A Shift from Democratic to Tri-bimaximal Neutrino Mixing with Relatively Large θ_{13}

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Abstract

Recent neutrino oscillation data hint that the smallest neutrino mixing angle θ_{13} is possible to lie in the range $5^\circ \lesssim \theta_{13} \lesssim 12^\circ$. We show that reasonable perturbations to the democratic mixing pattern, which is geometrically related to the tri-bimaximal mixing pattern through an equal shift $\theta_* \simeq 9.7^\circ$ of two large mixing angles, can naturally produce a nearly tri-bimaximal neutrino mixing matrix V with sufficiently large θ_{13}. Two especially simple but viable scenarios of V are proposed and their phenomenological consequences are discussed.

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$\boxed{\mathbf{1}}$ Recent solar, atmospheric, reactor and accelerator neutrino oscillation experiments have provided us with very convincing evidence that neutrinos are massive and lepton flavors are mixed [1]. The mixing of lepton flavors is effectively described by a 3×3 unitary matrix V, whose nine elements can be parametrized in terms of three rotation angles and three CP-violating phases. Defining three rotation matrices in the (1,2), (1,3) and (2,3) planes as

$$
\begin{aligned}
R_{12}(\theta_{12}) &= \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \\
R_{13}(\theta_{13}) &= \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} , \\
R_{23}(\theta_{23}) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} ,
\end{aligned} \tag{1}
$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $1 \leq i < j \leq 3$), one may parametrize V in nine topologically different ways [2]. The so-called standard parametrization takes the form

$$
\begin{aligned}
V &= R_{23}(\theta_{23}) \otimes P_\delta \otimes R_{13}(\theta_{13}) \otimes P_\delta^\dagger \otimes R_{12}(\theta_{12}) \otimes P_\nu \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu ,
\end{aligned} \tag{2}
$$

in which $P_\delta = \mathrm{Diag}\{1, 1, e^{i\delta}\}$ and $P_\nu = \mathrm{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ are two diagonal phase matrices containing three CP-violating phases. A recent global analysis of current neutrino oscillation data yields $\theta_{12} = 34.5^\circ \pm 1.0^\circ$, $\theta_{13} = 5.1^{+3.0^\circ}_{-3.3^\circ}$ and $\theta_{23} = 42.8^{+4.7^\circ}_{-2.9^\circ}$ at the 1σ level [3], but three phases of V remain entirely unconstrained. The ongoing and forthcoming neutrino oscillation experiments will measure θ_{13} and δ. On the other hand, the neutrinoless double-beta decay experiments will help to probe or constrain ρ and σ.

The smallness of θ_{13} and the largeness of θ_{12} and θ_{23} have motivated some speculations about a constant neutrino mixing matrix with $\theta_{13} = 0^\circ$, such as the "democratic" pattern

$$
U_0 = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} \tag{3}
$$

with $\theta_{12}^{(0)} = 45^\circ$, $\theta_{13}^{(0)} = 0^\circ$ and $\theta_{23}^{(0)} = \arctan(\sqrt{2}) \simeq 54.7^\circ$ [4] or the "tri-bimaximal" pattern

$$
V_0 = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \tag{4}
$$

with $\vartheta_{12}^{(0)} = \arctan(1/\sqrt{2}) \simeq 35.3^\circ$, $\vartheta_{13}^{(0)} = 0^\circ$ and $\vartheta_{23}^{(0)} = 45^\circ$ [5]. Either of them can be obtained in the limit of a certain flavor symmetry (e.g., the discrete S(3) flavor symmetry for U_0 or the discrete A_4 symmetry for V_0 [6]), and the latter has to be broken in order to generate nonzero θ_{13} and CP violation. Since V_0 is much closer to the best-fit values of current data on three neutrino mixing angles, it has recently attracted much more interest.

Note that the entries of V_0 are actually the same as those of U_0, although their positions are essentially different. Hence it is interesting to explore not only an intrinsic relationship between U_0 and V_0 but also how to link them to the realistic neutrino mixing matrix V via reasonable perturbations. Note also that there are some preliminary hints that the smallest neutrino mixing angle θ_{13} might not be very small. For example, $\theta_{13} \simeq 5.1^{+3.0^\circ}_{-3.3^\circ}$ (1σ) by Gonzalez-Garcia *et al* [3], $\theta_{13} \simeq 7.3^{+2.0^\circ}_{-2.9^\circ}$ (1σ) by Fogli *et al* [7], and $\theta_{13} \simeq 8.1^{+2.8^\circ}_{-4.5^\circ}$ as the best-fit value by the KamLAND Collaboration [8]. Although the statistical significance of these results remains quite low, they *do* imply that θ_{13} is possible to lie in the range $5^\circ \lesssim \theta_{13} \lesssim 12^\circ$. On the theoretical side, it is certainly likely that θ_{13} may take a value in the above range [9]. So it makes sense to discuss how to confront a constant neutrino mixing pattern with a relatively large value of θ_{13}. On the other hand, a definite determination of θ_{13} may serve as a crucial turning-point of experimental neutrino physics to the era of precision measurements, in which the detection of leptonic CP violation and the search for new physics will become feasible [10].

