

## Are scalar mesons visible in $B^\pm \rightarrow \pi^+ \pi^- \pi^\pm$ decays? \*

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Two pion effective mass and helicity angle distributions in the charged  $B$ -meson decays into three charged pions are studied. The weak decay amplitudes are calculated in the framework of the QCD factorization approximation. The strong interactions between the pairs of pions are taken into account using scalar and vector pion form factors. The scalar form factors are constrained by data on pion-pion, kaon-antikaon and four pion production incorporated into a multichannel model of the coupled amplitudes. The vector form factor is obtained from the Belle Collaboration analysis of the  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$  decays. The theoretical distributions of the dipion effective masses are compared with the corresponding results of the recent Dalitz plot analysis of  $B^\pm \rightarrow \pi^+ \pi^- \pi^\pm$  decays performed by the BABAR Collaboration. We find that the  $S$ -wave dipion amplitude, although much smaller than the  $P$ -wave amplitude, plays an important role even in the  $\rho(770)$  mass range. We show that the helicity angle distribution is strongly asymmetric in the  $\rho$ -meson range. This effect can be attributed to the broad  $f_0(600)$  (or  $\sigma$ ) meson. The fact that the signal of the  $B^\pm \rightarrow f_0(980) \pi^\pm$  decay has not been found in the experimental analysis can be easily explained in our model since the relevant  $B$  decay amplitude is proportional to the scalar form factor which has a dip at the  $f_0(980)$  mass. We obtain a unified unitary description of the contribution of the three scalar resonances  $f_0(600)$ ,  $f_0(980)$  and  $f_0(1400)$  in terms of the pion non-strange scalar form factor.

*35th International Conference on High Energy Physics,  
July 22 - 28 2010  
Paris, France*

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\*This work has been supported in part by the Polish Ministry of Science and Higher Education (grant No N N202 248135) and by the IN2P3-Polish Laboratories Convention (project No 08-127).

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## 1. Introduction

Studies of the charged  $B$  mesons decays into three charged pions constitute an important part of the analyses of the three-body charmless hadronic  $B$  decays (see, for example [1] and references given therein). Investigation of the Dalitz diagrams with theoretically well constrained meson-meson strong interaction amplitudes can lead to a better extraction of the weak interaction parameters from experimental data. Recently the BABAR Collaboration has published results of an isobar model analysis of the  $B^\pm$  decays into  $\pi^\pm \pi^\mp \pi^\pm$  [2]. The  $\pi^+ \pi^-$  effective mass spectrum is largely dominated by the  $\rho(770)$  meson, yet the authors of Ref. [2] find some contribution of the scalar resonance  $f_0(1370)$  but none of the  $f_0(980)$ . Here we present the results of a theoretical model in which the  $S$ -wave pion-pion strong interaction amplitude is constrained using a multichannel unitary approach including the  $\pi^+ \pi^-$  coupling to  $K^+ K^-$ ,  $K^0 \bar{K}^0$  and  $4\pi$  (effective  $(2\pi)(2\pi)$ ) states. Application of this model to the partial wave analysis of the BABAR data shows that the scalar meson contributions, in particular that of the  $f_0(600)$ , are needed to explain the effective  $\pi\pi$  mass distributions.

## 2. Theoretical model

The weak decay amplitudes of  $B^\pm$ , corresponding to the quark transitions  $b \rightarrow u\bar{u}d$  and  $b \rightarrow d\bar{d}d$ , are derived in the QCD quasi-two body factorization approach for the limited range of the effective  $\pi^+ \pi^-$  masses less than about 1.7 GeV. One assumes that only two of the three produced pions interact strongly, forming either an  $S$ - or  $P$ -wave state denoted by  $R_S$  or  $R_P$ . The  $\pi\pi$  strong interaction amplitudes are constrained by chiral symmetry, by QCD and by experimental data on meson-meson interactions. The matrix elements of the effective weak Hamiltonian involve the pion non-strange scalar and vector form factors.

The  $\pi^-(p_1)\pi^+(p_2)$   $S$ -wave contribution to the  $B^- \rightarrow \pi^-(p_1)\pi^+(p_2)\pi^-(p_3)$  decay amplitude reads:

$$\mathcal{M}_S^-(s_{12}) = \frac{G_F}{\sqrt{3}} \left[ -\chi_S f_\pi (M_B^2 - s_{12}) F_0^{BR_S}(m_\pi^2) u(R_S \pi^-) + B_0 \frac{M_B^2 - m_\pi^2}{m_b - m_d} F_0^{B\pi}(s_{12}) v(\pi^- R_S) \right] \Gamma_1^{n^*}(s_{12}), \quad (2.1)$$

