Magnetars as Highly Magnetized Quark Stars: an analytical treatment

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ABSTRACT

We present an analytical model of a magnetar as a high density magnetized quark bag. The effect of strong magnetic fields ($B > 5 \times 10^{16}$ G) in the equation of state is considered. An analytic expression for the Mass-Radius relationship is found from the energy variational principle in general relativity. Our results are compared with observational evidences of possible quark and/or hybrid stars as well as with numerical results.

1. Introduction

The fundamental aspects of the physics involved in the description of the matter inside a white dwarf are well understood (Shapiro & Teukolsky 1983), but in the case of neutron stars the situation is rather different because the equation of state (EoS) of neutron matter at very high densities is still unknown.

The interior of a neutron star is an astrophysical laboratory in which matter is compressed to high densities. The compression of matter several times the saturation nuclear matter density, ρ_0, may produce a phase transition from nuclear to quark matter, *i.e.*, an unconfined quark-gluon plasma. Besides, under suitable circumstances, a conversion $d \to s$ quarks may happen through weak interactions, leading to what has been called strange quark matter (SQM). It has been stated that SQM may be the absolute ground state of strong interactions (Bodmer 1971; Witten 1984), although such hypothesis has not been confirmed yet. The natural scenario where SQM could occur is the inner core of neutron stars. Hence,

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if the SQM hypothesis is true, some neutron stars could be either quark stars or hybrid stars (stars which have quark cores surrounded by a hadronic shell).

On the other hand, it is well known that at the surface of neutron stars there exist magnetic fields of the order of $10^{12} - 10^{13}$ G. Compact stars with ultra strong magnetic fields ($10^2 - 10^3$ larger than those of a typical neutron star) are called magnetars. In such objects the magnetic field at the surface could be higher than 10^{18} G (de la Incera 2009).

The knowledge of the magnetars composition would help explain some astrophysical phenomena. Soft gamma ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) have been interpreted as evidence of magnetars. However some authors (Cheng & Daib 2002; Ouyed et al. 2007) claims that hybrid or magnetized quark stars could be the real sources of SGRs and AXPs. The M-R relationship tells us how matter composing the star behaves under compression, providing information about its composition. Several EoS for neutron, hybrid and quark stars have been proposed but none of them are conclusive (Douchin & Haensel 2001; Lattimer & Prakash 2001, 2007; Özel & Psaltis 2009). Each EoS produces a different Mass-Radius (M-R) relationship which can be contrasted with the available observational data in order to test their range of validity and/or set bounds on some parameters. At this particular point astrophysical studies become of great importance since they could shed some light in understanding fundamental aspects of matter: microphysics could be inferred from macrophysics. Here lies the great importance of studies related to ultra compact objects. For instance Lattimer & Prakash (2001, 2007) contrast some M-R relationships obtained theoretically for different EoS. Varying some parameters a difference of $4-10\%$ and $10-15\%$ in determining the maximum radius R_{max} and mass M_{max}, respectively, is shown for the same EoS.

Several papers study the M-R relationship of highly magnetized quark stars (HMQS) (Chakrabarty & Sahu 1996; González Felipe & Pérez Martínez 2009; Pérez Martínez et. al. 2010) through numerical integration of the Tolman-Oppenheimer-Volkoff (TOV) equations for different EoS. Although most studies of quark stars properties have used such method, Banerjee et. al (2000) had obtained a maximum mass and radius for unmagnetized quark stars analytically by using a non-relativistic gravitational treatment.

Approximate analytical solutions play an important role in astrophysical analysis, giving keener insight than the numerical solutions. Moreover, they may be used as a testing point to check if the numerical scheme is accurate and also they are the first step in the comparison between theory and observation. Indeed, an approximate analytical solution for M-R relationship may be all that is required when comparisons with observational limits that determine the confidence contour for the mass and radius are performed. Besides, in the high density EoS the uncertainties are of the same order, or larger, that the errors in the variational method.

The appropriate treatment for quark stars should be relativistic, since the existence of a maximum mass is associated to the behavior of a relativistic gas and general relativistic corrections are dominant (Weinberg 1972). In this paper we shall use the general relativistic energy variational principle described by Naurenberg & Chapline (1973) to obtain an analytic approximate formulae for the mass, radius and baryonic number of a highly magnetized quark star (HMQS). Quark stars are particularly suitable for a variational treatment since their density profile resembles a constant mass density star. We shall model a HMQS assuming quark matter within high density regime in the framework of a modified MIT Bag model EoS. We also assume that the magnetic field B is low enough to be treated like a correction in the EoS ($B << \mu^2$, with μ the baryon chemical potential). Although, as we will see in the following Sections, this is not a strong restriction.

The paper is organized as follow. In Section 2 we calculate the thermodynamical quantities of the system and we analyze the stability of quark matter with respect to decomposition in baryons. In Section 3 we provide the analytic relativistic M-R relationship and we compare our results with numeric ones from TOV equation (Pérez Martínez et. al. 2010) and observational data. We also determine the adiabatic index and the speed of sound analyzing the dynamical stability of the star. In Section 4 we present a summary of our main results and conclusions.

2. High density quark matter within a strong magnetic field

In this section we shall discuss the analytic approximations to the strange quark matter EoS in the presence of an uniform magnetic field $\boldsymbol{B} \parallel \hat{\boldsymbol{z}}$. Within the framework of the MIT Bag model, we assume three massless quarks u, d and s, neglecting mediated interactions between them. We also consider that the strong magnetic field is a small contribution to the total energy, a fact that will be checked later.