Let us pose two immediate and interesting questions: (1) what is the geometric relation between the democratic and tri-bimaximal neutrino mixing patterns? (2) which of them is more natural to receive relatively significant perturbations in order to accommodate relatively large θ_{13}? In this paper we shall point out that two large mixing angles predicted in the democratic mixing pattern U_0 are intrinsically related to their counterparts in the tri-bimaximal mixing pattern V_0 through an equal shift

$$\theta_* \equiv \theta_{12}^{(0)} - \vartheta_{12}^{(0)} = \theta_{23}^{(0)} - \vartheta_{23}^{(0)} \simeq 9.7^\circ \ . \tag{5}$$

This geometric relation keeps unchanged if a universal perturbation is imposed onto three mixing angles of U_0 or V_0. Although it is likely to generate relatively large θ_{13} by perturbing V_0, the perturbation term has to be adjusted in such a way that two large mixing angles of V_0 are slightly modified but the smallest angle of V_0 is significantly modified. In contrast, it is more natural to produce a nearly tri-bimaximal neutrino mixing matrix V with sufficiently large θ_{13} by introducing comparable perturbations to three mixing angles of U_0. We shall propose two especially simple but viable scenarios of V — scenario A is based on the standard parametrization of U_0 and scenario B relies on a very useful parametrization proposed by Fritzsch and Xing (FX) [11]. Scenario A predicts $\theta_{12} \simeq 35.3^\circ$, $\theta_{13} \simeq 9.7^\circ$ and $\theta_{23} = 45^\circ$ together with the maximal strength of leptonic CP violation $\mathcal{J}_{\rm max} = (\sqrt{2}+1)/(36\sqrt{3}) \simeq 3.9\%$; and scenario B predicts $28.3^\circ \lesssim \theta_{12} \lesssim 42.2^\circ$, $\theta_{13} \simeq 6.9^\circ$ and $\theta_{23} \simeq 44.6^\circ$ together with $\mathcal{J}_{\rm max} = 1/36 \simeq 2.8\%$. Both scenarios are in good agreement with current data, and they can soon be tested in a variety of neutrino oscillation experiments.

$\boxed{\mathbf{2}}$ First of all, the following relation between the democratic mixing matrix U_0 and the tri-bimaximal mixing matrix V_0 comes into our notice:

$$V_0 = R_{23}^T(\theta_*) \otimes U_0 \otimes R_{12}^T(\theta_*) \ , \tag{6}$$

where "T" means a transpose, and θ_* has been defined in Eq. (5). As a matter of fact, U_0 and V_0 can be decomposed into

$$
\begin{aligned}
U_0 &= R_{23}(45^\circ + \theta_*) \otimes R_{12}(45^\circ) \; , \\
V_0 &= R_{23}(45^\circ) \otimes R_{12}(45^\circ - \theta_*) \; .
\end{aligned} \tag{7}
$$

Two nonzero mixing angles of U_0 turn out to be $\theta_{12}^{(0)} = 45^\circ$ and $\theta_{23}^{(0)} = 45^\circ + \theta_*$, and those of V_0 are $\vartheta_{12}^{(0)} = 45^\circ - \theta_*$ and $\vartheta_{23}^{(0)} = 45^\circ$. Their geometrical relations in the real plane are shown in FIG. 1. So $\theta_{12}^{(0)}$ and $\theta_{23}^{(0)}$ are intrinsically related to $\vartheta_{12}^{(0)}$ and $\vartheta_{23}^{(0)}$ via an equal shift $\theta_* \simeq 9.7^\circ$. Although the size of θ_* is not small, it is smaller than the Cabibbo angle of quark mixing (i.e., $\theta_{\rm C} \simeq 13^\circ$ [1]). In this sense we argue that V_0 can be regarded as a consequence of U_0 whose (1,2) and (2,3) mixing angles are corrected by θ_* in a destructive way.

