SLAC-PUB-14309
BABAR-PROC-10/098
November 2010

Inclusive Measurements of $B \to X_c \ell \nu$ and $B \to X_s \gamma$ Decays: Mini-review

Kyle J. Knoepfel
University of Notre Dame[1]
E-mail: knoepfel@fnal.gov
(from the BABAR Collaboration)

Abstract

A mini-review of measurements of inclusive semileptonic $B \to X_c \ell \nu$ and radiative $B \to X_s \gamma$ decays is presented. The semileptonic X_c mass moments are presented from Belle and BABAR. The inclusive $B \to X_s \gamma$ branching fraction is presented from Belle as well as a preliminary measurement of the direct CP-asymmetry of $B \to X_{s+d} \gamma$ decays at BABAR. Fundamental Standard Model parameters and heavy quark parameters can be derived from the X_c mass and lepton energy moments from $B \to X_c \ell \nu$ decays and from the photon energy moments from $B \to X_s \gamma$ decays. The values of the CKM matrix element $|V_{cb}|$, the b-quark mass m_b, and the Fermi motion of the b-quark inside the B-meson are presented based on a global fit to these moments by the Heavy Flavor Averaging Group.

Contributed to the Proceedings of the Xth Nicola Cabibbo International Conference on Heavy Quarks and Leptons,
October 11–15, 2010, Frascati (Rome), Italy

[1] now at Fermi National Accelerator Laboratory

1 Introduction

Inclusive semileptonic $B \to X_c \ell \nu$ and radiative $B \to X_s \gamma$ decays, where $X_{c(s)}$ represents any final hadronic state with unit charm (strangeness), are powerful laboratories for new and Standard Model (SM) physics. The optical theorem can be used to related the inclusive decay rates to the forward-scattering of the B-meson. The resulting expression is the basis for an operator product expansion (OPE) in powers of Λ/m_B, where Λ is the scale of the momentum transfer of the decay, and m_B is the B-meson mass.

Since the lowest-order non-perturbative term arises at roughly $1/m_B^2$, calculations of inclusive measurements are typically precise (less than 10% uncertainty) as the theoretical predictions are free from large uncertainties that can arise from hadronic form factors present in exclusive decays. The precision on inclusive calculations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cb}|$ [1] and the $B \to X_s \gamma$ branching fraction [2] has reached the 2% and 7% levels, respectively. In order for any unambiguous statement to be made regarding the presence of new physics, the inclusive experimental measurements must be equally precise.

Although not sensitive to new physics contributions, the various spectra from $B \to X_c \ell \nu$ and $B \to X_s \gamma$ decays can be used to extract fundamental SM parameters. The OPE cannot be used in predicting the spectral shapes, however, as the expansion breaks down at the phase space end-points, where the non-perturbative terms become significant. To avoid this complication, integrated quantities (moments) instead of the spectrum itself can be used to compare theory and experiment.

We will present a mini-review of inclusive measurements of $B \to X_c \ell \nu$ and $B \to X_s \gamma$ decays, primarily at the B-factories, and particularly focusing on the X_c mass moments, the $B \to X_s \gamma$ branching fraction, and the $B \to X_{s+d} \gamma$ direct *CP*-asymmetry A_{CP}, which is predicted to be nearly zero in the SM [3]. We will also discuss the extraction of heavy quark parameters from the moments measurements.

2 Mass Moments from Semileptonic $B \to X_c \ell \nu$ Decays

The methods used in reconstructing $B \to X_c \ell \nu$ decays at Belle and BABAR are similar. To suppress backgrounds from continuum events ($e^+e^- \to q\overline{q}$, where $q = u, d, s, c$), one of the B-mesons (B_{reco}) is reconstructed in fully hadronic final states. The large semileptonic branching fraction of the signal B (B_{SL}) makes the hadronic reconstruction method a statistically feasible approach. The signature of the semileptonic B decay is the presence of a lepton with high energy, typically greater than 0.7 or 0.8 GeV in the B rest frame. The remaining tracks and calorimeter clusters in the event not used in tag reconstruction are combined to form the final state X_c hadronic system.

After the event selection, the remaining background can be classified into three categories: combinatorial backgrounds, where B_{reco} has been misreconstructed with particles from the B_{SL}; continuum backgrounds; and residual backgrounds, where B_{reco} was properly reconstructed, but B_{SL} is reconstructed with non-signal decays from misidentified leptons, cascade leptons, or $B \to X_u \ell \nu$ decays.

