K-mesic nuclei versus eta-mesic nuclei *

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The nuclear states of $\overline{K}$ and η are bound by a similar mechanism - the excitations of nucleons to $\Lambda(1405)$ and $N^*(1535)$ resonant states. The observed large differences in binding energies are understood in terms of separation of the involved energies and the resonance positions. The other experimental findings: broad $\overline{K}$-mesic and narrow η-mesic states are more difficult to understand. A phenomenological model for η-N interactions is used to explain the suppression of the η absorption in light nuclei.

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1. Introduction

The possibility of η-nuclear quasi-bound states was first discussed by Haider and Liu [1, 2] , when it was realized that the η-nucleon interaction is attractive. Nevertheless, the early (π,p) experiment looking for these effects was not conclusive [3]. The reasons for uncertainties in the interpretation of those results is a high background and apparently large widths of those states due to predominantly to the (η, π) conversion. Recent detailed calculations [4] indicate the states to be wide and difficult to extract experimentally.

The view that the widths of nuclear η states are large is widespread. Calculations [2] and especially those based on chiral models [4],[5], predict large widths. If the quasi-bound states exist, one expects these to be narrower, and easier to detect in the few-nucleon systems. Indeed, an indirect evidence was suggested by Wilkin [6], who interpreted a rapid slope of the $pd \to \eta^3$He low energy amplitude as a signal of a quasi-bound state. Later, a very strong three-body $d\eta$ correlations were found in measurements of the $np \to d\eta$ cross section in the threshold region [7]. Calculations indicate that the $d\eta$ system forms a virtual state [8]. The status of η^3He state is still unsettled.

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Some of these results have been superseded by new experiments:

• The final state interactions in the η^3He system obtained by COSY-ANKE determines large scattering length $A(\eta^3He) = \pm 10.7(\pm 0.8^{(+0.1)}_{(-0.5)}) + i1.5(\pm 2.6^{(+1.0)}_{(-0.9)})$ [9].

• The final state interactions in the η^4He system obtained by GEM determines scattering length $A(\eta^4He) = \pm 3.1(5) + i0.0(5)$ fm [10].

• The reaction $^{27}Al + p \rightarrow ^3He + p + \pi^- + X$ studied by COSY-GEM attributed X to the state of $\eta^{25}Mg$ nucleus. The energy $E_B \approx -13 - i5$ MeV was found [11].

These findings require rather weak absorption and contradict many theoretical calculations based on the single channel η-N or multiple nucleon absorption models. In this paper the latest Helsinki K-matrix model incorporating the η-N,π-N,γ-N channels is used [12]. It offers two characteristic features : large scattering length $a_{\eta N} = 0.91(6) + i0.27(2) fm$ and a rapid decrease of the absorptive scattering amplitude in the subthreshold region.

The essential point of this work is the observation that in the few nucleon systems the relevant η-N, scattering amplitude involves subthreshold energies in the η-N center of mass system. The quantity of interest is

$$T_{\eta N}(E_{cm} = -E_N - E_\eta - E_{recoil}) \tag{1}$$

where E_N and E_η are binding energies of a nucleon and the meson. To obtain the center of mass energy the recoil energy of the meson-nucleon pair with respect to the residual system has to be subtracted in the argument of $T_{\eta N}$. Such amplitudes are the standard input in the three-body Faddeev equations for the bound states and low energy scattering. There are other situations where the subthreshold amplitudes are appropriate: the interactions at nuclear surfaces and the tightly bound residual systems. To certain degree these situations are met in the states of η mesons bound to light nuclei.

The absorptive part Im $T_{\eta N}(E_{cm})$ determines the dominant part of the level widths. It is proportional to the phase space in the π-N decay channel given by the center of mass energy E_{cm}. Thus the argument E_{cm} given in Eq.(1) is proper, at least in the Im $T_{\eta N}(E_{cm})$, for a much wider class of systems.