Since V_0 itself is very close to the best-fit result obtained from current experimental data on three neutrino mixing angles [3], any possible perturbations to V_0 must be small enough (see, e.g., Refs. [12–14]). In the standard parametrization the overall perturbation matrix can be expressed as

$$
\Omega_\varepsilon = R_{23}^T(\varepsilon_{23}) \otimes P_\delta \otimes R_{13}(\varepsilon_{13}) \otimes P_\delta^\dagger \otimes R_{12}^T(\varepsilon_{12}) \tag{8}
$$

with $|\varepsilon_{ij}| \ll 1$ (for $ij = 12, 13, 23$), and thus the overall neutrino mixing matrix is given by

$$
\begin{aligned}
V &= R_{23}(45^\circ) \otimes \Omega_\varepsilon \otimes R_{12}(45^\circ - \theta_*) \otimes P_\nu \\
&= R_{23}(45^\circ - \varepsilon_{23}) \otimes P_\delta \otimes R_{13}(\varepsilon_{13}) \otimes P_\delta^\dagger \otimes R_{12}(45^\circ - \theta_* - \varepsilon_{12}) \otimes P_\nu
\end{aligned} \tag{9}
$$

with $\theta_{12} = 45^\circ - \theta_* - \varepsilon_{12}$, $\theta_{13} = \varepsilon_{13}$ and $\theta_{23} = 45^\circ - \varepsilon_{23}$. In view of $\theta_{12} = 34.5^\circ \pm 1.0^\circ$, $\theta_{13} = 5.1^{+3.0^\circ}_{-3.3^\circ}$ and $\theta_{23} = 42.8^{+4.7^\circ}_{-2.9^\circ}$ (1σ) extracted from a global analysis of current neutrino oscillation data [3], one immediately obtains $\varepsilon_{12} = 0.8^\circ \pm 1^\circ$, $\varepsilon_{13} = 5.1^{+3.0^\circ}_{-3.3^\circ}$ and $\varepsilon_{23} = 2.2^{+4.7^\circ}_{-2.9^\circ}$. One might argue that it would be somewhat unnatural if a perturbation to the smallest mixing angle of V_0 were much larger than the ones to two large angles of V_0. In this sense $|\varepsilon_{13}| \lesssim |\varepsilon_{12}| \lesssim |\varepsilon_{23}|$ seems to be a natural choice of three perturbation parameters, just corresponding to the fact $\theta_{13} < \theta_{12} < \theta_{23}$. Then $\theta_{13} = \varepsilon_{13}$ is expected to be very small, and it is most likely to lie in the range $0^\circ \lesssim \theta_{13} < 5^\circ$. In other words, it seems rather unlikely to obtain $\theta_{13} \gtrsim 5^\circ$ by introducing *natural* perturbations to three mixing angles of V_0. Note, however, that such arguments might not work when the neutrino mixing matrix is derived from a lepton mass model. From the point of view of model building, one is not subject to the assumption of $|\varepsilon_{13}| \lesssim |\varepsilon_{12}| \lesssim |\varepsilon_{23}|$ because three mixing angles may receive contributions from both the charged-lepton and neutrino sectors at the tree level [15] and they can also receive appreciable quantum corrections at the loop level [16].

Different from V_0, U_0 is not so close to the best-fit neutrino mixing pattern extracted from a global analysis of current neutrino oscillation data. Hence large perturbations to three mixing angles of U_0 can naturally be allowed, in order to bring U_0 to a phenomenologically favored form. In this case even $\theta_{13} \sim \theta_*$ may be achieved from reasonable perturbations to U_0, as we shall see later on.

$\boxed{\mathbf{3}}$ We proceed to discuss reasonable perturbations to the democratic mixing pattern U_0 so as to produce a realistic neutrino mixing matrix V with naturally large θ_{13}. To illustrate, we propose two simple but viable scenarios of V in two different parametrizations of U_0. They are based on the standard and FX parametrizations of U_0, leading respectively to the predictions $\theta_{13} \simeq 9.7^\circ$ and $\theta_{13} \simeq 6.9^\circ$ in the standard parametrization of V.

Scenario A

The standard parametrization of U_0 has been given in Eq. (7). Here we assume a universal angle $\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = \theta_*$ to perturb three mixing angles of U_0, in order to obtain a nearly tri-bimaximal neutrino mixing matrix V with sufficiently large θ_{13}. Such a perturbation term is reasonable in the sense that its magnitude is actually smaller than the Cabibbo angle $\theta_{\rm C}$. Let us recall that the realistic Cabibbo-Kobayashi-Maskawa quark mixing matrix [1] can be regarded as a result obtained from small perturbations to the identity matrix, in which the maximal perturbation term is just characterized by $\sin\theta_{\rm C}$. Now a realistic neutrino mixing matrix V arises from a universal perturbation of $\mathcal{O}(\theta_*)$ to three mixing angles of U_0. In this case the overall perturbation matrix reads

$$\Omega_* = R^T_{23}(\theta_*) \otimes P_\delta \otimes R_{13}(\theta_*) \otimes P^\dagger_\delta \otimes R^T_{12}(\theta_*) \; . \tag{10}$$

