

Masses of tetraquarks with open charm and bottom

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The masses of the heavy tetraquarks with open charm and bottom are calculated within the diquark-antidiquark picture in the framework of the relativistic quark model. The dynamics of the light quarks and diquarks is treated completely relativistically. The diquark structure is taken into account by calculating the diquark-gluon form factor. New experimental data on charmed and charmed-strange mesons are discussed. Our results indicate that the anomalous scalar $D_{s_0}^*(2317)$ and axial vector $D_{s_1}(2460)$ mesons could not be considered as diquark-antidiquark bound states. On the other hand, $D_s(2632)$ and $D_{s_J}^*(2860)$ could be interpreted as scalar and tensor tetraquarks, respectively. The predictions for masses of the corresponding bottom counterparts of the charmed tetraquarks are given.

PACS numbers: 12.40.Yx, 14.40.Gx, 12.39.Ki

I. INTRODUCTION

In last few years a significant experimental progress has been achieved in charmonium and charmed meson spectroscopy. Many new states, such as $X(3872)$, $Y(4260)$, $Y(4360)$, $Y(4660)$, $Z(4248)$, $Z(4430)$, $D_{s_0}^*(2317)$, $D_{s_1}(2460)$, $D_s(2632)$ and $D_{s_J}^*(2860)$ etc., were observed [1] which cannot be simply accommodated in the quark-antiquark ($c\bar{c}$ and $c\bar{s}$) picture. These states and especially the charged charmonium-like ones can be considered as indications of the possible existence of exotic multiquark states.

The open charm mesons, both with and without open strangeness, represent a special interest. Even seven years after the discovery of the charmed-strange $D_{s_0}^*(2317)$ and $D_{s_1}(2460)$ mesons their nature remains controversial in the literature. The abnormally light masses of these mesons put them below DK and D^*K thresholds, thus making these states narrow, since the only allowed strong decays $D_{s_J}^{(*)} \rightarrow D_s^{(*)}\pi$ violate isospin symmetry. The peculiar feature of these mesons is that they have masses almost equal or even lower than the masses of their charmed counterparts $D_0^*(2400)$ and $D_1(2427)$ [2–4]. Most of the theoretical approaches including lattice QCD [5], QCD sum rule [6] and different quark model [7, 8] calculations give masses of the 0^+ and 1^+ P -wave $c\bar{s}$ states significantly heavier (by 100-200 MeV) than the measured ones. Different theoretical solutions of this problem were proposed including consideration of these mesons as chiral partners of 0^- and 1^- states [9], $c\bar{s}$ states which are strongly influenced by the nearby DK thresholds [10], DK or $D_s\pi$ molecules [11], a mixture of $c\bar{s}$ and tetraquark states [12–16]. However the universal understanding of their nature is still missing. Therefore it is very important to observe their bottom counterparts. The unquenched lattice calculations of their masses can be found in Ref. [17].

TABLE I: Predictions [24] for the masses of charmed ($q = u, d$) and charmed-strange mesons (in MeV) in comparison with available experimental data [4, 25].