The Belle result uses 152×10^6 $B\overline{B}$ events [4] and removes continuum backgrounds by using data taken 60 MeV below the $\Upsilon(4S)$ resonance (off-resonance data). The signal and remaining backgrounds are modeled with Monte Carlo (MC) samples. Before the X_c mass moments are measured, detector resolution effects are deconvoluted from the spectrum using a Singular Value Decomposition (SVD) algorithm. The moments and corresponding uncertainties are calculated

$E^*_{\min}$ (GeV)	Belle Analysis (GeV^2/c^4)	*BABAR* Analysis (GeV^2/c^4)
0.7	4.403 ± 0.036 ± 0.052	———
0.9	4.353 ± 0.032 ± 0.041	4.416 ± 0.027 ± 0.063
1.1	4.293 ± 0.028 ± 0.029	4.354 ± 0.026 ± 0.063
1.3	4.213 ± 0.027 ± 0.024	4.281 ± 0.027 ± 0.061
1.5	4.144 ± 0.028 ± 0.022	4.220 ± 0.031 ± 0.070
1.7	4.056 ± 0.033 ± 0.022	4.158 ± 0.040 ± 0.094
1.9	3.996 ± 0.041 ± 0.021	4.136 ± 0.069 ± 0.142

Table 1: Second-order mass moments for various minimum lepton energy requirements ($E^*_{\min}$). The *BABAR* analysis measures moments in 100 MeV increments, starting at a minimum lepton energy of 0.8 GeV–the other measurements are omitted for brevity.

using the formulae

$$\left\langle M_X^k \right\rangle = \frac{\sum_i (M_X^k)_i n_i}{\sum_i n_i} \quad \text{and} \quad \sigma^2\left(\left\langle M_X^k \right\rangle\right) = \frac{\sum_{i,j} (M_X^k)_i X_{ij} (M_X^k)_j}{\left(\sum_i n_i\right)^2} \tag{1}$$

where $(M_X^k)_i$ is the central value of bin i of the unfolded spectrum, n_i is the contribution to the spectrum at bin i, and X is the covariance matrix. Belle measures moments corresponding to $k = 2, k = 4$, and also the centralized mass moment $(M_X^2 - \langle M_X^2 \rangle)^2$.

The *BABAR* analysis is performed with 232×10^6 $B\overline{B}$ pairs [5]. Instead of using off-resonance data (as in the Belle analysis), the *BABAR* analysis uses a threshold function to parameterize continuum as well as combinatorial background. The residual background and signal are modeled using MC samples. Detector resolution effects are taken into account through explicit calibrations to the invariant X_c mass. Using MC samples, the calibrations are parameterized using the form $M^k_{X,reco} = A + B \times M^k_{X,calib}$, where A and B are constants that depend on the energy imbalance in the event, the X_c multiplicity, the minimum lepton energy, and moment order k. The masses $M^k_{X,reco}$ and $M^k_{X,calib}$ correspond to the reconstructed, and calibrated mass quantities, respectively. A separate MC control sample of exclusive semileptonic decays is used to validate the calibration procedure, the results of which show good agreement between $M^k_{X,reco}$ and $M^k_{X,calib}$.

A comparison of the results from both experiments for $k = 2$ is shown in Table 1. The systematic errors are similar in both analyses. They arise from uncertainties in the assumption of the background normalization, the variations of the $B \to D^{(*)}\ell\nu$ branching fraction and form factors, and the normalization of the other signal contributions, including non-resonant final states. Whereas Belle assigns an uncertainty due to the unfolding parameter from the SVD algorithm, the *BABAR* analysis takes into account the uncertainty of the calibration procedure. The X_c moments, combined with the lepton energy moments and the $B \to X_s\gamma$ photon energy moments, are used to extract the heavy quark parameters (Section 4).