The resultant neutrino mixing matrix is

$$\begin{aligned} V &= R_{23}(45^\circ + \theta_*) \otimes \Omega_* \otimes R_{12}(45^\circ) \otimes P_\nu \\ &= R_{23}(45^\circ) \otimes P_\delta \otimes R_{13}(\theta_*) \otimes P^\dagger_\delta \otimes R_{12}(45^\circ - \theta_*) \otimes P_\nu \\ &= \begin{pmatrix} \sqrt{\frac{2}{3}}\, c_* & \sqrt{\frac{1}{3}}\, c_* & s_* e^{-i\delta} \\ -\sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}}\, s_* e^{i\delta} & \sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}\, s_* e^{i\delta} & \sqrt{\frac{1}{2}}\, c_* \\ \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}}\, s_* e^{i\delta} & -\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}\, s_* e^{i\delta} & \sqrt{\frac{1}{2}}\, c_* \end{pmatrix} P_\nu \; , \end{aligned} \tag{11}$$

where $c_* \equiv \cos\theta_* = (\sqrt{2}+1)/\sqrt{6}$ and $s_* \equiv \sin\theta_* = (\sqrt{2}-1)/\sqrt{6}$. This ansatz apparently predicts $\theta_{12} = 45^\circ - \theta_* \simeq 35.3^\circ$, $\theta_{13} = \theta_* \simeq 9.7^\circ$ and $\theta_{23} = 45^\circ$. So it is a nearly tri-bimaximal neutrino mixing pattern with a very appreciable value of θ_{13}. The Jarlskog parameter of leptonic CP violation [17] is given by

$$\mathcal{J} = c_{12} s_{12} c^2_{13} s_{13} c_{23} s_{23} \sin\delta = \frac{\sqrt{2}+1}{36\sqrt{3}} \sin\delta \lesssim 0.039 \sin\delta \tag{12}$$

in this scenario. The relatively large θ_{13} and (perhaps) $|\mathcal{J}|$ make scenario A easily testable in a variety of neutrino oscillation experiments in the near future.

Scenario B

In the FX parametrization a generic 3×3 neutrino mixing matrix V is expressed as

$$\begin{aligned} V &= R_{12}(\theta_l) \otimes R_{23}(\theta) \otimes P^\dagger_\phi \otimes R^T_{12}(\theta_\nu) \otimes P_\nu \\ &= \begin{pmatrix} s_l s_\nu c + c_l c_\nu e^{-i\phi} & s_l c_\nu c - c_l s_\nu e^{-i\phi} & s_l s \\ c_l s_\nu c - s_l c_\nu e^{-i\phi} & c_l c_\nu c + s_l s_\nu e^{-i\phi} & c_l s \\ -s_\nu s & -c_\nu s & c \end{pmatrix} P_\nu \; , \end{aligned} \tag{13}$$

where $P_\phi = {\rm Diag}\{e^{i\phi}, 1, 1\}$ and $P_\nu = {\rm Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ together with $c_l \equiv \cos\theta_l$, $s_l \equiv \sin\theta_l$, $c_\nu \equiv \cos\theta_\nu$, $s_\nu \equiv \sin\theta_\nu$, $c \equiv \cos\theta$ and $s \equiv \sin\theta$. This representation of V has proved to be more convenient and useful than the standard one in deriving the one-loop renormalization-group equations of three neutrino mixing angles and three CP-violating phases [18] and in linking flavor mixing parameters to the ratios of charge-lepton and neutrino masses [19]. It coincides with the standard parametrization in the $\theta_l \to 0$ limit (up to a rearrangement of the phase convention), in which $\theta_{12} = \theta_\nu$ and $\theta_{23} = \theta$ exactly hold. Hence the democratic mixing pattern U_0 can also be decomposed into a product of $R_{23}(45^\circ + \theta_*)$ and $R_{12}(45^\circ)$ in the FX parametrization with $\theta_\nu = 45^\circ$ and $\theta = 45^\circ + \theta_*$. Here again we assume a universal angle θ_* to perturb three mixing angles of U_0 in the form of

$$\Omega_l = R_{12}(\theta_*) \quad \text{and} \quad \Omega_\nu = R^T_{23}(\theta_*) \otimes P^\dagger_\phi \otimes R^T_{12}(\theta_*) \; . \tag{14}$$

Then we obtain

$$\begin{aligned} V &= \Omega_l \otimes R_{23}(45^\circ + \theta_*) \otimes \Omega_\nu \otimes R_{12}(45^\circ) \otimes P_\nu \\ &= R_{12}(\theta_*) \otimes R_{23}(45^\circ) \otimes P^\dagger_\phi \otimes R_{12}(45^\circ - \theta_*) \otimes P_\nu \\ &= \begin{pmatrix} \sqrt{\frac{1}{6}}\, s_* + \sqrt{\frac{2}{3}}\, c_* e^{-i\phi} & \sqrt{\frac{1}{3}}\, s_* - \sqrt{\frac{1}{3}}\, c_* e^{-i\phi} & \sqrt{\frac{1}{2}}\, s_* \\ \sqrt{\frac{1}{6}}\, c_* - \sqrt{\frac{2}{3}}\, s_* e^{-i\phi} & \sqrt{\frac{1}{3}}\, c_* + \sqrt{\frac{1}{3}}\, s_* e^{-i\phi} & \sqrt{\frac{1}{2}}\, c_* \\ -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P_\nu \; , \end{aligned} \tag{15}$$