State		Theory	Experiment		Theory	Experiment	
$n^{2S+1}L_J$	J^P	mass ($c\bar{q}$)	meson	mass	mass ($c\bar{s}$)	meson	mass
1^1S_0	0^-	1871	D	1869.62(16)	1969	D_s	1968.47(33)
1^3S_1	1^-	2010	$D^*(2010)$	2010.25(14)	2111	D_s^*	2112.3(5)
1^3P_0	0^+	2406	$D_0^*(2400)$	$\begin{cases} 2403(40)(^\pm) \\ 2318(29)(^0) \end{cases}$	2509	$D_{s0}^*(2317)$	2317.8(6)
$1P_1$	1^+	2469	$D_1(2430)$	2427(40)	2574	$D_{s1}(2460)$	2459.5(6)
$1P_1$	1^+	2426	$D_1(2420)$	2423.4(3.1)	2536	$D_{s1}(2536)$	2535.29(20)
1^3P_2	2^+	2460	$D_2^*(2460)$	2460.1($^{+2.6}_{-3.5}$)	2571	$D_{s2}(2573)$	2572.6(9)
2^1S_0	0^-	2581	$D(2550)^\dagger$	2539.4(8.1)(0)	2688		
2^3S_1	1^-	2632	$\begin{cases} D^*(2600)^\dagger \\ D^*(2640) \end{cases}$	$\begin{cases} 2621.3(5.6)(^+) \\ 2608.7(3.5)(^0) \\ 2637(6)? \end{cases}$	2731	$D_{s1}(2710)$	2709($^{+9}_{-6}$)
1^3D_1	1^-	2788	$\begin{cases} D^*(2760)^\dagger \\ D(2750)^\dagger \end{cases}$	$\begin{cases} 2769.7(4.1)(^+) \\ 2763.3(3.3)(^0) \\ 2752.4(3.2) \end{cases}$	2913		
$1D_2$	2^-	2850			2961		
$1D_2$	2^-	2806			2931		
1^3D_3	3^-	2863			2971	$D_{sJ}^*(2860)$	2862($^{+6}_{-3}$)
2^3P_0	0^+	2919			3054		
$2P_1$	1^+	3021			3154		
$2P_1$	1^+	2932			3067	$D_{sJ}(3040)$	3044($^{+31}_{-9}$)
2^3P_2	2^+	3012			3142		
3^1S_0	0^-	3062			3219		
3^3S_1	1^-	3096			3242		

† new states recently observed by BaBar [25]

Another unexpectedly narrow charmed-strange meson $D_s(2632)$ was discovered by SELEX Collaboration [18]. Its unusual decay properties triggered speculations about its possible exotic origin [1]. However, the status of this state remains controversial since FOCUS [19], BaBar [20] and Belle [21] reported negative results in their search for this state.

Three other charmed-strange mesons $D_{s1}(2710)$, $D_{sJ}^*(2860)$ and $D_{sJ}(3040)$ were discovered at B -factories by Belle and BaBar [22, 23]. Not all of them could be simply accommodated in the usual $c\bar{s}$ picture. Their decay pattern implies that $D_{sJ}^*(2860)$ should have natural parity, while $D_{s1}(2710)$ and $D_{sJ}(3040)$ should have unnatural parity. Our recent calculation of the heavy-light quark-antiquark meson spectra [24] have shown that $D_{s1}(2710)$ and $D_{sJ}(3040)$ are good candidates for the 2^3S_1 and $2P_1$ states, respectively. They nicely fit to the corresponding Regge trajectories, while $D_{s0}^*(2317)$, $D_{s1}(2460)$, $D_{sJ}^*(2860)$ and $D_s(2632)$ have anomalously low masses (lower than expected by 100-200 MeV) and do not lie on the respective Regge trajectories.

In the charmed sector very recently the BaBar Collaboration discovered four new signal

peaks $D(2550)$, $D^*(2600)$, $D(2750)$ and $D^*(2760)$ [25]. The last two signals, observed in $D^*\pi$ and $D\pi$ modes, have close mass and width values (which differ by 2.6σ and 1.5σ , respectively) and therefore could belong to the same state. The angular analysis shows that these signals can be considered as candidates for the radially excited 2^1S_0 , 2^3S_1 states and orbitally excited 1^3D_1 state, respectively. Their mass values are in a good agreement with the results of our model. We summarize our predictions for the masses of the $c\bar{q}$ ($q = u, d$) and $c\bar{s}$ mesons in Table I and confront them with the available experimental data [4, 25]. As it is clearly seen there are indications of the possible existence of the exotic states especially in the charmed-strange sector.

In our papers [26, 27] we calculated masses of the hidden heavy-flavour tetraquarks and light tetraquarks in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. Here we extend this analysis to the consideration of heavy tetraquark states with open charm and bottom. This study could help in revealing the nature of the anomalous charmed-strange mesons.

