Shear and bulk viscosities for pure glue matter

A.S. Khvorostukhin,[1,2,3] V.D. Toneev,[1,2] and D.N. Voskresensky[1,4]

[1] *GSI, Helmholtzzentrum für Schwerionenforschung GmbH, Planckstrasse 1, 64291 Darmstadt, Germany*
[2] *Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[3] *Institute of Applied Physics, Moldova Academy of Science, MD-2028 Kishineu, Moldova*
[4] *National Research Nuclear University "MEPhI", Kashirskoe sh. 31, Moscow 115409, Russia*

Shear η and bulk ζ viscosities are calculated in a quasiparticle model within a relaxation time approximation for pure gluon matter. Below T_c the confined sector is described within a quasiparticle glueball model. Particular attention is paid to behavior of the shear and bulk viscosities near T_c. The constructed equation of state reproduces the first-order phase transition for the glue matter. It is shown that with this equation of state it is possible to describe the temperature dependence of the shear viscosity to entropy ratio η/s and the bulk viscosity to entropy ratio ζ/s in reasonable agreement with available lattice data but absolute values of the ζ/s ratio underestimate the upper limits of this ratio in the lattice measurements typically by an order of magnitude.

PACS numbers: 25.75.-q, 25.75.Ag

I. INTRODUCTION

The high-energy heavy-ion collisions at SPS and RHIC energies have shown evidence for a new state of matter characterized by very low shear viscosity to entropy density ratio η/s similar to a nearly ideal fluid [1–4]. Lattice calculations indicate that the crossover region between hadron and quark-gluon matter has been reached in these experiments. On the other hand lattice calculations performed in gluodynamics (GD) clearly demonstrate that there occurs the first order phase transition.

The shear η and bulk ζ viscosities are parameters which quantify dissipative processes in the hydrodynamic evolution of a fluid. It is known that behavior of transport coefficients is sensitive to the presence of phase transitions in a medium (see papers [5–11] and references therein). Values of the bulk and shear viscosities near the phase transition critical temperature T_c affect the hydrodynamic evolution of the medium and may influence observables.

Lattice QCD is the most powerful technique to extract nonperturbative information on equation of state (EoS). When experimental data are lacking, lattice data are often used to fit model parameters. For pure gluon $SU(3)$ theory the EoS has been computed on the lattice more than a decade ago [12]. Recently much more accurate data have been obtained [13].

Among various existing phenomenological approaches, quasiparticle (QP) models are used to reproduce results obtained in the lattice QCD. In the case of GD the QP models rely on the assumption that for the temperature T above the critical one, $T > T_c$, the system consists of a gas of massive deconfined gluons. In the confined phase, at $T < T_c$, the glue matter is considered as a gas of massive glueballs.

In this paper we aim to investigate the behavior of viscosity coefficients for a gluon system which exhibits deconfinement phase transition. The phenomenological QP model is applied to describe available lattice data on EoS. Shear and bulk viscosities are calculated within a relaxation time approximation.

II. EQUATION OF STATE OF GLUE MATTER

In the QP approach the system of interacting gluons is treated as a gas of noninteracting quasiparticles with an effective mass $m_g(T)$, which depends on T as [4]

$$m_g^2(T) = \frac{N_c}{6}\, g^2(T)\, T^2 \tag{1}$$

with the temperature-dependent strong interaction constant

$$g^2(T) = \frac{48\pi^2}{11N_c\ \ln[\lambda(T-T_s)/T_c)]^2}\ , \tag{2}$$

where parameters T_s/T_c =0.5853, λ =3.3 are taken to fit the new lattice data, see below, and a number of colors N_c =3. The energy density and the pressure acquire then the following forms:

$$\begin{aligned}\varepsilon_g(T) &= \tfrac{d_g}{2\pi^2}\int_0^\infty p^2dp\tfrac{E}{\exp(E/T)-1} + B(T)\\ &\equiv \varepsilon_g^{id}(T, m_g(T)) + B(T),\end{aligned} \tag{3}$$

$$\begin{aligned}P_g(T) &= \tfrac{d_g}{6\pi^2}\int_0^\infty p^2dp\tfrac{p^2}{E}\tfrac{1}{\exp(E/T)-1} - B(T)\\ &\equiv P_g^{id}(T, m_g(T)) - B(T),\end{aligned} \tag{4}$$

where the degeneracy factor $d_g = 2(N_c^2-1) = 16$ for the $SU(3)$ gluodynamics, ε_g^{id} and P_g^{id} are the energy density and the pressure of the ideal gas of massive gluons. The temperature-dependent function $B(T)$ in Eq. (3) results from the thermodynamical identity, see Ref. [14],

$$T\frac{dP}{dT} - P(T) = \varepsilon(T), \tag{5}$$

which leads to the equation for $B(T)$:

$$\frac{dB(T)}{dT} = -\frac{\varepsilon_g^{id} - 3P_g^{id}}{m_g}\frac{dm_g}{dT}. \qquad (6)$$

Dealing only with gluon degrees of freedom one assumes that the matter at $T < T_c$ (the "hadronic" phase) consists of glueballs. While the meson scattering amplitude is parametrically suppressed as $1/N_c$, the scattering amplitude between glueballs scales as $1/N_c^2$ [15] and therefore the system can be considered as a noninteracting Bose gas of glueballs. Expected glueball masses are high, $m_{gb} \gtrsim 1$ GeV, and thereby only lowest-lying glueball states contribute to the EoS at temperatures of our interest. It is difficult to single out which states of the observed hadronic spectrum are glueballs because of a lack of knowledge of decay properties and existence of a strong mixing between glueballs and quark states [16]. However, using typical constant values for lowest-lying glueball masses within a statistical model one fails to reproduce the strong increase of thermodynamical variables near T_c [17]. The T-behavior of masses for two lowest-lying scalar 0^{++} and tensor 2^{++} glueballs was investigated on the lattice in [18]. Therefore, below we follow the $SU(3)$ lattice GD results. It was shown that the pole mass $m_{gb}(T)$, the Breit-Wigner mass $\tilde{m}_{gb}(T)$ and the thermal width Γ_{gb} are linked as follows :

$$m_{gb}(T) \approx \tilde{m}_{gb}(T) - 2T + \sqrt{4T^2 - \Gamma_{gb}^2(T)}\ . \qquad (7)$$