Extension to subthreshold energies reduces absorption in the η systems. On the other hand there are additional effects which enhance the level width. One is the two nucleon absorption of the meson. In the eta case there is an experimental check on the related rate coming from the η formation in the two nucleon collisions. This rate is low [13]. Another effect is the multiple scattering in the π-N decay channel. It goes beyond the optical potential approach and it is known to be significant on the K-mesic nuclei

[14, 15]. These questions are discussed in the main body of this paper with the special reference to the three recent experimental results.

In conclusion: (1) there are models of η-N interactions which generate fairly narrow η states in light nuclei. (2) Some systems in particular the η^3He are difficult to calculate precisely. Due to the apparently large scattering length all secondary effects become significant. (3) The nuclear states of $\overline{K}$ mesons indicate a need for explicit description of the decay channels. This question should be approached also in the η meson case.

2. Subthreshold eta-nucleon scattering amplitude

The latest Helsinki K-matrix model incorporates the η-N, π-N, γ-N channels. It is presented in refs. [12] and only the main points are indicated here. The scattering data are parameterized in terms of a phenomenological $K_{i,j}$ - a matrix in the channel indices i, j. Next, linear equations for the scattering matrix T

$$T_{i,j} = K_{i,j} + i\Sigma_m \; K_{i,m} \; Q_m \; T_{m,j} \tag{2}$$

are solved with Q_m being the diagonal matrix of the CM momenta in each channel. The energy region of interest for the few body eta physics spans from about 40 MeV below the eta-nucleon threshold to some 20 MeV above it. This region is dominated, in both channels, by the $N^*(1535)$. The model used here supposes this state to be determined by some short range interactions. Next, this state is coupled to the channel states which change its properties. However, to obtain a better restriction of the parameters the region of the K matrix description is extended to about 200 MeV below and above the threshold. So, the higher N(1650) resonance is also included.

The K matrix is parameterized as

$$K_{i,j} = \Sigma \; \frac{\sqrt{\gamma_i \gamma_j}}{E_o - E} + B_{i,j} \tag{3}$$

where the sum extends over two resonances represented by the pole terms. The γ_i couple these to the channels. The additional background matrix B_{ij} describes other forms of the interactions. These change the bare resonance energies E_o to those observed in the scattering experiments. The free parameters γ_π, E_o, B_{ij} obtained by the best fit to the data may be found in refs. [12]. One obtains several sets depending on the choice of the input data. The best result for the elastic η-N amplitude is plotted in Fig.1. It is only marginally better than other possibilities. The main difference happens in the value of $B_{\eta N,\eta N}$ reflected in different strength of the spike in Re $T_{\eta,N}$ at the threshold i.e. the scattering length. On the other hand the absorptive part Im $a_{\eta,N}$ stays close to the "canonical" value of 0.27(2) fm. The rapid decrease of Im $T_{\eta,N}$ occurs in all solutions.

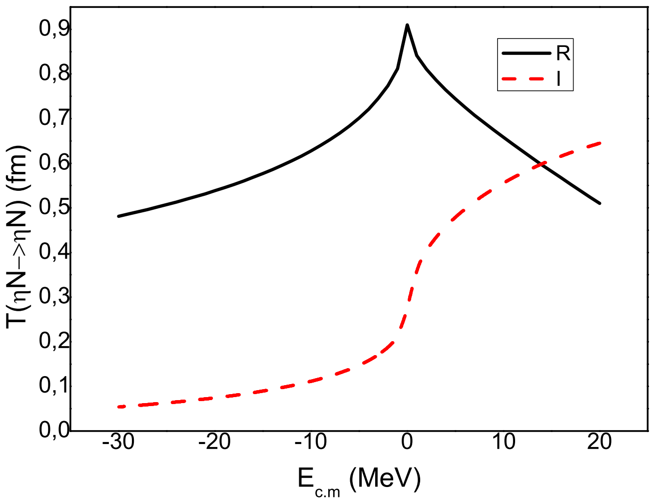

Fig. 1. The elastic η-N scattering amplitude plotted against the C.M. kinetic energy. Real part - continuous line, absorptive part - dashed line. This amplitude is well represented by the effective range expansion $T^{-1}_{\eta,N} + iq_\eta = 1/a + r_0/2q_\eta^2 + sq_\eta^4$, – with q_η being the momentum in the ηN center-of-mass and $a = 0.91(6) + i0.27(2), r_0 = -1.33(15) - i0.30(2), s = -0.15(1) - i0.04(1)$. All in *fermi* units.