As previously [26, 27], we use the diquark-antidiquark picture to reduce a complicated relativistic four-body problem to two subsequent more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed from light and heavy quarks. At the second step, a tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that we do not consider the diquark as a point particle but explicitly take into account its structure by calculating the form factor of the diquark-gluon interaction in terms of the diquark wave functions.

II. RELATIVISTIC MODEL OF TETRAQUARKS

In the quasipotential approach and diquark-antidiquark picture of heavy tetraquarks the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the tetraquark wave function (Ψ_T) of the bound diquark-antidiquark state, respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [28]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_{d,T}(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \quad (3)$$

Here, $M = E_1 + E_2$ is the bound-state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks (q and Q) which form the diquark or of the diquark (d) and antiquark (\bar{d}) which form the heavy tetraquark (T), and \mathbf{p} is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (4)$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption of an octet structure of the interaction from the difference in the qq and $q\bar{q}$ colour states. An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we assumed that the effective interaction is the sum of the usual one-gluon exchange term and a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli term. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark. The quasipotential is then defined as follows [7, 26].

(a) For the quark-quark (Qq) interactions, $V(\mathbf{p}, \mathbf{q}; M)$ reads

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q), \quad (5)$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2} \left[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V_{\text{conf}}^V(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right].$$

Here, α_s is the QCD coupling constant; $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge,

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2}, \quad D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2} \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right), \quad D^{0i} = D^{i0} = 0, \quad (6)$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{\epsilon(p) + m} \end{pmatrix} \chi^\lambda, \quad (7)$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

The effective long-range vector vertex of the quark is defined by [7]

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu, \quad \tilde{k} = (0, \mathbf{k}), \quad (8)$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In configuration space the vector and scalar confining potentials in the nonrelativistic limit [28] reduce to

$$\begin{aligned} V_{\text{conf}}^V(r) &= (1 - \varepsilon)V_{\text{conf}}(r), \\ V_{\text{conf}}^S(r) &= \varepsilon V_{\text{conf}}(r), \end{aligned} \quad (9)$$

with

$$V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B, \quad (10)$$

where ε is the mixing coefficient.

(b) For the diquark-antidiquark ($d\bar{d}'$) interaction, $V(\mathbf{p}, \mathbf{q}; M)$ is given by

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) &= \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d E_d}} \frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k}) \frac{\langle d'(P')|J_\nu|d'(Q')\rangle}{2\sqrt{E_{d'} E_{d'}}} \\ &+ \psi_d^*(P)\psi_{d'}^*(P') \left[J_{d;\mu} J_{d'}^\mu V_{\text{conf}}^V(\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right] \psi_d(Q)\psi_{d'}(Q'), \end{aligned} \quad (11)$$

where $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the finite size of the diquark $[P^{(\prime)} = (E_{d^{(\prime)}}, \pm\mathbf{p})$ and $Q^{(\prime)} = (E_{d^{(\prime)}}, \pm\mathbf{q})$, $E_d = (M^2 - M_d^2 + M_d^2)/(2M)$ and $E_{d'} = (M^2 - M_d^2 + M_d^2)/(2M)]$.

The diquark state in the confining part of the diquark-antidiquark quasipotential (11) is described by the wave functions

$$\psi_d(p) = \begin{cases} 1 & \text{for a scalar diquark,} \\ \varepsilon_d(p) & \text{for an axial-vector diquark,} \end{cases} \quad (12)$$

where the four-vector

$$\varepsilon_d(p) = \left(\frac{(\boldsymbol{\varepsilon}_d \cdot \mathbf{p})}{M_d}, \boldsymbol{\varepsilon}_d + \frac{(\boldsymbol{\varepsilon}_d \cdot \mathbf{p})\mathbf{p}}{M_d(E_d(p) + M_d)} \right), \quad \varepsilon_d^\mu(p)p_\mu = 0, \quad (13)$$

is the polarization vector of the axial-vector diquark with momentum \mathbf{p} , $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$, and $\varepsilon_d(0) = (0, \boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu} = \begin{cases} \frac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for a scalar diquark,} \\ -\frac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \frac{i\mu_d}{2M_d} \Sigma_\mu^\nu \tilde{k}_\nu & \text{for an axial-vector diquark.} \end{cases} \quad (14)$$

Here, the antisymmetric tensor Σ_μ^ν is defined by

$$(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu), \quad (15)$$

and the axial-vector diquark spin \mathbf{S}_d is given by $(S_{d;k})_{il} = -i\varepsilon_{kil}$; μ_d is the total chromomagnetic moment of the axial-vector diquark.