With the help of the ansatz $\tilde{m}_{gb}(T) = m_{gb}^0$, i.e. that the gluon Breit-Wigner masses are given by the PDG values,

$$\Gamma_{gb} = b_{gb}(T - T_{gb})\ \Theta(T - T_{gb}) \quad \text{for} \quad T_{gb} < T < T_c \qquad (8)$$

and recommended parameters $b_{gb}(0^{++})$ =4.23 and $b_{gb}(2^{++})$ =7.152, the relation (7) reproduces quite well the lattice results in the measured range $0.5T_c < T < T_c = 265$ MeV [18]. In our consideration we limit ourselves by the two above-mentioned spices of glueballs, the only ones which lattice data are available for.

With the temperature-dependent glueball masses a statistical treatment of glueballs needs an additional requirement of thermodynamic consistency. It has been satisfied in the same way as outlined above for gluons, see Eqs. (5) and (6) above.

To describe glue matter in the whole range of temperatures we use the first order phase transition model in accordance with lattice results for GD. Thus, one should conjugate the pure gluon (g) and the glueball (gb) phases by making use of the Gibbs conditions at the transition:

$$T_c^g = T_c^{gb} \equiv T_c\ , \qquad P_g(T_c) = P_{gb}(T_c)\ . \qquad (9)$$

We use the value T_c =265 MeV for the first order phase transition in agreement with the lattice $SU(3)$ GD [12, 13, 19].

In Fig. 1 we compare the model results for the pressure and the energy density with the lattice data. Values

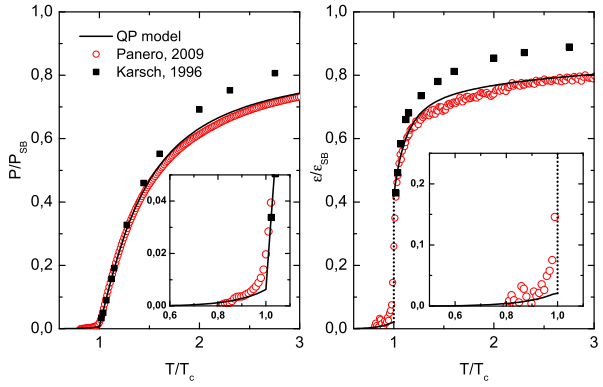

FIG. 1: The reduced pressure (left panel) and the energy density (right panel) of the glue matter. The solid line corresponds to taking into account two glueballs, scalar 0^{++} with $m_{gb}^0 = 1470$ MeV and tensor 2^{++} with $m_{gb}^0 = 2150$ MeV. Experimental points are the old Karsch's (filled squares) [20] and the new Panero's (circles) [13] lattice results. The region near T_c is zoomed in the insertion.

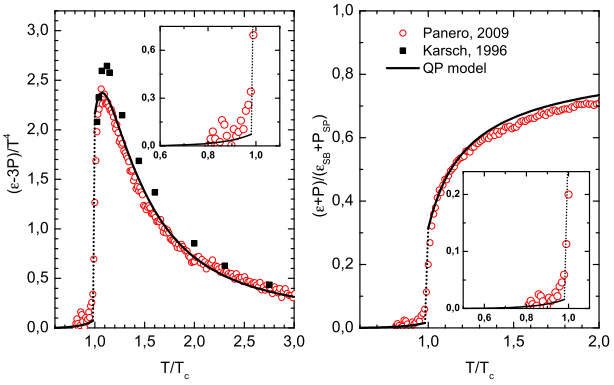

FIG. 2: The trace anomaly (left panel) and the reduced entalpy/entropy (right panel) of the glue matter. All notations are the same as in Fig. 1.

are normalized to those in the Stefan-Boltzmann (SB) limit. Both $\varepsilon/\varepsilon_{\rm SB}$ and $P/P_{\rm SB}$ increase fast and monotonically with the temperature above T_c but, as we see, the Stefan-Boltzmann limit is not saturated up to $3T_c$. Two sets of lattice data are similar qualitatively but old data [20] are appreciably higher than the new ones [13] at $T \gtrsim 1.5T_c$. Note that the new lattice data are extended to the region of the glueball phase, $T < T_c$. Our QP model gives a reasonable agreement with the new lattice data, except narrow vicinity to the left of T_c where model predictions are evidently below lattice points (see insertions in the left and right panels). Due to large glueball masses, this result is not changed if one adds next 2-3 glueball states to our two lowest-lying glueball states, although there exist statements that a good agreement with lattice data near T_c can be reached only if the whole high-lying glueball spectrum of the Hagedorn-type [21] or glueball condensate [17] are additionally included.

The interaction measure or trace anomaly $(\varepsilon - 3P)/T^4$

and the reduced enthalpy $(\varepsilon+P)/(\varepsilon_{\rm SB}+P_{\rm SB})$ are demonstrated in Fig. 2, in the left and right panels, respectively. Presence of a nonzero trace for the energy-momentum tensor relates with the breaking of the scale and conformal invariances. Again, a nice agreement is observed between the QP model and the new set of the lattice data [13], for a narrow region near T_c. The reduced enthalpy in the right panel of Fig. 2 for pure gluon system is just the reduced entropy, $s/s_{\rm SB}$, which is thereby also reproduced by our QP model. Thus, we see that the developed QP model successfully describes thermodynamic properties of the glue matter.