where c_* and s_* have been given below Eq. (11). It is obvious that this ansatz can predict $\theta_l = \theta_* \simeq 9.7^\circ$, $\theta_\nu = 45^\circ - \theta_* \simeq 35.3^\circ$ and $\theta = 45^\circ$. The Jarlskog invariant of leptonic CP violation turns out to be

$$\mathcal{J} = c_l s_l c_\nu s_\nu c s^2 \sin\phi = \frac{1}{36} \sin\phi \lesssim 0.028 \sin\phi \tag{16}$$

in this scenario, and its maximal magnitude corresponds to $\phi = \pm 90^\circ$.

Translating the results of three neutrino mixing angles from the FX parametrization to the standard parametrization in scenario B, we arrive at

$$\begin{aligned} \theta_{13} &= \arcsin\left(\sqrt{\frac{1}{2}}\, s_*\right) = \arcsin\left[\frac{1}{2\left(\sqrt{3}+\sqrt{6}\right)}\right] \simeq 6.9^\circ \; , \\ \theta_{23} &= \arctan\left(c_*\right) = \arctan\left(\frac{1}{2\sqrt{3}-\sqrt{6}}\right) \simeq 44.6^\circ \; , \end{aligned} \tag{17}$$

and

$$\theta_{12} = \arctan\left[\sqrt{2}\left|\frac{1 - t_* e^{i\phi}}{2 + t_* e^{i\phi}}\right|\right] \in \left[\arctan\left(\frac{4}{6+\sqrt{2}}\right), \arctan\left(\frac{4}{3+\sqrt{2}}\right)\right] \simeq [28.3^\circ, 42.2^\circ] \tag{18}$$

with $t_* \equiv \tan\theta_* = (\sqrt{2}-1)/(\sqrt{2}+1)$. We observe that the value of θ_{12} in the standard parametrization depends on the CP-violating phase ϕ in the FX parametrization, and its minimal (or maximal) value corresponds to $\phi = 0^\circ$ (or $\phi = 180^\circ$). Once θ_{12} is experimentally determined to a good degree of accuracy, it will be possible to calculate ϕ from Eq. (18). Given $\theta_{12} \simeq 34.5^\circ$ [3] for example,

$$\phi = \arccos\left[\frac{2\,(1+t_*^2) - (4+t_*^2)\tan^2\theta_{12}}{4\, t_*\,(1+\tan^2\theta_{12})}\right] \simeq \pm 61.3^\circ \; , \tag{19}$$

which in turn leads to $|\mathcal{J}| \simeq 2.5\%$. This amount of CP violation can in principle be measured in the future long-baseline neutrino oscillation experiments. No doubt, the simplest experimental way to distinguish between scenarios A and B is just to measure the smallest neutrino mixing angle θ_{13}.

$\boxed{\mathbf{4}}$ At present both scenarios A and B are compatible with the available neutrino oscillation data. In either scenario the values of three neutrino mixing angles are independent of four independent mass ratios of charged leptons and neutrinos (i.e., m_e/m_μ, m_μ/m_τ, m_1/m_2 and m_2/m_3). Of course, this kind of consequence is more or less contrived and it implies some quite special textures of the charged-lepton and neutrino mass matrices which might result from certain flavor symmetries [20]. Instead of going into any details of model building, here we discuss a few possibilities of reconstructing the charged-lepton mass matrix M_l and the neutrino mass matrix M_ν from a given pattern of the neutrino mixing matrix V. For the sake of simplicity, we only concentrate on scenario B to illustrate the salient features of our phenomenological treatment.

Let us define M_l and M_ν in the following lepton mass terms by assuming neutrinos to be the Majorana particles:

$$-\mathcal{L}_{\rm mass} = \overline{(e \quad \mu \quad \tau)_{\rm L}}\, M_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm R} + \frac{1}{2}\,\overline{(\nu_e \quad \nu_\mu \quad \nu_\tau)_{\rm L}}\, M_\nu \begin{pmatrix} \nu^c_e \\ \nu^c_\mu \\ \nu^c_\tau \end{pmatrix}_{\rm R} + {\rm h.c.}\,, \tag{20}$$

where M_ν is symmetric but M_l is arbitrary. One may diagonalize M_l and M_ν via the transformations $O^\dagger_l M_l O^\prime_l = \widehat{M}_l \equiv {\rm Diag}\{m_e, m_\mu, m_\tau\}$ and $O^\dagger_\nu M_\nu O^*_\nu = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$, where O_l, $O^\prime_l$ and O_ν are all unitary. Then the flavor mixing matrix $V \equiv O^\dagger_l O_\nu$ will show up in the weak charged-current interactions