The constituent quark masses $m_c = 1.55$ GeV, $m_b = 4.88$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV² and $B = -0.3$ GeV have values typical in quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [28] and the heavy-quark expansion [29]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia 3P_J -states [28]. In this case, the long-range chromomagnetic interaction of quarks vanishes in accordance with the flux-tube model.

Since we deal with diquarks and tetraquarks containing light quarks and diquarks, respectively, we adopt for the QCD coupling constant $\alpha_s(\mu^2)$ the simplest model with freezing [30], namely

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_B^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad (16)$$

where the scale is taken as $\mu = 2m_1 m_2 / (m_1 + m_2)$, the background mass is $M_B = 2.24\sqrt{A} = 0.95$ GeV [30], and the parameter $\Lambda = 413$ MeV was fixed from fitting the ρ mass [31]. Note that the other popular parametrization of α_s with freezing [32] leads to close values.

TABLE II: Masses M and form factor parameters of diquarks. S and A denote scalar and axial vector diquarks which are antisymmetric $[\cdot\cdot\cdot]$ and symmetric $\{\cdot\cdot\cdot\}$ in flavour, respectively.

Quark content	Diquark type	M (MeV)	ξ (GeV)	ζ (GeV ²)
$[u, d]$	S	710	1.09	0.185
$\{u, d\}$	A	909	1.185	0.365
$[u, s]$	S	948	1.23	0.225
$\{u, s\}$	A	1069	1.15	0.325
$\{s, s\}$	A	1203	1.13	0.280
$[c, q]$	S	1973	2.55	0.63
$\{c, q\}$	A	2036	2.51	0.45
$[c, s]$	S	2091	2.15	1.05
$\{c, s\}$	A	2158	2.12	0.99
$[b, q]$	S	5359	6.10	0.55
$\{b, q\}$	A	5381	6.05	0.35
$[b, s]$	S	5462	5.70	0.35
$\{b, s\}$	A	5482	5.65	0.27

III. DIQUARK AND TETRAQUARK MASSES

At the first step, we calculate the masses and form factors of the heavy and light diquarks. Since the light quarks are highly relativistic a completely relativistic treatment of the light quark dynamics is required. To achieve this goal, we closely follow our consideration of diquarks in heavy baryons and adopt the same procedure to make the relativistic potential local by replacing $\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + \mathbf{p}^2} \rightarrow E_{1,2} = (M^2 - m_{2,1}^2 + m_{1,2}^2)/2M$. Solving numerically the quasipotential equation (1) with the complete relativistic potential, which depends on the diquark mass in a complicated highly nonlinear way [33], we get the diquark masses and wave functions. In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, we calculate the corresponding matrix element of the quark current between diquark states. Such calculation leads to the emergence of the form factor $F(r)$ entering the vertex of the diquark-gluon interaction [33]. This form factor is expressed through the overlap integral of the diquark wave functions. Our estimates show that this form factor can be approximated with a high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}. \quad (17)$$

The values of the masses and parameters ξ and ζ for light and heavy scalar diquark $[\cdot\cdot\cdot]$ and axial vector diquark $\{\cdot\cdot\cdot\}$ ground states were calculated previously [26, 33] and are given in Table II.