III. CALCULATION OF VISCOSITY COEFFICIENTS

In principle it is possible to compute the shear and bulk viscosities directly from GD at finite temperature using Kubo formulae. However in practice, this is quite difficult task because GD is generally a strongly interacting theory with unknown mechanism of the confinement. Essential assumptions of our kinetic approach are that quasiparticles are well defined, elementary interactions are local and the dynamics may be described in the relaxation time approximation.

Derivation of viscosity coefficients starts with the expression for the energy-momentum tensor for quasi-free [41] boson quasiparticles of spices a :

$$T_a^{\mu\nu} = \int d\Gamma \left\{ \frac{p_a^\mu p_a^\nu}{E_a} F_a \right\} \ , \qquad (10)$$

$$d\Gamma \ = \ d_a \frac{d^3\vec{p}_a}{(2\pi)^3} \ , \quad p_a^\mu = (E_a(\vec{p}_a,\vec{r}),\vec{p}_a) \ ,$$

d_a is the degeneracy factor. The QP distribution function F_a fulfills the QP kinetic equation. We assume that gluon and glueball masses are given by Eqs. (1) and (7), respectively. The QP energy is determined by

$$E_a(\vec{p}) = \sqrt{\vec{p}^{\,2} + m_a^2(T,F_a)} \ . \qquad (11)$$

Below we consider only collisional sources of the viscosity. Applying the relaxation time approximation to the relativistic QP kinetic equation we arrive at the expression for the variation of the energy-momentum tensor (10) near the local equilibrium state:

$$\delta T^{\mu\nu} = -\sum_a \int d\Gamma \left\{ \tau_a \frac{p_a^\mu p_a^\nu}{E_a^2} \ p_a^\kappa \partial_\kappa F_a \right\}_{\rm loc.eq.} \ , \qquad (12)$$

where τ_a denotes the relaxation time of the given species which generally depends on the QP momentum $\vec{p}_a$. The local equilibrium distribution function for a boson is as follows:

$$F_a^{\rm loc.eq.}(p_a,x_a) = \left[e^{p_a^\mu u_\mu/T} - 1 \right]^{-1} \ , \qquad (13)$$

$u^\mu \simeq (1,\vec{u})$ for $|\vec{u}| \ll 1$. Performing variation in (12) we did not vary quantities which may depend on the distribution function only implicitly, like E_a, since only doing this one may arrive at the relaxation time form of the QP kinetic equation. Besides, in the gluon-glueball model used here only equilibrium values $m_a(T)$ are known and we are actually not able to find $\delta E_a[F]$.

The shear and bulk viscosities can be expressed through the variation of the energy-momentum tensor as follows:

$$\begin{aligned} \delta T_{ij} \ &= \ -\zeta \ \delta_{ij} \vec{\nabla}\cdot\vec{u} - \eta \ W_{ij}, \qquad (14) \\ \text{with} \quad W_{kl} \ &= \ \partial_k u_l + \partial_l u_k - \frac{2}{3}\delta_{kl} \ \partial_i u^i \ . \end{aligned}$$

Here and below Latin indices run $1,2,3$. To find the shear viscosity, we put $i \neq j$ in (14). To find the bulk viscosity, we substitute $i=j$ in (14) and use that $T^{ii}_{\rm loc.eq} = 3P_{\rm loc.eq.}$

Taking derivatives $\partial F_a^{\rm loc.eq.}/\partial x^\mu$ in Eq. (12) and using (14) as a definition of viscosity coefficients, by straightforward calculations we find expressions (see [22–25]) for the shear viscosity

$$\eta = \frac{1}{15T} \sum_a \int d\Gamma \, \tau_a \frac{\vec{p}_a^{\,4}}{E_a^2} \, F_a^{\rm eq} \ (1 \mp F_a^{\rm eq}) \qquad (15)$$

and for the bulk viscosity [42]

$$\zeta = -\frac{1}{3T} \sum_a \int d\Gamma \, \tau_a \ \frac{\vec{p}_a^{\,2}}{E_a} \, F_a^{\rm eq} \ (1 \mp F_a^{\rm eq}) \, Q_a, \qquad (16)$$

where the EoS-dependent Q_a factor is given by

$$Q_a = -\left\{ \frac{\vec{p}_a^{\,2}}{3E_a} - c_s^2 \left[E_a - T\frac{\partial E_a}{\partial T} \right] \right\} \qquad (17)$$

and $c_s^2 = \frac{\partial P}{\partial \epsilon}$ is the speed of sound squared.

Simplifying, instead of the momentum dependent value τ_a one may use the averaged partial relaxation time $\tilde{\tau}_a$ related to the cross section as

$$\tilde{\tau}_a^{-1}(T) = \sum_{a'} n_{a'}(T) \left\langle v_{aa'} \sigma^t_{aa'}(v_{aa'}) \right\rangle , \qquad (18)$$

where $n_{a'}$ is the particle density of a'-species, $\sigma^t_{aa'} = \int d\cos\theta \, d\sigma(aa' \to aa')/d\cos\theta \, (1-\cos\theta)$ is the transport cross section, in general, accounting for in-medium effects and $v_{aa'}$ is the relative velocity of two colliding particles a and a' in the case of binary collisions. Angular brackets denote a quantum mechanical statistical average over an equilibrated system. However one should bear in mind that averaged values $\tilde{\tau}_a^{-1}$ given by Eq. (18) yield only a rough estimate for the values τ_a^{-1}.

IV. RESULTS FOR VISCOSITIES

Below the shear and bulk viscosities are calculated with the help of Eqs. (15) and (16), respectively. The only

quantity which should be still specified is the relaxation time $\tilde{\tau}_a$.

Calculations of the relaxation time $\tilde{\tau}_a$ of partons already in the lowest order in the running coupling constant g^2 require summation of infinitely many diagrams. Resummation of the hard thermal loops results in the width $\tilde{\tau}^{-1}$ of partons $\sim g^2 T \ln(1/g)$ [26]. Based on this fact, the following parametrization was used for gluons [4, 27]

$$\tilde{\tau}_g^{-1} = N_c \, \frac{g^2 T}{4\pi} \, \ln \frac{2c}{g^2} \, , \qquad (19)$$

with the strong interaction coupling constant (2) and a tunning parameter c. The relaxation time for a mixture of scalar and tensor glueballs was estimated according to Eq. (18) assuming the glueball scattering cross section σ_{gb} =30 mb to be isotropic.