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{(e^\prime \quad \mu^\prime \quad \tau^\prime)_{\rm L}}\,\gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}\,, \tag{21}$$

where $\alpha^\prime$ (for $\alpha = e, \mu, \tau$) and ν_i (for $i = 1, 2, 3$) stand respectively for the mass eigenstates of charged leptons and neutrinos. Now that both charged-lepton and neutrino sectors may in general contribute to V, the reconstruction of M_l and M_ν crucially depends on how one decomposes a given pattern of V into O_l and O_ν. Taking scenario B for example, we consider four typical possibilities as follows.

Possibility (1): $O_l = \mathbf{1}$ and $O_\nu = V$. In this case the mass eigenstates of three charged leptons are identified with their flavor eigenstates, and hence only the neutrino sector is responsible for the effect of flavor mixing. So we have $M_l = \widehat{M}_l$ and $M_\nu = V \widehat{M}_\nu V^T$. Most authors have taken such a flavor basis for building phenomenological models of M_ν [20].

Possibility (2): $O_\nu = \mathbf{1}$ and $O_l = V^\dagger$. In this case the mass eigenstates of three neutrinos are identified with their flavor eigenstates, and hence only the charged-lepton sector contributes to flavor mixing. So we have $M_\nu = \widehat{M}_\nu$ and $M_l = V^\dagger \widehat{M}_l O^{\prime\dagger}_l$. Because $O^\prime_l$ is in general unknown, it is actually difficult to fix the texture of M_l in this flavor basis. One usually assumes M_l to be Hermitian or symmetric so as to reduce the number of degrees of freedom. If M_l is Hermitian, it can be diagonalized via $O^\dagger_l M_l O_l = \widehat{M}^\prime_l \equiv {\rm Diag}\{\lambda_e, \lambda_\mu, \lambda_\tau\}$ with

$|\lambda_\alpha| = m_\alpha$ (for $\alpha =, e, \mu, \tau$). Then it is possible to reconstruct M_l through $M_l = V^\dagger \widehat{M}_l^\prime V$. If M_l is symmetric, we simply set $O_l^\prime = O_l^* = V^T$ and then arrive at $M_l = V^\dagger \widehat{M}_l V^*$. A simple example of this kind was originally given in Ref. [4].

Possibility (3): $O_l = P_\phi \otimes R_{12}^\dagger(\theta_*)$ and $O_\nu = R_{23}(45^\circ) \otimes R_{12}(45^\circ - \theta_*) \otimes P_\nu$. Since $P_\phi^\dagger$ commutes with $R_{23}(45^\circ)$, the product $O_l^\dagger O_\nu$ automatically reproduces the pattern of V shown in Eq. (15). Assuming M_l to be symmetric for simplicity, we obtain

$$\begin{aligned} M_l &= P_\phi R_{12}^\dagger(\theta_*) \widehat{M}_l R_{12}^*(\theta_*) P_\phi^T \ , \\ M_\nu &= R_{23}(45^\circ) R_{12}(45^\circ - \theta_*) P_\nu \widehat{M}_\nu P_\nu^T R_{12}^T(45^\circ - \theta_*) R_{23}^T(45^\circ) \ . \end{aligned} \tag{22}$$

To be more explicit,

$$\begin{aligned} M_l &= \begin{pmatrix} [m_e c_*^2 + m_\mu s_*^2] e^{2i\phi} & [m_e - m_\mu] c_* s_* e^{i\phi} & 0 \\ [m_e - m_\mu] c_* s_* e^{i\phi} & m_e s_*^2 + m_\mu c_*^2 & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \ , \\ M_\nu &= \begin{pmatrix} \frac{2}{3}\tilde{m}_1 + \frac{1}{3}\tilde{m}_2 & \frac{1}{3}[\tilde{m}_2 - \tilde{m}_1] & \frac{1}{3}[\tilde{m}_1 - \tilde{m}_2] \\ \frac{1}{3}[\tilde{m}_2 - \tilde{m}_1] & \frac{1}{6}\tilde{m}_1 + \frac{1}{3}\tilde{m}_2 + \frac{1}{2}m_3 & \frac{1}{2}m_3 - \frac{1}{6}\tilde{m}_1 - \frac{1}{3}\tilde{m}_2 \\ \frac{1}{3}[\tilde{m}_1 - \tilde{m}_2] & \frac{1}{2}m_3 - \frac{1}{6}\tilde{m}_1 - \frac{1}{3}\tilde{m}_2 & \frac{1}{6}\tilde{m}_1 + \frac{1}{3}\tilde{m}_2 + \frac{1}{2}m_3 \end{pmatrix} \ , \end{aligned} \tag{23}$$

where $\tilde{m}_1 \equiv m_1 e^{2i\rho}$ and $\tilde{m}_2 \equiv m_2 e^{2i\sigma}$. It is interesting to notice that the Dirac CP-violating phase ϕ is attributed to M_l while the Majorana CP-violating phases ρ and σ come from M_ν in this decomposition. One may derive both the textures of M_l and M_ν from certain flavor symmetries [20]. For instance, the non-Abelian A_4 flavor symmetry has been used to derive the form of M_ν in Eq. (23) [21].