where

$$u(R_S \pi^-) = \lambda_u \left[ a_1(R_S \pi^-) + a_4^u(R_S \pi^-) + a_{10}^u(R_S \pi^-) - (a_6^u(R_S \pi^-) + a_8^u(R_S \pi^-)) r_\chi^\pi \right] + \lambda_c \left[ a_4^c(R_S \pi^-) + a_{10}^c(R_S \pi^-) - (a_6^c(R_S \pi^-) + a_8^c(R_S \pi^-)) r_\chi^\pi \right], \quad (2.2)$$

$$v(\pi^- R_S) = \lambda_u \left[ -2a_6^u(\pi^- R_S) + a_8^u(\pi^- R_S) \right] + \lambda_c \left[ -2a_6^c(\pi^- R_S) + a_8^c(\pi^- R_S) \right] \quad (2.3)$$

and

$$B_0 = \frac{m_\pi^2}{m_u + m_d}, \quad r_\chi^\pi = \frac{2B_0}{m_b + m_u}. \quad (2.4)$$

In the above equations  $G_F$  denotes the Fermi coupling constant,  $f_\pi$  is the pion decay constant,  $s_{12} = (p_1 + p_2)^2$ ,  $p_1$  and  $p_2$  being the  $\pi^-$  and  $\pi^+$  momenta. The masses of the charged  $B$  mesons, of the charged pions, of the  $b$ ,  $u$  and  $d$  quarks are denoted by  $M_B$ ,  $m_\pi$ ,  $m_b$ ,  $m_u$  and  $m_d$ , respectively. The fitted parameter  $\chi_S$  is the proportionality factor which appears under the assumption that the

decay amplitude of  $R_S$  state into two pions is proportional to the pion non-strange scalar form factor  $\Gamma_1^{n^*}(s_{12})$ . The  $BR_S$  and  $B\pi$  transition form factors are denoted by  $F_0^{BR_S}(m_\pi^2)$  and  $F_0^{B\pi}(s_{12})$ , respectively. The symbols  $\lambda_u = V_{ub}V_{ud}^*$  and  $\lambda_c = V_{cb}V_{cd}^*$  are products of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix elements  $V_{qq'}$ . The effective Wilson coefficients  $a_j^{u,c}$ ,  $j = 1, 4, 6, 8, 10$ , are calculated to next-to-leading order in the strong coupling constant including the vertex and penguin corrections.

The pion non-strange scalar form factor is calculated in a unitary relativistic three coupled-channel model using the  $\pi\pi$ ,  $K\bar{K}$  and effective  $(2\pi)(2\pi)$  scattering  $T$  matrix of Refs. [3]. It is constrained at low energy by chiral perturbation theory. This form factor depends on two fitted parameters: the first one insures the convergence of the involved integrals and the second one controls the high-energy behaviour of  $\Gamma_1^{n^*}(s_{12})$ .

Explicit expressions for the  $P$ -wave contributions to the  $B$  decay amplitudes can be found in [4]. Here the pion vector form factor takes into account the contributions of the three vector resonances  $\rho(770)$ ,  $\rho(1450)$  and  $\rho(1700)$  and follows from the Belle Collaboration analysis of the semi-leptonic  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$  decays [5]. For the  $P$ -wave amplitude we introduce a fitted overall normalization factor  $N_P$ . Due to presence of two identical pions in the final state one has to symmetrize the decay amplitudes over the two possible  $\pi^+ \pi^-$  combinations.