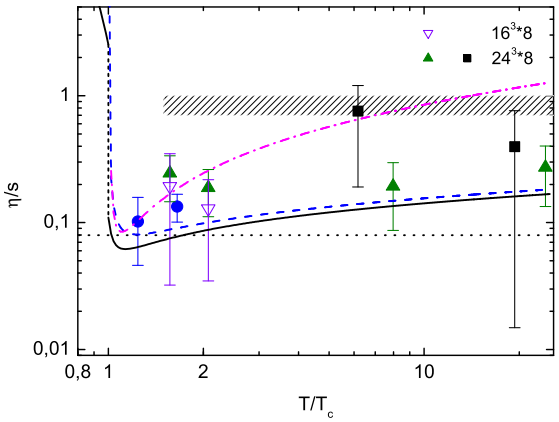

FIG. 3: The ratio of the shear viscosity to entropy density for a pure glue matter. Solid and dashed lines are for our QP two-phase gluon-glueball model with two different choices of the coefficient c in Eq. (19), see the text for a detail. The vertical short-dashed line joins two boundary points of the mixed gluon-glueball phase. Dot-dashed line shows viscosity calculations with the relaxation time $\tilde{\tau}_g$ given by Eq. (20). The horizontal dotted line is the $\eta/s = 1/4\pi$ bound. The lattice gauge $SU(3)$ data with $16^3 \cdot 8$ and $24^3 \cdot 8$ lattice are from Ref. [29] (triangles and squares) and [30] (filled circles). The shaded region corresponds to the perturbative result (cited from [28]).

Comparison between the GD lattice data [29, 30] and our QP results for the shear viscosity of the glue matter is presented in Fig. 3. The magnitude of the η/s ratio in our model is defined mainly by the value of the relaxation time (19). The solid line shows results of our calculation provided we use recommended value c =14.4, though our parameters of Eq. (2) are slightly different from those used in Ref. [4]. This c-value was tuned in Ref. [4] to the old lattice data for thermodynamic quantities [12]. As we see, the η/s ratio gets discontinuity at $T = T_c$ with more than by an order of magnitude lower value at $T \to T_c + 0$ (in gluon phase) than at $T \to T_c - 0$ (in the glueball phase). Also the solid curve lies reasonably close to the points (filled circles, triangles and squares) and its value in the minimum is slightly below the AdS/CFT $1/4\pi$ bound [31] (compare with the dotted curve). Preserving the form of the relaxation time (19) we can still increase η/s values by tuning the parameter c. Taking $c = 11.44$ we achieve the limit case $\tilde{\tau_g}^{-1} \to 0$ for $T \to T_c + 0$ (full transparency). In this case (see the dashed line in Fig. 3 for $T > T_c$) we may reach a slightly better overall agreement with the lattice data [29, 30], and the $1/4\pi$ bound is achieved at the minimum. Varying the c-value in the interval $11.44 < c < 14.4$ one may simulate different values of the η/s jump at $T = T_c$, but for temperatures $T \gtrsim 1.5T_c$ the η/s ratio changes only slightly demonstrating a slow increase with the growing temperature. Thus, bearing in mind large error bars in the lattice data we are able to conclude that the results of the developed here two-phase gluon-glueball model are consistent with the existing lattice results [29, 30]. Perturbative regime (see the shaded region) is not achieved up to very high temperatures.

The η/s ratio for the pure gluon phase in the range of $T \sim (1-2)T_c$ was also evaluated in Ref. [32]. The model employs the QP ansatz for EoS successfully tested to describe old lattice results [28]. In the paper [32] viscosity is treated by means of the kinetic theory for gluon quasiparticles. It is of interest that the model, being consistent with the old (and less accurate) lattice data for viscosity [28] and thermodynamics [20] which overestimate pressure at $T \gtrsim 1.5T_c$ (as follows from the comparison with new data, see Fig. 1), predicts a stronger temperature dependence of η/s at $T > T_c$ than our model, which in turn is consistent with the new lattice data [29, 30]. The crucial point here is that the gluon relaxation time is defined essentially differently:

$$\tilde{\tau}_{\rm BKR}^{-1} = a_\eta/(32\pi^2) T \; g^4 \; \log(a_\eta \pi/g^2) \, , \qquad (20)$$

where a_η =6.8. Here $\tilde{\tau}_{\rm BKR}^{-1} \propto g^4$, as it was estimated in early work of Hosoya and Kaiantie [34], whereas the above used $\tilde{\tau}_g^{-1} \propto g^2$ [4, 27].

The η/s ratio obtained with the relaxation time (20) is plotted in Fig. 3 by the dot-dashed line. This result for $T \lesssim 2T_c$ recovers that of Ref. [32] but it significantly differs from those calculated with Eq. (19). Using recent lattice results for higher T [29] it is possible to disentangle two parameterizations of relaxation times (19) and (20). Indeed, for $T \gtrsim 10T_c$ shear viscosity calculations with (20) demonstrate a noticeable growth exceeding lattice data and even a perturbative estimate $(\eta/s)_{\rm pert} \approx 0.8 - 1.0$. Contrary, predictions of our QP model with relaxation time (19) are in a reasonable agreement with the lattice results and do not contradict perturbative estimates. Recently the relaxation time $\tilde{\tau}_g$ was estimated in Ref. [33] according to Eq. (18) from analysis of cross sections of the $gg \to gg$ and $gg \to ggg$ processes. It was found that η/s=0.13 and 0.076 for values α_s =0.3 and 0.6, respectively (which correspond to temperatures T/T_c =2.6 and 1.36 provided Eq. (2) is used). Would these points be plotted in Fig. 3, they turned out to be quite consistent with our QP model results. This can be

considered as an additional numerical argument in favor of using Eq. (19) as rather appropriate phenomenological expression.

