 Introduction

Providing a promising solution to a long-standing problem in the Standard Model (SM), the gauge hierarchy problem, and motivated by the possibility of being tested at the Large Hadron Collider (LHC) and other future collider projects such as the International Linear Collider (ILC), supersymmetry (SUSY) has been intensively explored for the last several decades. In addition, under the R-parity conservation, the minimal supersymmetric extension of the SM (MSSM) provides neutralino LSP which is a good candidate for the dark matter, a mysterious block of the universe needed to explain the cosmological observation. Furthermore, in the MSSM, all the SM gauge couplings successfully unify at the GUT scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV, and this fact strongly supports the GUT paradigm.

The exact SUSY requires that the SM particles and their superpartners have equal masses. However, we have not yet observed any signal of sparticles in both direct and indirect experimental searches. This implies that not only SUSY should be broken at some energy, but also SUSY breaking should be transmitted to the MSSM sector in a clever way not to cause additional flavor changing neutral currents and CP-violations associated with supersymmetry breaking terms. There have been several interesting mechanisms for desirable SUSY breaking and its mediations.

In this paper, we consider one of the possibilities, the gaugino mediated SUSY breaking (gaugino mediation) [1]. With a simple 5D braneworld setup of this scenario, the SUSY breaking is first mediated to the gaugino sector, while sfermion masses and trilinear couplings are negligible at the compactification scale of the extra 5th dimension. At low energies, the sfermion masses and trilinear couplings are generated through RGE runnings with the gauge interactions, realizing the flavor-blind sfermion masses. However, the gaugino mediation in the context of the MSSM predicts stau LSP, and such a stable charged particle is disfavored in cosmological point of view. This problem can be naturally solved if the compactification scale is higher than the GUT scale and a GUT is realized there [2]. The RGE runnings as the GUT play the crucial role to push up stau mass, and neutralino LSP is realized at the electroweak scale, which is a suitable dark matter candidate as usual in SUSY models.

There are many possibilities of GUT models with different unified gauge groups and representations of the matter and Higgs multiplets in the groups. A question arising here is how we can discriminate GUT models by experiments carrying out at energies far below the GUT scale. Note that SUSY GUT models with SUSY breaking mediations at or above the GUT scale leave their footprints on sparticle mass spectra at low energies through the RGE evolutions. Typical sparticle mass spectrum, once observed, can be a probe of $SU(5)$ unification [3]. In a similar way, three different types of seesaw mechanism for neutrino masses can be distinguished at the

LHC and the ILC [4]. In this paper, based on the same idea, we investigate a possibility to discriminate different GUT models with the gaugino mediation. A remarkable feature of the gaugino mediation is that the model is highly predictive and sparticle masses are determined by only 2 free parameters, the compactification scale (M_c) and the input gaugino mass (M_G) at M_c , with a fixed $\tan\beta$ and the sign of the μ -parameter.

The structure of this paper is as follows: In Section 2, we briefly discuss the basic setup of the gaugino mediation and introduce two examples of GUT models, the minimal $SU(5)$ model and a simple $SO(10)$ model. In Section 3, we analyze the RGE evolutions of sparticle masses and the trilinear couplings for the two GUT models from the compactification scale to the electroweak scale, and find sparticle mass spectra which are consistent with a variety of theoretical and experimental constraints. Fixing parameters in the both models to result in the same neutralino LSP mass, giving the observed dark matter relic abundance, we compare sparticle mass spectra. We find sizable sparticle mass differences which can be a probe to discriminate the GUT models. The last section is devoted for conclusions.

2 Model setup

In the gaugino mediation scenario [1], we introduce a 5-dimensional flat spacetime in which the extra 5th dimension is compactified on the S^1/Z_2 orbifold with a radius $r = 1/M_c$. The SUSY breaking sector resides on a $(3 + 1)$ -dimensional brane at one orbifold fixed point, while the matter and Higgs sectors on another brane at the other orbifold fixed point. Since the gauge multiplet propagates in the bulk, the gaugino can directly couple with the SUSY breaking sector and acquires the soft mass at the tree level. On the other hand, due to the sequestering between two branes, the matter superpartners and Higgs fields cannot directly communicate with the SUSY breaking sector and hence, sfermion and Higgs boson soft masses and also the trilinear couplings are all zero at the tree level. According to this structure of the gaugino mediation, in actual analysis of RGE evolutions for soft parameters, we set non-zero gaugino mass at the compactification scale and solve RGEs from M_c toward low energies. Soft masses of matter superpartners and Higgs fields are generated via the RGE evolutions.