3 Branching Fraction and A_{CP} of $B \to X_s\gamma$ Decays

The principal signature of an inclusive $B \to X_s\gamma$ decay is the high-energy photon. Two approaches have been used: a semi-inclusive method where multiple X_s final states are reconstructed to ap-

Background	Belle Analysis	*BABAR* Analysis
Process	$1.7 < E^*_\gamma < 2.8\,\mathrm{GeV}$	$1.8 < E^*_\gamma < 2.8\,\mathrm{GeV}$
$B \to X\pi^0$	0.597^*	0.613^*
$B \to X\eta$	0.199^*	0.192^*
$B \to X\omega$		0.027^*
$B \to X\eta'$	↑	0.008^*
$B \to XJ/\psi$	0.111	0.007
$B \to Xe^{\pm}(\gamma)$	↓	0.062
Final State Radiation (FSR)		0.019
Fake Photon: $e^{\pm}$	0.041	0.033
Fake Photon: K^0_L and $\overline{n}$	0.020	0.025^*
Other	0.032	0.014

Table 2: The $B\overline{B}$ background composition according to Monte Carlo simulation after all selection cuts in the signal regions for the Belle and *BABAR* analyses. The variable E^*_γ is the candidate photon energy in the $\Upsilon(4S)$ rest frame. Fractions followed by an asterisk represent backgrounds that are corrected using an appropriate data control sample. The backgrounds from $B \to X\omega$ to FSR are grouped together in the Belle analysis and comprise 11.1% of the total $B\overline{B}$ background. Note that the antineutron and not the K^0_L background component is corrected in the *BABAR* analysis.

proximate an inclusive process; and the fully-inclusive method where the X_s is not reconstructed. Although the semi-inclusive method results in small backgrounds, assumptions must be made regarding the fraction of final states that were not reconstructed, introducing an unavoidable model dependence. In contrast, the fully-inclusive method suffers from large experimental backgrounds, but is sensitive to all hadronic final states within the measured energy region.

The fully-inclusive analyses at *BABAR* and Belle are similar [6]. Both analyses impose a minimum photon energy cut to remove significant backgrounds, principally from photon daughters of π^0 mesons in continuum and $e^+e^- \to \tau^+\tau^-$ events, and non-signal B decays. To remove photons from π^0 and η decays, the invariant mass of the photon candidate and any other photon in the event is calculated; the event is rejected if the invariant mass is consistent with the nominal π^0 or η mass. Photons from continuum events are suppressed by making requirements on high-momentum leptons (lepton tagging), which are unlikely to come from continuum events, and by exploiting the different event topologies between continuum and $B\overline{B}$ events by using multivariate algorithms.

To remove the remaining background, continuum events are subtracted from the photon energy spectrum using luminosity-scaled off-resonance data—data taken 60 (40) MeV below the $\Upsilon(4S)$ resonance at Belle (*BABAR*). Backgrounds from B decays are removed using data-corrected MC simulations. The contribution from $B \to X_d\gamma$ decays must be removed from the resulting energy spectrum, and various procedures must be employed to remove effects from the boost of the B to the $\Upsilon(4S)$ rest frame, and from the detector resolution, which is convoluted with the true photon energy spectrum.

The breakdown of B backgrounds in the *BABAR* and Belle analyses is shown in Table 2. The fractions are given in the signal regions of the respective analyses. The majority of $B\overline{B}$ background photons after the event selection are from $\pi^0(\eta)$ decays where the photon partner was not reconstructed, allowing the photon candidate to slip past the $\pi^0(\eta)$ vetoes. Belle corrects for photon

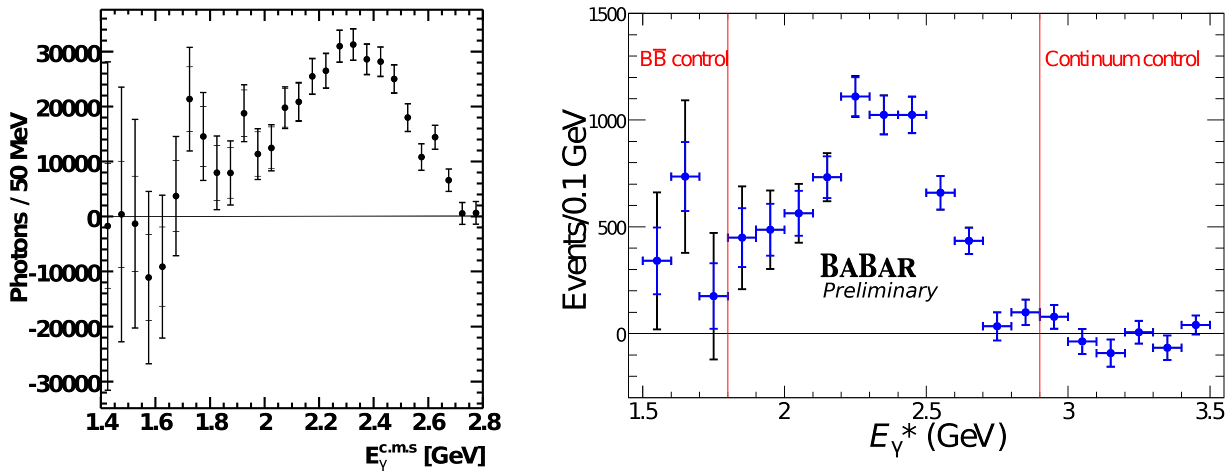

Figure 1: **(left)** Measured photon energy spectrum from Belle, corrected for efficiency and detector effects; **(right)** preliminary photon spectrum at *BABAR* after all selection cuts. Inner error bars, where visible, represent the statistical contribution to the total error (outer error bars).