The measured lattice points for the ratio of the bulk viscosity to the entropy density are plotted in Fig. 4 together with different model results. In a broad range of temperatures the global behavior of lattice data can be roughly approximated as $\zeta/s = 0.02/\sqrt{T/T_c - 1}$ (see short dashed curve). The reduced bulk viscosity ζ/s calculated in our two-phase gluon-glueball model following Eqs. (16) and (19) is shown by the solid line for $c = 14.4$ and by the dashed line for $\tilde{\tau}_g^{-1}$ vanishing at T_c. Values of ζ/s for both curves noticeably underestimate the corresponding values on the approximating short-dashed curve, typically by an order of magnitude. Nevertheless the shape of the curves is similar to that given by the approximating curve. Singularity at $T \to T_c + 0$ demonstrated by the dashed line (see insertion in Fig. 4) is due to the divergence of $\tilde{\tau}_a$ in this limiting case.

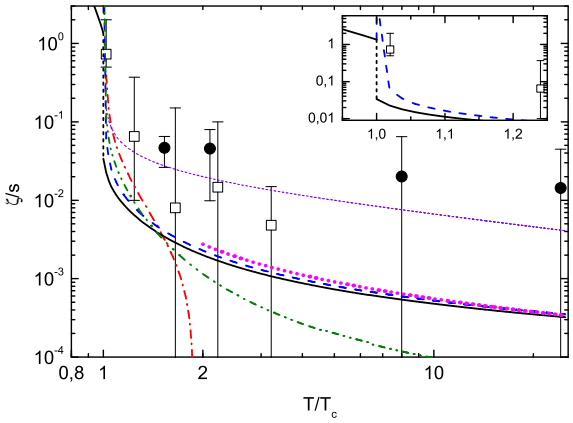

FIG. 4: The bulk viscosity to entropy density ratio for a glue matter. Solid and dashed lines are results of our QP two-phase gluon-glueball model with two relaxation times as in Fig. 3. (the vertical short-dashed line joins boundary points of the mixed phase). The dash-double-dotted line is the calculation result with Eqs. (22) and (19) and dot-dashed one is calculated according to Eq. (23) and (20). The perturbative estimate (24) is plotted by the dotted line. Experimental points are from [35] (empty squares) and [29] (filled circles). Thin short-dashed curve corresponds to a simple approximating dependence $\zeta/s = 0.02/\sqrt{T/T_c - 1}$ to guide the eye.

The bulk viscosity (16) includes rather complicated factor Q_a depending on the EoS used. Using the energy conservation for a system with temperature-independent masses of particles one may present the result (16) as follows:

$$\zeta = \sum_a \frac{d_a}{T} \int \frac{d^3p}{(2\pi)^3} \tau_a F_a^{eq}(1 \mp F_a^{eq}) \left[\frac{\vec{p}^2}{3E_a} - c_s^2 E_a \right]^2 \quad (21)$$

For a single-component gas this expression exactly coincides with the 25-years old result of Gavin [36].

Chakraborty and Kapusta [37] presented another expression [43]

$$\zeta_{\rm ChK} = \sum_a \frac{d_a}{T} \int \frac{d^3p}{(2\pi)^3} \bar{\tau}_a F_a^{eq}(1 \mp F_a^{eq}) Q_a^2 \quad (22)$$

which differs from (16) but also reduces to (21) for $m_a = const$. Note that they also disregard the QP interaction term in the energy-momentum tensor, see Eq. (99) of their work. The reasons of differences between (16) and (22) are discussed in Appendix. The dash-double-dotted line in Fig. 4 demonstrates the $\zeta_{\rm ChK}/s$ ratio following Eq. (22) with the relaxation time $\bar{\tau}_g = \tilde{\tau}_g$ given by Eq. (19) (for $\tilde{\tau}_g^{-1} \to 0$ at $T \to T_c + 0$). We see that Eq. (22) yields a strong T suppression of the bulk viscosity at $T \gtrsim 1.5T_c$, as compared to that given by Eq. (16) (compare dash-double-dotted and dashed lines in Fig. 4).

A somewhat different expression for ζ was used by Bluhm, Kämpfer and Redlich [32]. In their model the bag constant B is a functional of the non-equilibrium distribution function. They found

$$\zeta_{\rm BKR} = \sum_a \frac{d_a}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a}{E_a} F_a^{eq}(1 \mp F_a^{eq}) \times Q_a \left[m_a^2(T) - T \frac{dm_a^2(T)}{dT} \right] . \quad (23)$$

Here it was assumed that the QP interaction contributes to the energy-momentum tensor. Thereby compared to (16) there appeared the second term $T dm_a^2(T)/dT$ in the square bracket of Eq. (23). For constant masses the latter equation is also reduced to (21). Numerical calculations with (23) (see dot-dashed line in Fig. 4) give rise to the $\zeta_{\rm BKR}/s$ ratio which dramatically falls down, being in large discrepancy with both used above models and the lattice data for $T > 1.5T_c$. For $T > 1.9T_c$ Eq. (23) becomes invalid providing negative values.

A perturbative estimate [35] gives

$$(\zeta/s)_{\rm pert} \approx 0.02\alpha_s^2 \quad (24)$$

for $0.06 \le \alpha_s \le 0.3$. Applying T-dependent coupling constant (2) for $\alpha_s = g^2(T)/4\pi$ we get a perturbative estimate of the bulk viscosity to entropy density ratio (plotted by the dotted line in Fig. 4). As is seen, in the region of its applicability Eq. (24) produces only slightly larger values of ζ/s than those given by our QP model.