When the compactification scale is lower than M_{GUT} , the detailed study on MSSM sparticle masses in the gaugino mediation showed that the LSP is stau in most of the parameter space [2]. Clearly, this result is disfavored in cosmological point of view. However, it has been shown that this drawback can be ameliorated if we assume a GUT model and $M_c > M_{GUT}$ [2]: the RGE evolutions from M_c to M_{GUT} push up stau mass and realize neutralino LSP. In other words, the grand unification is crucial to realize phenomenologically viable sparticle mass spectrum in the gaugino mediation. In order to suppress sfermion masses compared to gaugino masses at

the compactification scale, the spatial separation between two branes should not be too small, equivalently, the compactification scale should not be too large. In the following analysis, we set the reduced Planck scale (M_P) as the upper bound on M_c :

$$M_c \leq M_P = 2.43 \times 10^{18} \text{ GeV}. \quad (1)$$

There have been many GUT models proposed based on different unified gauge groups such as $SU(5)$, $SO(10)$ and E_6 . In this paper, we consider two GUT models as examples, namely, the minimal $SU(5)$ model and a simple $SO(10)$ model [5].

In the minimal $SU(5)$ model, the matter multiplets of the i -th generation are arranged in 2 representations, $\bar{\mathbf{5}}_i$ and $\mathbf{10}_i$. Two Higgs doublets in the MSSM are embedded in the representations of $\bar{\mathbf{5}}_H + \mathbf{5}_H$, while the $\mathbf{24}_H$ Higgs multiplet plays the role of breaking the $SU(5)$ gauge symmetry to the SM one. The particle contents of the minimal $SU(5)$ model along with the Dynkin index and the quadratic Casimir for corresponding multiplets are listed in Table 1.

Table 1: Particle contents of the minimal $SU(5)$ GUT

$SU(5)$	Particles	Dynkin Index	$C_2(\mathbf{R})$
$\bar{\mathbf{5}}_i$	D_i^c, L_i	1/2	12/5
$\mathbf{10}_i$	Q_i, U_i^c, E_i^c	3/2	18/5
$\bar{\mathbf{5}}_H$	H_d	1/2	12/5
$\mathbf{5}_H$	H_u	1/2	12/5
$\mathbf{24}_H$	additional Higgs	5	5

In $SO(10)$ GUT models, all the matter multiplets of the i -th generation are unified into a single $\mathbf{16}_i$ representation. In a simple $SO(10)$ model investigated in [5], Higgs multiplets of the representations $\mathbf{10}_H + \mathbf{10}'_H + \bar{\mathbf{16}}_H + \mathbf{16}_H + \mathbf{45}_H$ are introduced. The up-type (down-type) Higgs doublets in the MSSM is realized as a linear combination of two up-type (down-type) Higgs doubles in $\mathbf{10}_H + \mathbf{10}'_H$, while the Higgs multiplets of $\bar{\mathbf{16}}_H + \mathbf{16}_H + \mathbf{45}_H$ representations work to break the $SO(10)$ gauge symmetry to the MSSM one. Similarly to Table 1, the particle contents of this model are listed in Table 2.