Possibility (4): $O_l = P_\phi \otimes R_{23}^\dagger(\theta_*) \otimes R_{12}^\dagger(\theta_*)$ and $O_\nu = R_{23}(45^\circ - \theta_*) \otimes R_{12}(45^\circ - \theta_*) \otimes P_\nu$. Here again the product $O_l^\dagger O_\nu$ can reproduce the pattern of V in Eq. (15). Assuming M_l to be symmetric, we analogously arrive at

$$\begin{aligned} M_l &= P_\phi R_{23}^\dagger(\theta_*) R_{12}^\dagger(\theta_*) \widehat{M}_l R_{12}^*(\theta_*) R_{23}^*(\theta_*) P_\phi^T \ , \\ M_\nu &= R_{23}(45^\circ - \theta_*) R_{12}(45^\circ - \theta_*) P_\nu \widehat{M}_\nu P_\nu^T R_{12}^T(45^\circ - \theta_*) R_{23}^T(45^\circ - \theta_*) \ . \end{aligned} \tag{24}$$

More explicitly, we have

$$\begin{aligned} M_l &= \begin{pmatrix} [m_e c_*^2 + m_\mu s_*^2] e^{2i\phi} & [m_e - m_\mu] c_*^2 s_* e^{i\phi} & [m_e - m_\mu] c_* s_*^2 e^{i\phi} \\ [m_e - m_\mu] c_*^2 s_* e^{i\phi} & [m_e s_*^2 + m_\mu c_*^2] c_*^2 + m_\tau s_*^2 & [m_e s_*^2 + m_\mu c_*^2 - m_\tau] c_* s_* \\ [m_e - m_\mu] c_* s_*^2 e^{i\phi} & [m_e s_*^2 + m_\mu c_*^2 - m_\tau] c_* s_* & [m_e s_*^2 + m_\mu c_*^2] s_*^2 + m_\tau c_*^2 \end{pmatrix} \ , \\ M_\nu &= \begin{pmatrix} \frac{2}{3}\tilde{m}_1 + \frac{1}{3}\tilde{m}_2 & \frac{2}{3}\sqrt{\frac{1}{3}}[\tilde{m}_2 - \tilde{m}_1] & \frac{1}{3}\sqrt{\frac{2}{3}}[\tilde{m}_1 - \tilde{m}_2] \\ \frac{2}{3}\sqrt{\frac{1}{3}}[\tilde{m}_2 - \tilde{m}_1] & \frac{2}{9}\tilde{m}_1 + \frac{4}{9}\tilde{m}_2 + \frac{1}{3}m_3 & \frac{\sqrt{2}}{3}m_3 - \frac{\sqrt{2}}{9}\tilde{m}_1 - \frac{2\sqrt{2}}{9}\tilde{m}_2 \\ \frac{1}{3}\sqrt{\frac{2}{3}}[\tilde{m}_1 - \tilde{m}_2] & \frac{\sqrt{2}}{3}m_3 - \frac{\sqrt{2}}{9}\tilde{m}_1 - \frac{2\sqrt{2}}{9}\tilde{m}_2 & \frac{1}{9}\tilde{m}_1 + \frac{2}{9}\tilde{m}_2 + \frac{2}{3}m_3 \end{pmatrix} \ , \end{aligned} \tag{25}$$

where $\tilde{m}_1$ and $\tilde{m}_2$ have been defined above. In this case the textures of M_l and M_ν are more or less parallel to each other, implying that they could arise from a common flavor symmetry or dynamic mechanism.

Indeed, there are infinite possibilities of reconstructing M_l and M_ν for a given pattern of V which is consistent with current experimental data. From a phenomenological point of view, we hope to make the textures of M_l and M_ν as simple as possible, or much easier to link with an underlying flavor symmetry. In this sense the above examples just serve for illustration. Theoretically, a viable neutrino mass model should predict or constrain the proper forms of M_l and M_ν from which the flavor mixing matrix V can be derived. But the inverse approach discussed above (i.e., starting from V to reconstruct the textures of M_l and M_ν based on a few assumptions) remains very useful because it is at least possible to help give a ballpark estimate of the flavor structure that a viable model ought to possess.