Above only the collisional source of the bulk viscosity has been considered. An another source is associated with a soft mode, being present near the second order phase transition point [44], see [9, 10]. The new lattice GD calculations demonstrate a significant increase of the ratio ζ/s at approaching the critical point ($\zeta/s \simeq 0.5 \div 2$ at $T = 1.02\ T_c$). These values are neither reproduced by our QP model using relaxation time Eq. (19) nor by the approximating short-dash curve exploiting a simple T dependence of the ζ/s ratio. Might be one needs to include a soft mode and other extra sources of the bulk viscosity to fit the lattice data on the bulk viscosity.

Comparing results presented in Fig. 3 and Fig. 4 we see that in the gluon phase in a narrow vicinity of the critical point (for $(T-T_c)/T_c \lesssim 0.1$) the ratio $\zeta/\eta \gtrsim 0.1$ reaching the value $\zeta/\eta \simeq 0.3$ for $T \to T_c + 0$. The ratio sharply decreases with increase of the temperature up to values $\zeta/\eta \sim 10^{-2} \div 10^{-3}$ for $T > 2T_c$. Smallness of this ratio controls the violation of the conformal symmetry.

V. CONCLUSIONS

A quasipartical approach has been applied to the $SU(3)$ glue matter with temperature-dependent masses. Matching the pure gluon and glueball phase descriptions by means of the Gibbs conditions allows one to describe successfully this system in a thermodynamically consistent way both above and below the critical temperature T_c. For thermodynamic characteristics the quasiparticle model results are in good agreement with the latest lattice data.

The constructed equation of state was used to calculate the shear and bulk viscosities in the relaxation time approximation in a wide temperature range. The magnitudes of the shear and bulk viscosities are mainly determined by the value of the relaxation time which in our case is evaluated in the hard thermal loop approximation. With the chosen value of the relaxation time the shear viscosity to entropy density ratio η/s fits rather well the scant lattice data. We found that the ratio η/s undergoes a discontinuity at the critical temperature $T = T_c$. At T slightly above T_c the ratio η/s has a minimum, which value is close to the AdS/CFT bound $1/4\pi$. Then η/s increases with subsequent growth of the temperature. The bulk viscosity to entropy density ratio ζ/s also has a break at T_c. Then it monotonically decreases with the temperature increase. Although the calculated ζ/s ratio essentially underestimates the upper limits given by the corresponding lattice data, its temperature dependence is well described.

Within our model the ratio $\zeta/\eta \simeq 0.3$ at $T \to T_c + 0$ and it sharply decreases with the rising temperature till values $\zeta/\eta \sim 10^{-2} \div 10^{-3}$ for $T > 2T_c$.

We point out that our QP model, including only the collisional source of the viscosity (with the relaxation time estimated within the hard thermal loop picture), disregards other possible sources [10]. One of such sources is associated with presence of a soft mode [9] in the vicinity of the second and weak first order phase transition critical points. Being included these sources could allow one to increase the resulting ζ/s ratio. However, since statistical error bars are very large, new more certain lattice data are required in order to draw a more definite conclusion on the agreement or disagreement of the calculated ζ/s ratio with the lattice results.

Acknowledgements

We are thankful to D. Blaschke, W. Cassing, M. Gorenstein, Yu. Ivanov, E. Kolomeitsev, N. Koshelev and V. Skokov for useful discussions. This work was supported in part by the DFG grant WA 431/8-1, the RFFI grants 08-02-01003-a, the Ukrainian-RFFI grant 09-02-90423-ukr-f-a and Heisenberg-Landau grant.

VI. APPENDIX

Deriving kinetic coefficients authors of [22–25] used the relaxation time approximation to the kinetic equation presenting collision integral as

$$StF_a = -\delta F_a/\tau_a[F^{\rm loc.eq.}], \tag{25}$$

where

$$\delta F_a = F_a(E_a[F]) - F_a^{\rm loc.eq.}(E_a[F^{\rm loc.eq.}]), \tag{26}$$

see e.g., Eqs. (38) and (40) in [25]. Here it was assumed that the collision term should be zero for the global and local equilibrium states, i.e. for $F_a = F_a^{\rm loc.eq.}(E_a[F^{\rm loc.eq.}])$.

Then after setting $F_a = F_a^{\rm loc.eq.}(E_a[F^{\rm loc.eq.}])$ in the l.h.s. of the kinetic equation one finds

$$\delta F_a = -\frac{\tau_a[F^{\rm loc.eq.}]}{E_a[F^{\rm loc.eq.}]} p_a^\mu \frac{\partial F_a^{\rm loc.eq.}(E_a[F^{\rm loc.eq.}])}{\partial x_a^\mu}, \tag{27}$$

see Eq. (2.3) in [22] and Eq. (42) in [25]. We stress that all quantities in the r.h.s. of this equation including the relaxation time τ are expressed in terms of the *local equilibrium distribution functions*.

To derive expressions for the shear and bulk viscosities (15), (16) one presents spatial components of the variation of the energy-momentum tensor of quasiparticles (10) as

$$\delta T^{ik} = \sum_a \int d\Gamma \frac{p_a^i p_a^k}{E_a[F]} \delta F_a - \sum_a \int d\Gamma \frac{p_a^i p_a^k F_a^{\rm loc.eq.}}{E_a^2[F^{\rm loc.eq.}]} \delta E_a$$
$$\to \sum_a \int d\Gamma \frac{p_a^i p_a^k}{E_a[F^{\rm loc.eq.}]} \delta F_a. \tag{28}$$

To avoid cumbersome expressions we omitted antiparticle terms. The reduction done in the second line in Eq. (28) is actually an ansatz: we vary only the distribution function and do not vary quantities which depend on the distribution function implicitly (through phase-space integrals incorporating the distribution function), i.e., $\delta E_a[\delta F]$ are put zero. This reduction is in spirit of the relaxation time approximation to the kinetic equation, where the momentum dependent relaxation time parameter is replaced in actual calculations by an averaged value. Note that dropping the δE term we actually ignore a sub-leading term in case of a weak coupling constant and/or for a very dilute system, see Eq. (5.22) of [38]. The distribution function δF_a counted from the local equilibrium value enters expression for δT^{ik}, expression for $\delta T^{00} = 0$ (see (45) and (50) in [25]) and expressions for the viscosities. Thus one can easily compute

kinetic coefficients knowing thermodynamic quantities in the local rest frame $\vec{u}=0$.