3 Sparticle masses in two models

Now we analyze sparticle mass spectrum at low energy for each GUT model. In the gaugino mediation, gaugino mass is a unique input at the compactification scale $M_c > M_{GUT}$. For a given GUT model, solving the RGEs from M_c to M_{GUT} with the gaugino mass input, we obtain

Table 2: Particle contents of a simple $SO(10)$ GUT

$SO(10)$	Particles	Dynkin Index	$C_2(\mathbf{R})$
$\mathbf{16}_i$	i - th generation	2	45/8
$\mathbf{10}_H$	H_u^1, H_d^1	1	9/2
$\mathbf{10}'_H$	H_u^2, H_d^2	1	9/2
$\mathbf{16}_H$	additional Higgs	2	45/8
$\mathbf{16}_H$		2	45/8
$\mathbf{45}_H$		8	8

a set of soft parameters at the GUT scale, with which we solve the MSSM RGEs for the soft parameters toward low energies. General 1-loop RGE formulas for the soft parameters in a GUT model are given by [2]:

$$\frac{d\alpha_U}{dt} = -\frac{b_U}{2\pi}\alpha_U^2, \quad (2)$$

$$\frac{d}{dt} \left(\frac{M}{\alpha_U} \right) = 0, \quad (3)$$

$$\frac{dm^2}{dt} = -2C_2(\mathbf{R})\frac{\alpha_U}{\pi}M^2, \quad (4)$$

$$\frac{dA}{dt} = \left(\sum_i C_2(\mathbf{R}_i) \right) \frac{\alpha_U}{\pi}M, \quad (5)$$

where α_U is the unified gauge coupling, b_U is the beta function coefficient, M is the running gaugino mass, m is the running mass of a scalar field in the \mathbf{R} representation under the GUT gauge group, and C_2 is the quadratic Casimir. For the boundary conditions in the gaugino mediation scenario,

$$M(M_c) = M_G \neq 0, \quad m^2(M_c) = 0, \quad A(M_c) = 0, \quad (6)$$

we can easily find the solutions:

$$\alpha_U(\mu)^{-1} = \alpha_U(M_c)^{-1} + \frac{b_U}{2\pi} \ln(\mu/M_c), \quad (7)$$

$$m^2(\mu) = 2\frac{C_2(\mathbf{R})}{b_U}M^2(\mu) \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(\mu)} \right)^2 \right], \quad (8)$$

$$A(\mu) = -\frac{2}{b_U} \left(\sum_i C_2(\mathbf{R}_i) \right) M(\mu) \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(\mu)} \right) \right]. \quad (9)$$

We now apply the above solutions to the minimal $SU(5)$ GUT model with the particle

contents as in Table 1. Since the beta function coefficient of the model is $b_U = 3$, we have

$$\alpha_U(M_{GUT})^{-1} = \alpha_U(M_c)^{-1} + \frac{3}{2\pi} \ln(M_{GUT}/M_c), \quad (10)$$

$$m_{\mathbf{10}}^2(M_{GUT}) = \frac{12}{5} M_{1/2}^2 \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right)^2 \right], \quad (11)$$

$$m_{\mathbf{5}}^2(M_{GUT}) = m_{\mathbf{5}}^2(M_{GUT}) = \frac{8}{5} M_{1/2}^2 \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right)^2 \right], \quad (12)$$

$$A_u(M_{GUT}) = -\frac{32}{5} M_{1/2} \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right) \right], \quad (13)$$

$$A_d(M_{GUT}) = -\frac{28}{5} M_{1/2} \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right) \right], \quad (14)$$

where $M_{1/2} = M(M_{GUT})$ is the universal gaugino mass at the GUT scale. Note that the sfermion masses at the GUT scale are not universal, but the relation between soft masses of different representation fields are fixed by C_2 .

For the simple $SO(10)$ model with the particle contents in Table 2, the beta function coefficient is $b_U = 4$ and we have

$$\alpha_U(M_{GUT})^{-1} = \alpha_U(M_c)^{-1} + \frac{2}{\pi} \ln(M_{GUT}/M_c), \quad (15)$$

$$m_{\mathbf{16}}^2(M_{GUT}) = \frac{45}{16} M_{1/2}^2 \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right)^2 \right], \quad (16)$$

$$m_{\mathbf{10}}^2(M_{GUT}) = \frac{9}{4} M_{1/2}^2 \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right)^2 \right], \quad (17)$$

$$A(M_{GUT}) = -\frac{63}{8} M_{1/2} \left[1 - \left(\frac{\alpha_U(M_c)}{\alpha_U(M_{GUT})} \right) \right]. \quad (18)$$

In the $SO(10)$ model, the MSSM sfermion masses are universal at the GUT scale.