backgrounds just from π^0 and η decays using an uncorrelated data sample of inclusive $\pi^0(\eta)$ decays. In addition to using dedicated control samples for corrected $\pi^0(\eta)$ backgrounds, *BABAR* also corrects for photons that arise from ω and η' decays, as well as for false photon signatures by electrons and antineutrons.

The efficiency-corrected and unfolded photon energy spectrum as measured by Belle is shown in the left plot of Figure 1. The branching fraction obtained is $\mathcal{B}(B \to X_s\gamma) = (3.45 \pm 0.15_{stat.} \pm 0.40_{syst.}) \times 10^{-4}$ for a minimum photon energy of 1.7 GeV in the B rest frame. Reference [6] quotes branching fraction results for various minimum photon energy requirements, as well as the corresponding first and second-centralized energy moments. The dominant systematic errors arise from the uncertainty on the $B\overline{B}$ background estimation and data-based corrections. Additional systematic uncertainties enter from the unfolding algorithm, the photon detection efficiency, the removal of $B \to X_d\gamma$ contamination, and the transformation from the $\Upsilon(4S)$ to the B rest frame.

The preliminary photon energy spectrum measured at *BABAR* is shown in the right plot of Figure 1. The energy region above 2.9 GeV is used to validate the off-resonance subtraction procedure. The various $B\overline{B}$ MC corrections are validated in the energy region $1.53 < E^*_\gamma < 1.8$ GeV, which is composed almost entirely of $B\overline{B}$ background after continuum background subtraction. The flavor of the signal b-quark is identified by the charge of the tag lepton: $\ell^+(\ell^-) \implies b(\bar{b})$. The photon energy spectrum is then divided according to the lepton charge. The A_{CP} is then:

$$A_{CP}(B \to X_{s+d}\gamma) = \frac{1}{1-2\omega}\frac{N^+ - N^-}{N^+ + N^-} \tag{2}$$

where ω accounts for dilution effects, and $N^{+(-)}$ are the events tagged with an $\ell^+(\ell^-)$.

To reduce the sensitivity to the systematic uncertainties of the $B\overline{B}$ background, the A_{CP} is extracted with a photon energy cut of $E^*_\gamma > 2.1$ GeV. The dominant uncertainty is therefore due to the limited statistics of the off-resonance data subtraction. The measured yields are $N^{+(-)} =$