3. The absorption of η-mesons in light nuclei

3.1. Helium

In this section the η-He scattering lengths are calculated. The energy argument entering eq.(1) in ^{4}He is determined by the nucleon separation $E_N \approx -21$ MeV and $E_\eta = 0$. The meson-nucleon recoil energy is a function of total pair momentum P and the momentum distribution $f(P)$ is calculable with the meson and nucleon wave functions

$$f(P) = \int d\mathbf{r}\ \phi_N(r)\phi_\eta(r)\ \exp(i\mathbf{Pr}), \tag{4}$$

where $\mathbf{r}$ is the coordinate relative to the tritium (or ^{3}He) core. The $f(P)^2$ is peaked around the average momentum and the average recoil is given by $E_{recoil} =< P^2 > /(2M_{\eta N,R})$ where $M_{\eta N,R}$ is the corresponding reduced mass. For low energy mesons one has $E_\eta \approx 0$, $\phi_\eta(r) \approx const$ and $E_{recoil} =$ 16 MeV. The average subthreshold energy is $E_{cm} \approx -37$ MeV and the amplitude becomes $T_{\eta,N}(E_{cm}) = 0.45 + i0.051$. For the other isotope ^{3}He the corresponding values are $E_N \approx -7$, $E_{recoil} = 12$, $E_{cm} \approx -19$ MeV and $T_{\eta,N}(E_{cm}) = 0.54 + i0.077$ fm. The width of momentum distribution is about 10 MeV and inspection of Fig.1 indicates that the $T_{\eta,N}(E_{cm})$ is fairly constant in the regions of interest.

These effective scattering matrices $T_{\eta,N}(E_{cm})$ are now used to calculate the $A(\eta, He)$ scattering lengths. The η-He multiple scattering series is summed according to the prescription of ref.[17] equivalent to the calculation in terms of the optical potential. The latter is given in the standard way

$$V_N(r) = -\frac{2\pi}{\mu_{\eta N}}\ T_{\eta N}(E_{cm})\ \rho(r), \tag{5}$$

where ρ is the nuclear density, μ is the η-N reduced mass and the index N on V_N indicates the single nucleon origin of this potential.

• For ^{4}He this input generates $A(\eta,^4 He) = -2.90 + i0.35$ fm, which corresponds to a quasi-bound state of energy $E \approx -6$ MeV and width $\Gamma \approx$ 3.0 meV. This compares well with the experimental $A(\eta,^4 He) = \pm 3.1(5) + i0.0(5)$fm, [10].

• In ^{3}He both the theoretical and experimental situation is uncertain as apparently the singularity in the η^3He scattering matrix is close to the threshold. On the experimental side COSY-11 [16] obtains $|A(\eta,^3 He)| =$ 4.3(5) fm. This result is consistent with the phenomenological $A(\eta^3 He) =$ $4.24(29) + i0.72(81)$ fm based on older data [18]. In the latter case, the inclusion of (π, η) data allowed to establish the sign of the real part which signals a virtual state. These lengths indicate that the related singularity of the $T(\eta^3 He)$ matrix is located in the complex energy plane some 1.5-2 MeV away from the threshold.

On the other hand the COSY-ANKE solution is $A_{\eta,N} = \pm 10.7(\pm 0.8^{(+0.1)}_{(-0.5)}) + i1.5(\pm 2.6^{(+1.0)}_{(-0.9)})$, [9]. This value indicate the pole to be only 0.3 MeV away from the threshold. To obtain it one needs a strong suppression of the meson absorption.

The photo-production result [19] indicates a quasi-bound state of energy $E = -4.4(4.2) - i12.8(3.1)$ which corresponds to a much smaller scattering length.