For numerical calculation, we have only two free parameters, M_G and M_c , with fixed $\tan \beta$ and the sign of the μ -parameter. In MSSM RGE analysis below M_{GUT} , we choose $M_{1/2}$ as a free parameter and the other soft parameters are fixed once M_c fixed. In order to compare sparticle spectrum in the two GUT models, it is necessary to fix a common base for them. We choose the values of free parameters in such a way that two models give the same neutralino LSP mass. In the gaugino mediations, neutralino LSP is bino-like, so that the same $M_{1/2}$ inputs for two models give (almost) the same masses for neutralino LSP. The compactification scale M_c is still left as a free parameter, whose degree of freedom is used to fix another sparticle mass. Here we impose a cosmological constraint that the relic abundance of neutralino LSP is consistent with the (cold) dark matter abundance measured by the WMAP [6]:

$$\Omega_{CMD} h^2 = 0.1131 \pm 0.0034. \quad (19)$$

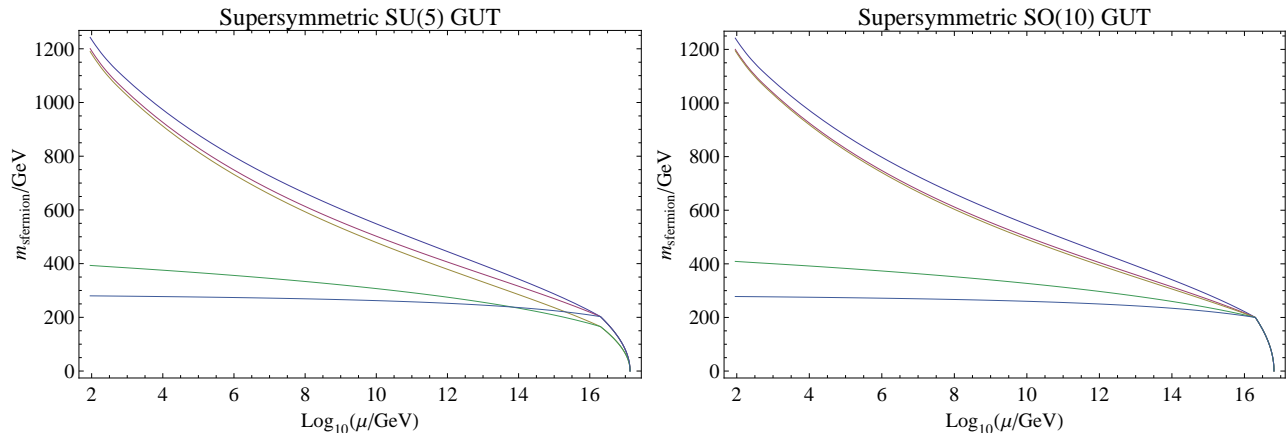


Figure 1: RGE evolution of the first two generation sfermion soft masses ($m_{\tilde{Q}}$, $m_{\tilde{U}^c}$, $m_{\tilde{D}^c}$, $m_{\tilde{L}}$ and $m_{\tilde{E}^c}$ from top to bottom) with $\tan\beta = 30$, $\mu > 0$ and $M_{1/2} = 500$ GeV for the $SU(5)$ and $SO(10)$ models, respectively.

This WMAP constraint dramatically reduces the viable parameter space of the models as in the constrained MSSM [7]. For a given $\tan\beta$ and a fixed $M_{1/2}$, the compactification scale is completely fixed by this cosmological constraint. As we will see, the right relic abundance is achieved by the neutralino co-annihilations with the next-to-LSP (mostly right-handed) stau almost degenerated with the LSP. For the two GUT models, the resultant next-to-LSP stau masses are found to be almost the same.