However we should note that in the QP Fermi liquid theory following the work of Abrikosov and Khalatnikov [39] one usually uses a different procedure to obtain transport coefficients, see [40] for detail. One exploits that in the original Landau collision term enters a combination

$$\delta\left(\sum_a E_a[F]\right)\cdot[F_1(E_1)F_2(E_2)(1\mp F_3(E_3))(1\mp F_4(E_4)) -F_3(E_3)F_4(E_4)(1\mp F_1(E_1))(1\mp F_2(E_2))], \quad (29)$$

where E_a are functionals of *the exact non-equilibrium distribution function*, $a=1,2,3,4$. The square bracketed term is zero not only for $F_a=F_a^{\rm loc.eq.}[E_a(F^{\rm loc.eq.})]$ but also for $F_a^{\rm loc.eq.}(E_a[F])$. Thereby $StF_a^{\rm loc.eq.}(E_a[F])=0$. Thus introducing

$$\delta\widetilde{F}_a=F_a(E_a[F])-F_a^{\rm loc.eq.}(E_a[F]) \quad (30)$$

in the relaxation time approximation we may rewrite the collision term as

$$StF_a=-\delta\widetilde{F}_a/\bar{\tau}_a[E(F)]. \quad (31)$$

The quantity $\bar{\tau}[F]$ entering Eqs. (31) and (22) depends on unknown exact non-equilibrium distribution function, since the δ-function term in the collision integral and the local equilibrium distributions there continue to depend on exact energies in this approach. If we want to calculate the value of $\tau[F^{\rm loc.eq.}]$ entering Eqs. (15), (16) using Eq. (31) we should still expand $E[F]$ in (31) near the known value $E[F^{\rm loc.eq.}]$ everywhere including the δ-function term in the collision integral.

From the l.h.s. of the kinetic equation one gets

$$\delta\widetilde{F}_a=-\frac{\bar{\tau}_a[F]}{E_a[F]}p_a^\mu\frac{\partial F_a^{\rm loc.eq.}(E_a[F])}{\partial x_a^\mu}. \quad (32)$$

Then one may use a simple expression for δT^{ik}

$$\delta T^{ik}=\sum_a\int d\Gamma\frac{p_a^ip_a^k}{E_a[F]}\delta\widetilde{F}_a \quad (33)$$

since now variations are everywhere performed at fixed E_a. As above the QP interaction term is omitted. Comparing second line of Eq. (28) and (33) we see that disregarding implicit dependence $E[\delta F]$ Refs [22–25] actually do not distinguish distributions δF and $\delta\widetilde{F}$.

Then in both considered approaches one uses exact relation $E_a[F]=\delta T^{00}/\delta F_a$, i.e. that the variation of the energy is determined through δF_a as

$$\begin{aligned}\delta T^{00} &= \sum_a\int d\Gamma E_a[F]\delta F_a \\ &= \sum_a\int d\Gamma E_a[F^{\rm loc.eq.}]\delta F_a+O((\delta F)^2). \quad (34)\end{aligned}$$

Following (26) and (30) we have

$$\delta F-\delta\widetilde{F}=\frac{\partial F^{\rm loc.eq.}(E[F^{\rm loc.eq.}])}{\partial E}\delta E. \quad (35)$$

Further instead of using a complicated implicit dependence $\delta E[\delta F]$ with δF given by Eq. (27), that would be fully correct procedure, Ref. [37] uses the ansatz relations (see Eq. (102) of that work)

$$\begin{aligned}\delta\widetilde{F} &= \exp\left\{-\frac{E[F^{\rm loc.eq.}]}{T[F^{\rm loc.eq.}]}\right\}\frac{E^{\rm loc.eq.}}{T^2_{\rm loc.eq.}}\delta T, \quad (36)\\ \delta E &= \frac{\delta T}{E}\frac{mdm}{dT}=\frac{\delta\widetilde{F}}{F}\frac{T^2}{E^2}\frac{mdm}{dT},\end{aligned}$$

which assume that the distribution function in non-equilibrium state has the form $F=e^{-E[F]/T[F]}$ in the Boltzmann limit $F\ll 1$. Thus although Ref. [37] distinguishes distributions δF and $\delta\widetilde{F}$, it uses very special relations (36) which might be incompatible with $\delta E(m[\delta\widetilde{F}])$ as it follows from Eqs. (32), (11).

In order to find bulk viscosity one further expresses $F_a=F_a^{\rm loc.eq.}(E_a[F])(1-A_a\partial_\rho u^\rho)$, see Ref. [37], and one observes that the shift of the solution $A(E)\to A(E)-bE$ generates new solutions of the Landau kinetic equation for arbitrary constant b. Then one chooses b to explicitly fulfill the Landau-Lifshitz condition $u_\mu\delta T^{\mu\nu}=0$. Note that this modification of the solution is quite not necessary provided one guarantees that the condition $\delta T^{00}=0$ holds in the local rest frame. We have checked that this condition is satisfied in our QP model.

Finally within this approach one arrives at the expression (22) for the bulk viscosity which is explicitly positive definite, whereas positive definiteness of Eq. (16) is not seen explicitly. However we stress once more that *all quantities in (22) still depend on exact energies while how the latter depend on unknown exact distribution functions is hidden.* Thus explicit positive definiteness of expression (22) for ζ presents actually only an apparent improvement. Any case, in order to use Eq. (22) in practical calculations, where only equilibrium quantities are known, one should replace $E[F]$ to $E[F^{\rm loc.eq.}]$.