$\boxed{\mathbf{5}}$ We have paid our attention to how to confront a constant neutrino mixing pattern, which may be motivated by a certain flavor symmetry and can predict $\theta_{13} = 0^\circ$ in the symmetry limit, with a relatively large value of θ_{13} (e.g., $5^\circ \lesssim \theta_{13} \lesssim 12^\circ$). The latter seems quite possible, at least not to be impossible, according to some preliminary experimental hints extracted from current neutrino oscillation data. We have shown that reasonable perturbations to the democratic mixing pattern U_0, which is geometrically related to the tri-bimaximal mixing pattern V_0 through an equal shift $\theta_* \simeq 9.7^\circ$ of two large mixing angles, can naturally produce a nearly tri-bimaximal neutrino mixing matrix V with relatively large θ_{13}. We have proposed two simple but viable scenarios of V for illustration: one of them is based on the standard parametrization of U_0 and predicts $\theta_{13} \simeq 9.7^\circ$, and the other relies on the FX parametrization of U_0 and predicts $\theta_{13} \simeq 6.9^\circ$. Both scenarios are in good agreement with current neutrino oscillation data, and they can soon be tested in a variety of more accurate neutrino oscillation experiments.

In this work we have tried not to go into any details of model building. But we have discussed a few phenomenological possibilities of reconstructing the charged-lepton mass matrix M_l and the neutrino mass matrix M_ν for a given neutrino mixing matrix V with a relatively large value of θ_{13}. Both the charged-lepton and neutrino sectors may in general have significant contributions to V, and hence a specific lepton flavor model should be able to determine the textures of M_l and M_ν so as to give testable predictions for both neutrino mixing angles and CP-violating phases.

Finally, we stress that it is not impossible to obtain a sufficiently large value of θ_{13} at the electroweak scale from finite quantum corrections to a given constant neutrino mixing pattern with $\theta_{13} = 0^\circ$ [16]. It is also possible to generate θ_{13} via the renormalization-group running effects from the conventional seesaw scales of $\mathcal{O}(10^{14})$ GeV down to the electroweak scale [22], in particular when the seesaw threshold effects are taken into account [23]. But it seems more likely to achieve relatively large θ_{13} from relatively significant symmetry breaking terms at a given scale where the constant neutrino mixing pattern can be derived on the basis of a certain flavor symmetry. In this sense our speculations and discussions are expected to be phenomenologically useful and suggestive.

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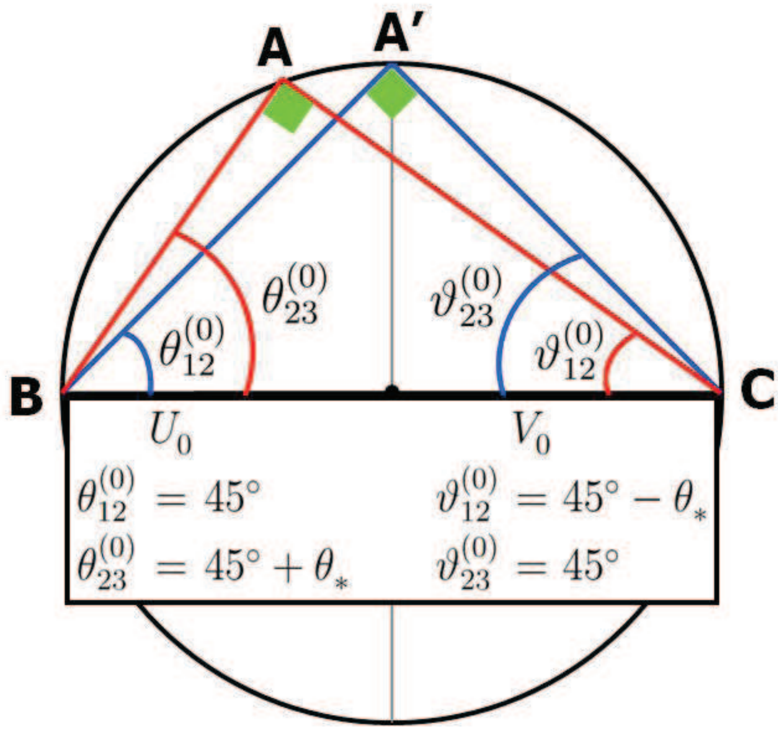

FIG. 1. A geometric relationship between the democratic mixing pattern U_0 (with $\theta_{12}^{(0)} = 45°$ and $\theta_{23}^{(0)} = 45° + \theta_*$) and the tri-bimaximal mixing pattern V_0 (with $\vartheta_{12}^{(0)} = 45° - \theta_*$ and $\vartheta_{23}^{(0)} = 45°$), where $\theta_* = \arctan(\sqrt{2}) - 45° = 45° - \arctan(1/\sqrt{2}) \simeq 9.7°$. Four nonzero mixing angles of U_0 and V_0 correspond to four inner angles of two right triangles $\triangle$ABC and $\triangle$A′BC in the real plane.