The RGE evolutions of the first two generations of squarks and sleptons are demonstrated in the case of $\tan\beta = 30$, $\mu > 0$ and $M_{1/2} = 500$ GeV for the $SU(5)$ and $SO(10)$ models in Figure 1. The compactification scales M_c for the two models are fixed to give the correct neutralino relic abundance: $M_c = 1.36 \times 10^{17}$ GeV and 6.53×10^{16} GeV for the $SU(5)$ and $SO(10)$ models, respectively. Here we can see characteristic features of running sfermion masses for the two GUT models, namely, sfermion masses are unified at two points in the $SU(5)$ model, on the other hand, one point unification in the $SO(10)$ model. The cosmological constraint requires the next-to-LSP stau, which is mostly the right-handed stau, is almost degenerated with the neutralino LSP, and we find $m_{\mathbf{10}}^{SU(5)} \approx m_{\mathbf{16}}^{SO(10)}$ at the GUT scale. However, there is a sizable mass splitting between $m_{\mathbf{5}}^{SU(5)}$ and $m_{\mathbf{16}}^{SO(10)}$. This is the key to distinguish the two GUT models. In terms of sparticles in the MSSM, the difference appears in masses of down-type squarks and the left-handed sleptons.

In our numerical analysis, we employ the SOFTSUSY 3.1.4 package [8] to solve the MSSM RGEs and produce mass spectrum. While running this program, we always set $sign(\mu) = +1$, for simplicity. The relic abundance of the neutralino dark matter is calculated by using the

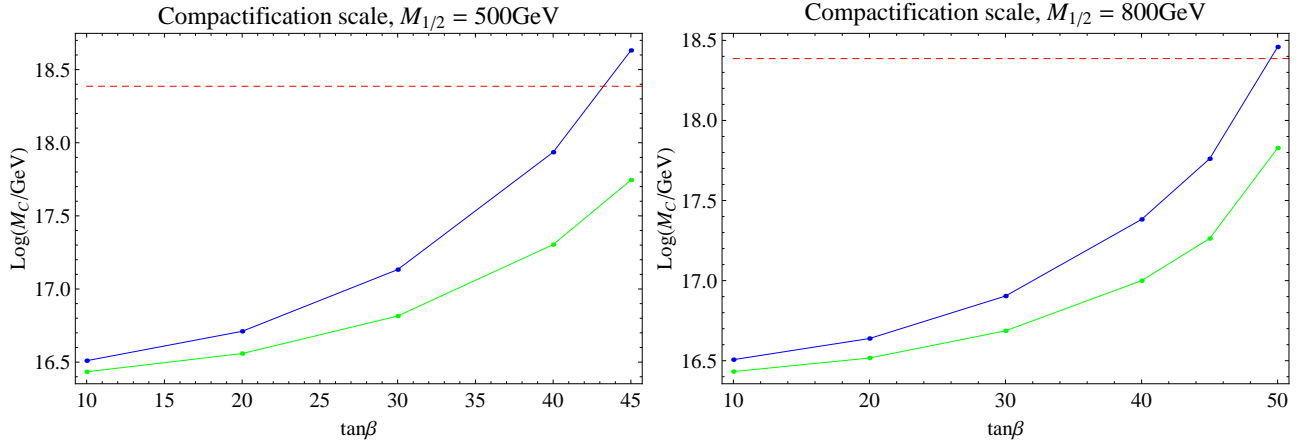


Figure 2: Compactification scale as a function of $\tan\beta$ in the case $M_{1/2} = 500$ GeV and 800 GeV. In each plot, the upper (blue) and lower (green) solid lines correspond to the $SU(5)$ and $SO(10)$ models, respectively. The horizontal dashed (red) line indicates the theoretical constraint (1).

micrOMEGAs 2.4 [9] with the output of SOFTSUSY in the SLHA format [10]. In addition to the cosmological constraint, we also take into account other phenomenological constraints such as the lower bound on Higgs boson mass [11]:

$$m_h \geq 114.4 \text{ GeV}, \quad (20)$$

the constraints on the branching ratios of $b \rightarrow s\gamma$, $B_s \rightarrow \mu^+\mu^-$ and the muon anomalous magnetic moment $\Delta a_\mu = g_\mu - 2$:

$$2.85 \times 10^{-4} \leq BR(b \rightarrow s + \gamma) \leq 4.24 \times 10^{-4} \quad (2\sigma) \quad [12], \quad (21)$$

$$BR(B_s \rightarrow \mu^+\mu^-) < 5.8 \times 10^{-8} \quad [13], \quad (22)$$

$$3.4 \times 10^{-10} \leq \Delta a_\mu \leq 55.6 \times 10^{-10} \quad (3\sigma) \quad [14]. \quad (23)$$