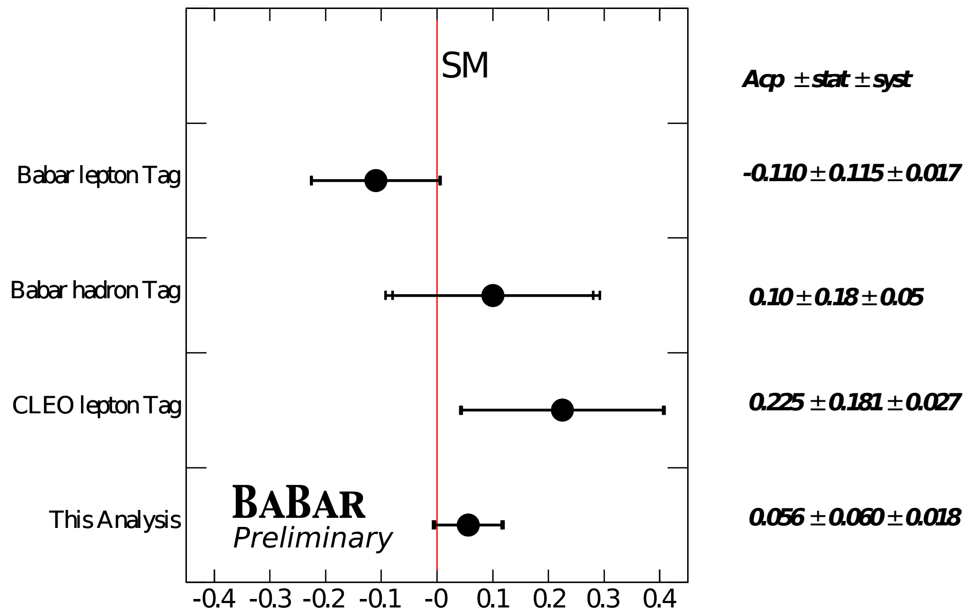

Figure 2: Comparison of previous $A_{CP}(B \rightarrow X_{s+d}\gamma)$ measurements to the preliminary *BABAR* result. Inner error bars, where visible, represent the statistical contribution to the total error (outer error bars).

$2397 \pm 151_{stat.}(2623 \pm 158_{stat.})$, giving rise to a raw A_{CP} of $0.045 \pm 0.044_{stat.}$. The dilution term $\omega = 0.131 \pm 0.0064_{syst.}$ arises from wrong-sign leptons, which result from B^0-$\overline{B}^0$ oscillations, $B \rightarrow D \rightarrow X\ell\nu$ cascade decays, and lepton misidentification. Accounting for these effects and also additional potential biases from the $B\overline{B}$ subtraction, the preliminary result at *BABAR* is $A_{CP}(B \rightarrow X_{s+d}\gamma) = 0.056 \pm 0.063$, consistent with SM expectation. A comparison to previous measurements is shown in Figure 2.

4 Extraction of Heavy Quark Parameters

Due to its heavy mass, the b-quark field can be expanded non-relativistically. In the context of the Heavy Quark Expansion (HQE), various parameters arise, which characterize the motion of the b-quark inside the B meson. In the kinetic scheme, at the lowest orders appear expectation values of dimension-five and -six operators: μ_π^2 (Fermi motion), μ_G^2 (B- B^* splitting), ρ_{LS}^3 (spin-orbit coupling) and ρ_D^3 (Darwin term).

To extract the values of these parameters, along with the value of the b-quark mass m_b and the CKM matrix element $|V_{cb}|$, the X_c mass moments and lepton energy moments from $B \rightarrow X_c\ell\nu$ decays and the photon energy moments from $B \rightarrow X_s\gamma$ decays are combined into a fit, based on a χ^2 minimization technique [7]. A vector of experimental moments ($\mathbf{M}_{\rm exp}$) is compared with the analytic predictions from the HQE ($\mathbf{M}_{\rm HQE}$). The constructed χ^2:

$$\chi^2 = (\mathbf{M}_{\rm exp} - \mathbf{M}_{\rm HQE})^T \, C_{\rm tot}^{-1} \, (\mathbf{M}_{\rm exp} - \mathbf{M}_{\rm HQE}) \tag{3}$$

is minimized to obtain the HQE parameters. The sum of the experimental and theoretical covariance matrices is represented by C_{tot}.

The fit results from measurements of the individual Belle and *BABAR* experiments are presented in References [5, 8]. The $|V_{cb}|$ *vs.* m_b and μ_π^2 *vs.* m_b results from a global fit by the Heavy Flavor Averaging Group (HFAG) are shown in Figure 3 [9]. The global fit includes measurements from various experiments, but excludes the most recent photon energy moments from Belle. Inclusion of the photon energy moments gives rise to a roughly 1σ tension between the results with and without the photon moments. This tension is a source of much discussion among the theory and experimental communities, and further discussion is beyond the scope of this Proceedings contribution.