### 3. Results and discussion

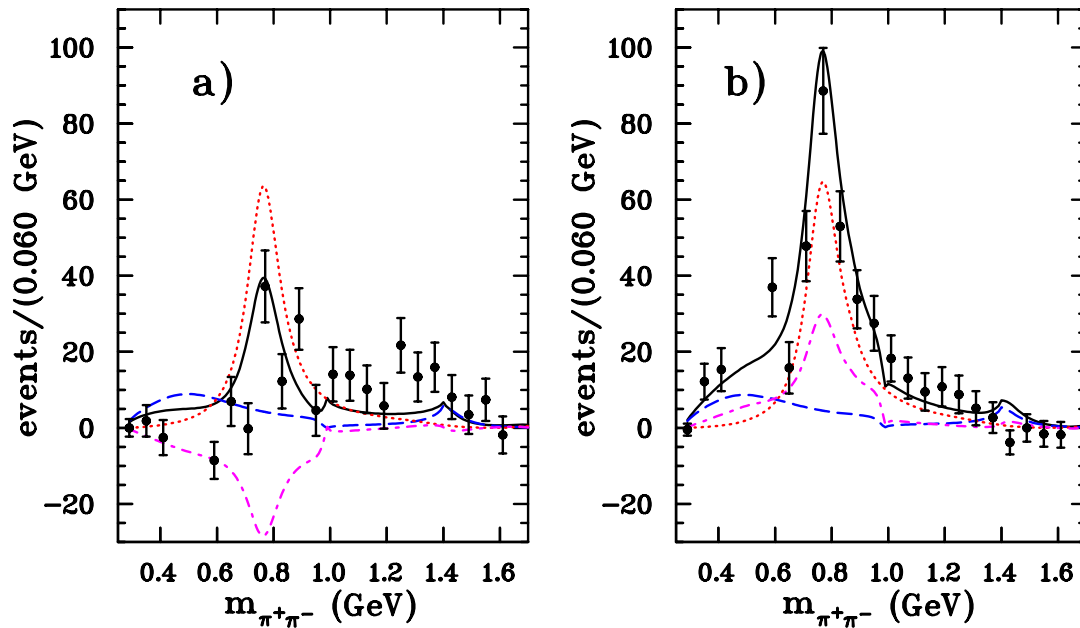
We obtain a good fit to the  $\pi\pi$  effective mass distributions of the BABAR Collaboration data of the  $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$  decays [2]. The value of the theoretical branching fraction for the  $B^\pm \rightarrow \rho(770)^0 \pi^\pm$  decays,  $(8.2 \pm 0.5) \times 10^{-6}$ , agrees well with that,  $(8.1 \pm 0.7 \pm 1.2_{-1.1}^{+0.4}) \times 10^{-6}$ , of the experimental analysis. The normalization factor  $N_P$  is found to be close to 1.

In Fig. 1 we show the  $\pi^+ \pi^-$  effective mass distributions for the  $B^+$  decays. In the left plot the value of the cosine of the pion helicity angle  $\theta$  is negative and in the right one it is positive. The  $\pi^+ \pi^-$  spectra are dominated by the  $\rho(770)^0$  resonance but at low effective masses the  $S$ -wave contribution is sizable. Here the  $f_0(600)$  resonance manifests its presence. Furthermore one observes a strong negative or positive interference of the  $S$  and  $P$  waves in the event distributions depending on the sign of  $\cos\theta$ . The interference term between the  $S$  and  $P$  dipion amplitudes can reach a value as high as 30% of the dominating  $\rho(770)$  contribution. The  $f_0(980)$  resonance is not directly visible as a peak, since the scalar form factor has a dip near 1 GeV. At 1.4 GeV the maximum of the  $S$ -wave distribution comes from the scalar resonance  $f_0(1400)$  [3]. Integrating the overall  $S$ -wave contribution in the whole available phase space one obtains as much as 25 % of the total  $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$  branching fraction. Similar results to those shown in Fig. 1 are obtained for the  $B^-$  decays. The absolute differences between the  $B^+$  and  $B^-$  distributions are small so the corresponding  $CP$  asymmetry is also small, about 3%.

Our model yields a unified description of the contribution of the three scalar resonances  $f_0(600)$ ,  $f_0(980)$  and  $f_0(1400)$  in terms of one function: the pion non-strange scalar form factor. This reduces strongly the number of needed free parameters to analyze the Dalitz plot. The functional form of our  $S$ -wave amplitude, proportional to  $\Gamma_1^{n^*}(s)$ , could be used in Dalitz-plot analyses and the table of  $\Gamma_1^{n^*}(s)$  values can be sent upon request.

Let us note that the strong interaction phases of the decay amplitudes are constrained by unitarity and meson-meson data. This should help to improve the precision of the weak interaction amplitudes extracted from Dalitz plot analyses, especially the value of the weak angle phase  $\gamma$  (or  $\phi_3$ ). Of course new experimental data with better statistics would be welcome. One expects new results on the  $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$  decays from the Belle Collaboration and in near future, from LHCb and super  $B$  factories.

In summary, we have shown that the scalar mesons are clearly visible in the  $B^\pm \rightarrow \pi^+ \pi^- \pi^\pm$  decays. Moreover, the introduction to the decay amplitudes of the pion non-strange scalar form factor, constrained by theory and other experiments than  $B$  decays, can be more economical in the description of data than the use of the isobar model with many free parameters fitted for each scalar resonance and, in addition, for the nonresonant term.



**Figure 1:** The  $\pi^+ \pi^-$  light effective mass distributions from the fit to the BABAR experimental data [2] for the  $B^+$  decays into  $\pi^+ \pi^- \pi^+$  a) with  $\cos\theta < 0$  and b) with  $\cos\theta > 0$ , where  $\theta$  is the helicity angle. The dashed line represents the  $S$ -wave contribution, the dotted line that of the  $P$  wave and the dot-dashed line that of the interference term. The solid line corresponds to the sum of these contributions.

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