Moreover, we should stress that values of the relaxation time in (16) and (22) are different. Since we do not perform complicated microscopic calculations of the relaxation time but only estimate its average value we actually cannot distinguish, which expression (16) or (22) is more preferable and may use both of them.

Note that Eq. (23) is derived for a different model, where the QP interaction contributes to the energy-momentum tensor. Also authors use different value for the relaxation time.

Thus different ansatze used in derivation of Eqs. (16), (22) and (23) lead to different values of the bulk viscosity, as it is shown in Fig. 4.

[1] E.V. Shuryak, Nucl. Phys. **A750**, 64 (2005).
[2] M. Gyulassy and L. McLerran, Nucl. Phys. **A750**, 30 (2005).
[3] U.W. Heinz, arXiv:nucl-th/0512051.
[4] A. Peshier and W. Cassing, Phys. Rev. Lett. **94**, 172301 (2005).
[5] J.I. Kapusta, arXiv:0809.3746.
[6] L. P. Csernai, J. I. Kapusta and L. D. McLerran, Phys. Rev. Lett. **97**, 152303 (2006).
[7] P. Zhuang, J. Hüfner, S. P. Klevansky and L. Neise, Phys. Rev. **D51**, 3728 (1995); P. Rehberg, S. P. Klevansky and J. Hüfner, Nucl. Phys. **A608**, 356 (1996).
[8] R.A. Lacey, N.N. Ajitanand, J.M. Alexander, P. Chung, J. Jia, A. Taranenko, P. Danielewicz, arXiv:0708.3512.
[9] L.I. Mandelstam and M.A. Leontovich, ZhETF **7**, 438 (1937).
[10] K. Paech and S. Pratt, Phys. Rev. **C74**, 014901 (2006).
[11] D. Kharzeev and K. Tuchin, JHEP **0809**, 093 (2008); F. Karsch, D. Kharzeev and K. Tuchin, Phys. Lett. **B663**, 217 (2008); K. Huebner, F. Karsch and C. Pica, Phys. Rev. **D78**, 094501 (2008).
[12] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lütgemeier and B. Petersson, Phys. Rev. Lett. **75**, 4169 (1995); Nucl. Phys. **B469**, 419 (1996).
[13] M. Panero, Phys. Rev. Lett. **103**, 232001 (2009); arXiv:0912.2448.
[14] M.I. Gorenstein and S.N. Yang, Phys. Rev. **D52**, 5206 (1995); V.V. Begun, M.I. Gorenstein, O.A. Mogilevsky, arXiv:1001.3139.
[15] E. Witten, Nucl. Phys. **B160**, 57 (1979).
[16] V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. **E18**, 1 (2009).
[17] F. Buisseret, EPJ **C68**, 473 (2010).
[18] N. Ishii, H. Suganuma and H. Matsufuru, Phys. Rev. **D66**, 014507 (2002); Phys. Rev. **D66**, 094506 (2002).
[19] D.E. Miller, Phys. Reps. **443**, 55 (2007).
[20] F. Karsch, *Simulating of quark-gluon plasma on lattice in Quark-Gluon Plasma*, edited by R.C. Hwa, 1989, World Scientific.
[21] H.B. Meyer, Phys. Rev. **D80**, 051502 (R) (2009).
[22] Ch. Sasaki and K. Redlich, Phys. Rev. **C79**, 055207 (2009).
[23] Ch. Sasaki and K. Redlich, Nucl. Phys. **A832**, 62 (2010).
[24] A.S. Khvorostukhin, V.D. Toneev and D.N. Voskresensky, arXiv:0912.2191 (Yad. Fiz. **74** in press).
[25] A.S. Khvorostukhin, V.D. Toneev and D.N. Voskresensky, Nucl. Phys. **A845**, 106 (2010).
[26] R.D. Pisarski, Phys. Rev. Lett. **63**, 1129 (1989); V.C. Lebedev and A.V. Smilga, Ann. Phys. (N.Y.) **202**, 229 (1990).
[27] A. Peshier, Phys. Rev. **D70**, 034016 (2004).
[28] A. Nakamura and S. Sakai, Phys. Rev. Lett. **94**, 072305 (2005).
[29] S. Sakai and A. Nakamura, **PoS LAT2007**, 221 (2007).
[30] H.B. Meyer, Phys. Rev. **D76**, 101701 (2007).
[31] P. Kovtun, T.D. Son and O.A. Starinets, JHEP **0310**, 064 (2003); Phys. Rev. **94**, 111601 (2005).
[32] M. Bluhm, B. Kämpfer and K. Redlich, Nucl. Phys. **A830**, 737c (2009).
[33] Z. Xu and C. Greiner, Phys. Rev. Lett. **100**, 172301 (2008).
[34] A. Hosoya and K. Kajantie, Nucl. Phys. **B250**, 666 (1985).
[35] H. B. Meyer, Phys. Rev. Lett. **100**, 162001 (2008).
[36] S. Gavin, Nucl. Phys. **A435**, 826 (1985).
[37] P. Chakraborty and J.I. Kapusta, arXiv:1006.0257.
[38] S. Jeon, Phys. Rev. **D52** (1995) 3591.
[39] A.A. Abrikosov and I.M. Khalatnikov, Rept. Progr. Phys. **22**, 329 (1959).
[40] G. Baym and Ch. Pethick, *Landau Fermi Liquid Theory: Concepts and Applications*, Ed. Willey and Sons, 1991.
[41] The contribution of the QP interaction to the variation of the energy-momentum is assumed to be small in the model under consideration and thereby it is disregarded.
[42] We have checked that the Landau-Lifshitz condition $\delta T^{00} = \sum_a \int d\Gamma \tau_a E_a Q_a = 0$ is satisfied in our QP model.
[43] Here we denote the relaxation time $\bar{\tau}_a$ to emphasize its difference from the quantity τ_a used in Eq. (16), see discussion in Appendix.
[44] The statement is also correct in the case of a weak first order phase transition.