At the second step, we calculate the masses of heavy tetraquarks considered as the bound states of a heavy-light diquark and light antidiquark. For the potential of the S -wave ($\langle \mathbf{L}^2 \rangle = 0$) diquark-antidiquark interaction (11) we get [27]

$$V(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{E_1 E_2} \left\{ \mathbf{p} \left[\hat{V}_{\text{Coul}}(r) + V_{\text{conf}}^V(r) \right] \mathbf{p} - \frac{1}{4} \Delta V_{\text{conf}}^V(r) \right\}$$

$$+\frac{2}{3}\left[\Delta\hat{V}_{\text{Coul}}(r)+\frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V_{\text{conf}}^V(r)\right]\mathbf{S}_1\cdot\mathbf{S}_2\Big\}, \quad (18)$$

where

$$\hat{V}_{\text{Coul}}(r)=-\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$$

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors $F_{1,2}(r)$. Here, $\mathbf{S}_{1,2}$ are the spin operators of diquark and antidiquark. In the following we choose the total chromomagnetic moment of the axial-vector diquark $\mu_d = 0$. Such a choice appears to be natural, since the long-range chromomagnetic interaction of diquarks proportional to μ_d then also vanishes in accordance with the flux-tube model.

The resulting quasipotential equation with the complete kernel (18) is solved numerically without any approximations.

IV. RESULTS AND DISCUSSION

Masses of the heavy tetraquark ground ($1S$) states calculated in the diquark-antidiquark picture are presented in Table III. In this table we also give possible experimental candidates for charmed and charmed-strange tetraquarks.

In the charmed meson sector the situation is rather complicated. Comparing results presented in Tables I and III we see that our model predicts very close masses for the scalar 0^+ orbitally excited $c\bar{q}$ meson (1^3P_0) and the ground state ($1S$) tetraquark, composed from the scalar $[cq]$ diquark and scalar $[\bar{q}\bar{q}]$ antidiquark. The same is true for the masses of the 1^+ axial vector $c\bar{q}$ meson ($1P_1$) and the tetraquark, composed from the axial vector $\{cq\}$ diquark and scalar $[\bar{q}\bar{q}]$ antidiquark. The calculated masses are consistent with the measured masses of the scalar $D_0^*(2400)$ and axial vector $D_1(2430)$ mesons. The mixing of the $c\bar{q}$ and tetraquark states could be responsible for the observed difference in masses of the charged and neutral $D_0^*(2400)$ mesons.

In the charmed-strange sector the 0^+ and 1^+ tetraquarks are predicted to have masses significantly (by 200-300 MeV) higher than experimentally measured masses of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons (cf. Tables I, III). This excludes the interpretation of these anomalously light D_s mesons as the heavy diquark-antidiquark (tetraquark) states in our model. Instead, we find that the lightest scalar 0^+ tetraquark, composed from the scalar $[cq]$ diquark and scalar $[\bar{s}\bar{q}]$ antidiquark, has a mass consistent with the controversial $D_s(2632)$ observed by SELEX [18]. The $D_{sJ}^*(2860)$ meson, observed by BaBar [23] both in DK and D^*K modes,¹ has mass coinciding within experimental error bars with the prediction for the mass of the tensor 2^+ tetraquark, composed from the axial vector $\{cq\}$ diquark and axial vector $\{\bar{s}\bar{q}\}$ antidiquark. The rather high value of its spin can explain the non-observation of this state by Belle in B decays.

In Table IV we compare our results for the masses of the charmed-strange diquark-antidiquark bound states with the predictions of Refs. [13–16]. From this table we see that only the model of Ref. [13] gives masses of scalar and axial vector tetraquarks compatible with the observed masses of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons. All other models predict

¹ This state should therefore have natural parity and total spin $J \geq 1$.

TABLE III: Masses of charmed and bottom diquark-antidiquark ground ($1S$) states (in MeV) and possible experimental candidates [4]. S and A denote scalar and axial vector diquarks.