We examine two typical values of $M_{1/2} = 500$ and 800 GeV for a variety of $\tan\beta = 10, 20, 30, 40, 45,$ and 50. The mass spectra of the two models are shown in Table 3 for the case of $M_{1/2} = 500$ GeV and in Table 4 for the case of $M_{1/2} = 800$ GeV. In Tables, we also list the values of the compactification scale M_c chosen to reproduce the observed dark matter abundance, the branching ratios of $b \rightarrow s\gamma$ and $B_s \rightarrow \mu^+\mu^-$, and the anomalous magnetic moment of muon Δa_μ .

Using the data in Tables 3 and 4, we plot the compactification scale as a function of $\tan\beta$ for $M_{1/2} = 500$ and 800 GeV, respectively, in Figure 2. The upper (blue) and lower (green) solid lines indicate the $SU(5)$ and $SO(10)$ models, respectively. The horizontal dashed (red)

line corresponds to the upper bound on the compactification scale (1). These figures show that the theoretical constraint (1) rules out a large $\tan\beta$ region for the $SU(5)$ model. We find the upper bounds $\tan\beta \lesssim 43$ for $M_{1/2} = 500$ GeV and $\tan\beta \lesssim 49$ for $M_{1/2} = 800$ GeV. Comparing the two plots in Figure 2, we see that the bound on $\tan\beta$ becomes more severe for smaller $M_{1/2}$ inputs.

For the sparticle spectra presented in Tables 3 and 4, phenomenological constraints of (19), (20), (22) and (23) are all satisfied. However, the predicted branching ratio $BR(b \rightarrow s\gamma)$ can be too small to satisfy the experimental bound (21) for a large $\tan\beta$. In Figure 3, we show the values of $BR(b \rightarrow s\gamma)$ for all the samples in Table 3 and 4, along with the experimental allowed region between two dashed (red) lines. We can see that for the case with $M_{1/2} = 500$ GeV, there is an upper bound on $\tan\beta \lesssim 38$. In general, for a smaller $M_{1/2}$ input, we will find a more severe bound on $\tan\beta$.

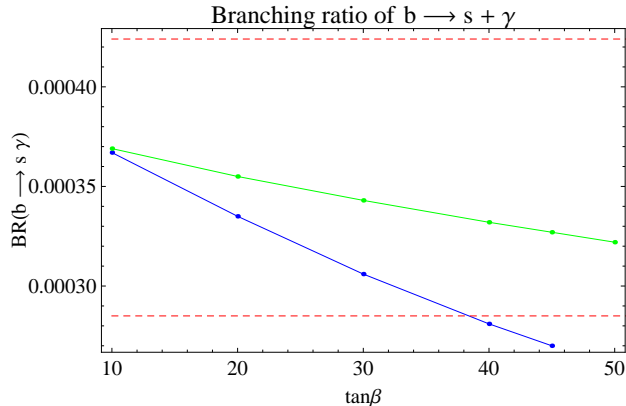


Figure 3: $BR(b \rightarrow s\gamma)$ as a function of $\tan\beta$ for $M_{1/2} = 500$ and 800 GeV. The lower (blue) and upper (green) solid lines correspond to $M_{1/2} = 500$ GeV and 800 GeV, respectively. The horizontal dashed (red) lines indicate the upper and lower bounds of the branching ratio (21).