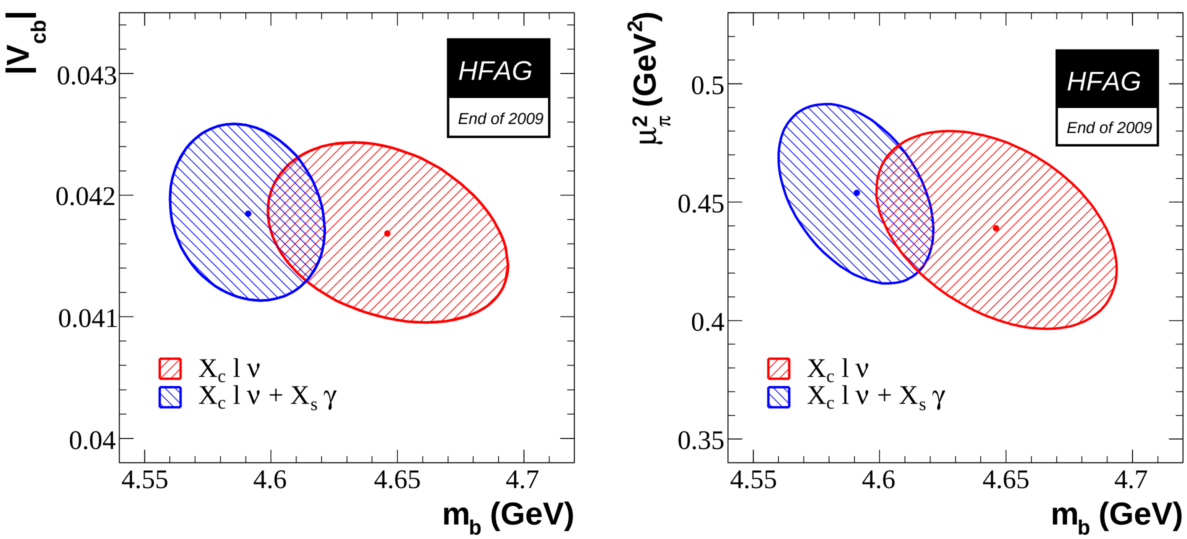

Figure 3: Global fit results by HFAG. Shown are the $\Delta\chi^2 = 1$ contours for the fit with just the semileptonic moments (labeled $X_c\ell\nu$) and the fit that includes the photon energy moments (labeled $X_c\ell\nu + X_s\gamma$).

5 Summary & Acknowledgments

We have argued that inclusive $B \to X_c\ell\nu$ and $B \to X_s\gamma$ decays are probes of the Standard Model in extracting fundamental SM parameters ($|V_{cb}|$ and m_b) and HQE parameters (μ_π^2, μ_G^2, ρ_{LS}^3 and ρ_D^3) through fits to the measured moments. Radiative $B \to X_s\gamma$ decays are also sensitive to new physics by new physics particles propagating in the penguin diagram. The $B \to X_s\gamma$ branching fraction and direct *CP*-asymmetry results presented, however, are consistent with SM expectations.

The author would like to thank the organizers of the Heavy Quarks and Leptons conference for an enjoyable experience, and also Vittorio Lubicz for explaining the status of the lattice determination of the hadronic form factor in $B \to K^*\gamma$ decays. He also thanks Kevin Flood for helpful discussions, and appreciates the support from Colin Jessop.

References

[1] D. Benson, et al, *Imprecated, yet impeccable: on the theoretical evaluation of* $\Gamma(B \to X_c \ell \nu)$, *Nucl. Phys. B* **665** 367 (2003)

[2] M. Misiak, et al, *Estimate of* $\mathcal{B}(\overline{B} \to X_s \gamma)$ *at* $\mathcal{O}(\alpha_s^2)$, *Phys. Rev. Lett.* **98** 022002 (2007)

[3] T. Hurth and T. Mannel, *CP asymmetries in* $b \to (s/d)$ *transitions as a test of CKM CP violation*, *Phys. Lett. B* **511** 196 (2001)

[4] C. Schwanda, et al, *Moments of the invariant mass spectrum in* $B \to X_c \ell \nu$ *decays at Belle*, *Phys. Rev. D* **75** 032005 (2007)

[5] B. Aubert, et al, *Measurement and interpretation of moments in inclusive semileptonic decays* $\overline{B} \to X_c \ell^- \overline{\nu}$, *Phys. Rev. D* **81** 032003 (2010)

[6] A. Limosani, *Measurement of Inclusive Radiative B-Meson Decays with a Photon Energy Threshold of 1.7* GeV , *Phys. Rev. Lett.* **103** 241801 (2009) `arXiv:0907.1384 [hep-ex]`

[7] O. L. Buchmüller and H. U. Flächer, *Fit to moments of inclusive* $B \to X_c \ell \overline{\nu}$ *and* $B \to X_s \gamma$ *decay distributions using heavy quark expansions in the kinetic scheme*, *Phys. Rev. D* **73** 073008 (2006)

[8] C. Schwanda, et al, *Measurement of the moments of the photon energy spectrum in* $B \to X_s \gamma$ *decays and determination of* $|V_{cb}|$ *and* m_b *at Belle*, *Phys. Rev. D* **78** 032016 (2008)

[9] The Heavy Flavor Averaging Group, *Average of b-hadron, c-hadron, and τ-lepton Properties* `arXiv:1010.1589 [hep-ex]`