State J^P	Diquark content	Theory Mass	Experiment		Theory Mass
			Meson	Mass	
$cq\bar{q}\bar{q}$					
0^+	$S\bar{S}$	2399	$D_0^*(2400)$	$\begin{cases} 2403(40)(^\pm) \\ 2318(29)(^0) \end{cases}$	5758
1^+	$S\bar{A}$	2558			5950
1^+	$A\bar{S}$	2473	$D_1(2430)$	2427(40)	5782
0^+	$A\bar{A}$	2503			5896
1^+	$A\bar{A}$	2580			5937
2^+	$A\bar{A}$	2698			6007
$cq\bar{s}\bar{q}$					
0^+	$S\bar{S}$	2619	$D_s(2632)$	2632.5(1.7)	5997
1^+	$S\bar{A}$	2723			6125
1^+	$A\bar{S}$	2678			6021
0^+	$A\bar{A}$	2689			6086
1^+	$A\bar{A}$	2757			6118
2^+	$A\bar{A}$	2863	$D_{sJ}^*(2860)$	$2862 \begin{pmatrix} +6 \\ -3 \end{pmatrix}$	6177
$cs\bar{s}\bar{q}$					
0^+	$S\bar{S}$	2753			6108
1^+	$S\bar{A}$	2870			6238
1^+	$A\bar{S}$	2830			6134
0^+	$A\bar{A}$	2839			6197
1^+	$A\bar{A}$	2901			6228
2^+	$A\bar{A}$	2998			6284
$cs\bar{s}\bar{s}$					
1^+	$S\bar{A}$	3025			6383
0^+	$A\bar{A}$	3003			6353
1^+	$A\bar{A}$	3051			6372
2^+	$A\bar{A}$	3135			6411

significantly higher mass values for these states. The main difference between these approaches consists in the substantial distinctions in treating quark dynamics in tetraquarks. The authors of Ref. [13] use a phenomenological approach, determining diquark masses and parameters of hyperfine interactions between quarks from adjusting their predictions to experimental observables. Contrary we describe diquarks and tetraquarks dynamically as quark-quark and diquark-antidiquark bound systems and calculate their masses and form

TABLE IV: Comparison of theoretical predictions for the masses of $cq\bar{s}\bar{q}$ tetraquarks (in MeV).

J^P	this paper	[13]	[14]	[15]	[16]
0^+	2619	2371	2840	2731	2616
1^+	2678	2410	2841		
1^+	2723	2462	2880	2841	
0^+	2689	2424	2503	2699	
1^+	2757	2571	2748		
2^+	2863	2648	2983		2854

factors in the model where all parameters were previously fixed from considerations of meson properties. Different dynamical approaches were applied in Refs. [14–16]. The authors of Ref. [14] calculate diquark-antidiquark mass spectra in the quark model employing the QCD potential found by means of the AdS/QCD correspondence. Tetraquark masses are calculated in Ref. [15] in the nonrelativistic quark model including both the confining interaction and meson exchanges, while in Ref.[16] the coupled-channel formalism is employed. In the latter two approaches the anomalous charmed-strange mesons could be only accommodated as a mixture of quark-antiquark and tetraquark states with a phenomenologically adjusted mixing interaction. Thus it seems to be unlikely that the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ could be pure diquark-antidiquark bound states.

V. CONCLUSIONS

We calculated the masses of heavy tetraquarks with open charm and bottom in the diquark-antidiquark picture using the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by the numerical solution of the quasipotential wave equation with the corresponding relativistic potentials. The diquark structure was taken into account in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of heavy and light hadron properties. It was found that the $D_0^*(2400)$, $D_s(2632)$ and $D_{sJ}^*(2860)$ mesons could be tetraquark states with open charm, while the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons cannot be interpreted as diquark-antidiquark bound states. The masses of the bottom counterparts of charmed tetraquarks were calculated. It is important to search for them in order to help revealing the nature of controversial charmed and charmed-strange mesons.

The authors are grateful to M. Müller-Preussker for support and to V. Matveev, V. Savrin and M. Wagner for discussions. This work was supported in part by the Deutsche Forschungsgemeinschaft (DFG) under contract Eb 139/6-1 and the Russian Foundation for Basic Research (RFBR) grants No.08-02-00582 and No.10-02-91339.

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