Taking into account all theoretical and phenomenological bounds, we compare the mass difference between the two GUT models. As mentioned before, in Tables 3 and 4 we see relatively large mass differences in left-handed slepton sector and right-handed down-type squark sector. This effect is not so clear in the third generation squark masses because of the large Yukawa contributions. Figure 4 shows the mass difference $\delta m = m^{SO(10)} - m^{SU(5)}$ between left-handed selectrons/smuons of the two models as a function of $\tan\beta$ for $M_{1/2} = 500$ GeV (lower solid line) and 800 GeV (upper solid line). As we have discussed above, the upper bound on M_c and the constraint from sparticle contributions to the $b \rightarrow s\gamma$ process provide us the upper bound on $\tan\beta$. The dashed vertical line and the left dot-dashed line correspond to the upper bound on $\tan\beta$ from $BR(b \rightarrow s\gamma)$ and $M_c \leq M_P$, respectively, applied to the case with $M_{1/2} = 500$ GeV

(lower solid line). The right dot-dashed line is the upper bound from $BR(b \rightarrow s\gamma)$ for the case with $M_{1/2} = 800$ GeV (upper solid line). Depending on values of $\tan\beta$, the mass differences for $M_{1/2} = 500$ GeV varies $\delta m = 5 - 25$ GeV, while $\delta m = 7 - 75$ GeV for $M_{1/2} = 800$ GeV. These mass differences can be sufficiently large compared to expected errors in measurements of sparticle masses at the LHC and the ILC [15].

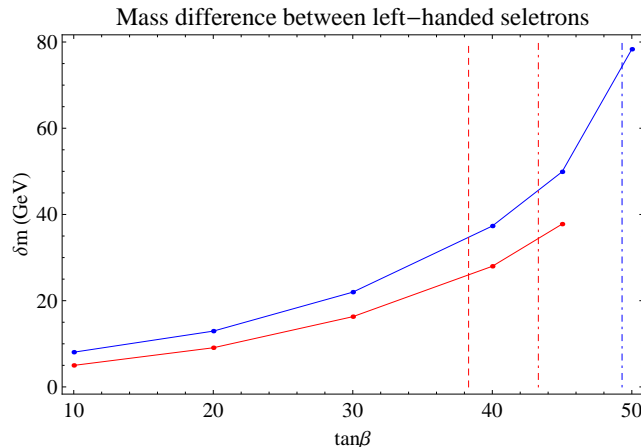


Figure 4: Mass difference $\delta m = m^{SO(10)} - m^{SU(5)}$ between left-handed selectrons/smuons of the two models is plotted as a function of $\tan\beta$ for $M_{1/2} = 500$ and 800 GeV. The lower (red) and upper (blue) solid lines correspond to Table 3 with $M_G = 500$ GeV and Table 4 with $M_G = 800$ GeV, respectively. The dashed line is the upper bound on $\tan\beta$ from the $b \rightarrow s\gamma$ constraint. The dot-dashed lines indicate the upper bounds on $\tan\beta$ by the theoretical constraint $M_c < M_P$. The right vertical bound applies to the case with $M_{1/2} = 800$ GeV, while two left vertical lines to the case with $M_{1/2} = 500$ GeV.

4 Conclusion

In the context of the gaugino mediation scenario, we have investigated supersymmetric grand unified theories. The gaugino mediation scenario, once applied to a GUT model, is highly predictive and all sparticle masses are determined by only two inputs, the unified gaugino mass and the compactification scale, with a given $\tan\beta$ and the sign of the μ -parameter. When we choose a particular GUT model with fixed particle contents, the relation among sparticle masses at the GUT scale is determined by the group theoretical factors, the Dynkin index and quadratic Casimir, associated with the representation of fields. Therefore, the difference of GUT models is reflected in sparticle mass spectrum at low energies. Taking two GUT models, the minimal $SU(5)$ GUT and a simple $SO(10)$ GUT models as examples, we have analyzed sparticle mass spectra together with theoretical and phenomenological constraints and

compared resultant sparticle masses in the two models. Because of the difference in unification of quarks and leptons into representations under the GUT gauge groups, a significant difference among sparticle masses appears in the left-handed slepton and right-handed down-type squark sectors. Fixing the input parameters in each model so as to give the same neutralino mass and to reproduce the observed neutralino dark matter relic abundance, we have found sizable differences in sparticle mass spectra in two models, which can be identified in the LHC and the ILC. Although we have considered only two GUT models, our strategy is general, and we conclude that precise measurements of sparticle mass spectrum can be a probe to discriminate various supersymmetric unification scenarios.

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