•• Calculations of large scattering lengths are unstable. Indeed, with equation (5) and the effective $T_{\eta,N}(E_{cm}) = 0.54 + i0.077$ obtained from fig1. one obtains a large length $A_{\eta,N} = -6.2 + i2.8$ fm. However, a simple correction introduced to the multiple scattering series, the replacement of A^2 by $A(A-1)$ in the double scattering term, changes the result to $A_{\eta,N} =$ $7.3 + i7.7$ fm. This shows the outcome to be unstable against second order effects. A better calculating techniques are also required.

3.2. Magnesium

In this section a crude estimate of the η mesic level in ^{25}Mg is given. With the characteristic value of Re $T_{\eta,N}(E_{cm}) = 0.52$ fm the optical potential generates the η binding of about 18 MeV. The radius m.s. of the meson density distribution becomes 3.2 fm comparable to the charge density radius of 3.11 fm [20]. Thus the meson is mostly located in the region beyond the half density radius which in this nucleus is 2.76 fm. The nucleon separation energies are determined mostly by the upper single particle levels with an average about 15-20 MeV. Following Fig.1 one obtains $T_{\eta,N}(E_{cm}) \approx 0.52 + i0.07$ fm. This generates narrow width $\Gamma \approx 6.0$ MeV.

One concludes that the suppression of the level widths can be understood at least on the semi-quantitative level. However, the binding offered by the K-matrix model seems to be excessive. In addition a number of higher order effects must be included:

1) Two nucleon ηNN capture. There is an experimental check on this effect to be discussed below. It adds some 1-2 MeV to the width.

2) Interactions in the decay channel

3) Nuclear medium effects change $T_{\eta,N}(E_{cm})$. In the light nuclei these are hard to calculate.

4. Other absorption modes

The η meson lifetime in a nucleus is determined by the basic reactions

$$\eta N \to \pi N \tag{6}$$

$$\eta N \to \pi\pi N \tag{7}$$

$$\eta(NN)^0 \to NN \tag{8}$$

$$\eta(NN)^1 \to NN. \tag{9}$$

Where the superfix denotes the spin of NN pairs. The first process is known fairly well, the second one is usually included into absorptive $T_{\eta,N}$ amplitude due to the two pion decay of the $N^*(1535)$.

The other two reactions (8) and (9) correspond to η absorption on two correlated NN pairs in either the spin singlet or spin triplet states. A phenomenological evaluation of the rates is possible as the cross sections for

$$pp \to pp\eta \tag{10}$$

$$pn \to d\eta \tag{11}$$

$$pn \to pn\eta \tag{12}$$

have been measured in the close to threshold region. The analysis based on the detailed balance corrected for final state interaction has been performed in ref. [13]. Absorptive potential of the $\rho(r)^2$ profile with a weak strength Im $W_{NN}(r = 0) = 3.2$ MeV was obtained.

An additional absorption mode exists if the decay channel is described explicitly. It has been studied in terms of Faddeev equations used to calculate the $\bar{\mathrm{K}}$ NN quasi-bound state energy [14]. An explicit treatment of the multiple scattering in the decay channel generates an additional binding and enlarges the width of the state. Similar effects are found in a variational calculations of the $\bar{\mathrm{K}}$- few- N levels [15].

4.1. Interactions in the decay channels

A simple model of the $\bar{\mathrm{K}}$ interacting with two fixed nucleons is used to explain the effect (a finer presentation may be found in ref.[15]). Consider scattering of the meson bound to two nucleons fixed at a separation $\mathbf{r}$. Let the amplitudes of the meson at each nucleon be ψ_1, ψ_2. The meson bounces off each nucleon and the multiple scattering equations are

$$\psi_1 + t\ G\ \psi_2 = 0, \qquad \psi_2 + t\ G\ \psi_1 = 0, \tag{13}$$

where t is the meson-nucleon scattering matrix and G is the propagator for the meson passing from one to the other nucleon

$$G = G(p, r) = \frac{1}{r}\ \exp(ipr) \tag{14}$$

One needs to regularize G at short ranges but for simplicity of the presentation this is suppressed. The consistency between the scattering and the bound state requires vanishing of the determinant

$$D = 1 - (t\ G)^2 = (1 + tG)(1 - tG) = 0. \tag{15}$$

This condition determines the *complex* eigen-momentum $p(r)$ which gives the energy and the width of the meson + fixed-NN system.

If the $\bar{\mathrm{K}}$N interaction is dominated by a resonance below the threshold, such as $\Lambda(1405)$, then $t = \gamma^2/(E - E_o + i\Gamma/2)$, where γ is a coupling constant and $E_o - i\Gamma/2$ is the $\Lambda(1405)$ complex energy. The solution of eigenvalue equation, $1 + tG = 0$, takes the form

$$E = E_o - i\Gamma/2 - \gamma^2 G(r, p). \tag{16}$$

The solution $E(r) \equiv E_B(r) - i\Gamma(r)/2$ depends on the N-N separation r. Since Re $G(r, p)$ close to the resonance is positive, the binding of $\bar{\mathrm{K}}$ to fixed two nucleons is stronger than the $\bar{\mathrm{K}}$ binding to one nucleon, $\mid E_B(r) \mid > \mid E_o \mid$. Increasing the separation $r \to \infty$ one obtains $G \to 0$ and $E(r) \to E_o$, i.e. the $\bar{\mathrm{K}}$ meson becomes bound to one of the nucleons. In the same limit the lifetime of $\bar{\mathrm{K}}$ becomes equal to the lifetime of $\Lambda(1405)$. Hence, the separation energy is understood here as the energy needed to split the $\bar{\mathrm{K}}$-N-N system into the $\Lambda(1405)$-N system. The last term in Eq.(16) constitutes a potential contracting the two nucleons. It is very strong and leads to large 50-100 MeV bindings of the system and the widths in the range of 40-80 MeV. The next step in the calculation (not presented) is to allow the nucleon degrees of freedom and use these results in a variational procedure.

The decay channel $\Sigma\pi$ coupled to the basic $\bar{\mathrm{K}}$ N channel may be introduced explicitly. The wave function has two components, one related to the $\bar{\mathrm{K}}$ N the other to the $\Sigma\pi$ channel. The scattering amplitudes are two dimensional vectors $\psi_i \to [\psi_i^K,\ \psi_i^\pi]$ at each nucleon. Now t becomes a matrix in two channel indices $t_{a,b} = \gamma_a \gamma_b\ /\ (E - E_o + i\Gamma/2)$, where $a, b = K, \pi$, and below the threshold $\Gamma/2 = (\gamma_\pi)^2\ p_\pi$. The multiple scattering equations are changed accordingly and the binding energy

$$Re\ E = E_o - (\gamma_K)^2 \frac{cos(p_R r)}{r} \exp(-p_I r) - (\gamma_\pi)^2 \frac{cos(p_\pi r)}{r} \tag{17}$$

becomes larger than the binding of the resonance but the collisions in the decay channel induce oscillations. This oscillatory behavior is also seen in the width of the system

$$Im\ E = -(\gamma_\pi)^2\ p_\pi\ [1 + \frac{sin(p_\pi r)}{p_\pi r}] - (\gamma_K)^2 \frac{sin(p_R r)}{r} \exp(-p_I r). \tag{18}$$

The contribution from multiple scattering in the decay channel is sizable in general but it oscillates and may under some conditions reduce the total width. That is an effect of interference in the decay channel. In the $\bar{\mathrm{K}}$ NN case the scattering in the decay channel turns out to be constructive and leads to about 25% stronger binding and larger widths.

Unfortunately, in the η meson case this method cannot be used as the $N^*(1535)$ is located above the η-N threshold. The solutions given above exist for N-N distances less than a critical value R_c. In the case of ^{4}He one has $R_c \approx 1.5$fm and the variational method of ref.[15] seems applicable. In the most interesting ^{3}He case it is not. Other methods should be tried as the effects might be sizable.

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