2.1. Quark matter in a magnetic field

Let us compute the grand canonical thermodynamic potential Ω in the high density regime. Due the Landau quantization the phase space volume integral in the momentum space is replaced by

$$\frac{1}{(2\pi)^3}\int d^3pf(p) = \frac{1}{(2\pi)^3}\int dp_z d^2p_\perp f(p) = \frac{qB}{4\pi^2}\sum_{\nu=0}^{\nu=\infty}(2-\delta_{\nu 0})\int_{-\infty}^{+\infty} dp_z f(\nu, p_z),$$

where $(2-\delta_{\nu 0})$ means that the zeroth Landau level is singly degenerate, whereas all other states are doubly degenerate. The grand canonical potential for each quark in the presence of a strong magnetic field is given by

$$\Omega_i = -\frac{q_i B g_i}{8\pi^2}\sum_{\nu=0}^{\nu_{max}}(2-\delta_{\nu 0})\left[\mu\sqrt{\mu^2-2\nu q_i B}-2\nu q_i B\ln\frac{\mu+\sqrt{\mu^2-2\nu q_i B}}{\sqrt{2\nu q_i B}}\right],$$

where $g_i = 2\times 3$, are spin and color degeneracy, and q_i is the absolute value of the charge of the particle, $q_u = 2|e|/3$ and $q_d = q_s = |e|/3$, with e the value of electronic charge.

For simplicity, we consider the quark masses $m_q = 0$, which implies that the electrons are not present and quarks chemical potential are, as a consequence of equilibrium conditions, all equal $\mu_u = \mu_d = \mu_s \equiv \mu$.

By imposing that

$$p_z^2 = \mu^2 - 2\nu q_i B \geq 0,$$

we can determine the upper limit of the sum ν_{max} from

$$\nu \leq \frac{\mu^2}{2q_i B} \equiv \nu_{max}.$$

Note that the presence of the magnetic field will affect the EoS only by a correction term, even considering magnetic fields of the order of $B \approx 10^{18}$ G because the magnetic energy density is small compared with the MIT Bag constant, B, $B^2 \ll \mathsf{B}$. Besides, in the high density regime we have $\mu^2 \gg q_i B$. Thus, the magnetic energy contribution may be treated as a perturbation.

The series can be summed with the Euler-MacLaurin formula

$$\sum_{j=0}^{n} f(j) = \int_0^n f(x)dx + \frac{1}{2}[f(n)+f(0)] + \frac{1}{12}[f'(n)-f'(0)] + R, \tag{1}$$

where R is the remainder term, usually expressed in terms of periodic Bernoulli polynomials (Spivey 2006), which can be estimated using

$$|R| \leq \frac{1}{12}\int_1^{\nu_{max}-1}\left|f''(\nu)\right|\, d\nu.$$

To avoid divergences that appear in the the limit of high densities or negligible quark masses, in the third term of equation (1) we apply the Euler-MacLaurin formula in the form

$$\begin{aligned}\Omega_i \simeq\ & \Omega_i(\nu_{max}) + \Omega_i(0) + \int_1^{\nu_{max}-1}\Omega_i(\nu)d\nu + \frac{1}{2}\left[\Omega_i(\nu_{max}-1)+\Omega_i(1)\right] \\ & + \frac{1}{12}\left[\partial_\nu \Omega_i\,|_{(\nu_{max}-1)} - \partial_\nu \Omega_i\,|_{(1)}\right] + R \end{aligned} \tag{2}$$

In the limit $\mu^2 \gg q_i B$ the thermodynamical potential can be found performing first the integral in equation (2) and then expanding in power series of B. The result is

$$\Omega_i = -\frac{\mu^4}{4\pi^2} + \frac{3\mu^2 q_i\, B}{2\pi^2} + \mathcal{O}(B^2). \tag{3}$$

Note that equation (3) has a linear dependence on the magnetic field B instead of a quadratic one. SQM behaves like a ferromagnet in the $B^2 \ll \mathsf{B}$ limit. The particle density $n_i = -\frac{\partial \Omega_i}{\partial \mu}$ is

$$n_i = \frac{\mu^3}{\pi^2} - \frac{3\mu\, q_i\, B}{\pi^2} + \mathcal{O}(B^2). \tag{4}$$

The remainder, including terms of order B^2 obtained by integrating equation (2) gives $R \leq 2\%$. Note that when $B = 0$ in equations (3, 4) we recover the usual expressions for a non-interacting massless quark gas at zero temperature and zero magnetic field.

2.2. Equation of state

With the above results, one can form the modified EoS of SQM in the MIT Bag model. Within this framework, the difference between the energy density of the perturbative and non-perturbative QCD vacuum is taken into account by the "bag constant" B. The charge neutrality condition

$$2n_u = n_d + n_s, \tag{5}$$

and the β-equilibrium condition

$$\mu_u = \mu_d = \mu_s \equiv \mu$$

are automatically satisfied.

Combining the results of Section 2 we obtain

$$\Omega = \sum_{i=u,d,s} \Omega_i + \mathsf{B} = -\frac{3\mu^4}{4\pi^2} + \frac{2B\mu^2}{\pi^2} + \mathsf{B} + \mathcal{O}(B^2).$$

Replacing equation (5) in the baryon number density condition, $n_B = \frac{1}{3}\sum_{i=u,d,s} n_i$, we obtain

$$n_B = \frac{\mu^3}{\pi^2} - \frac{2B\mu}{\pi^2} + \mathcal{O}(B^2). \tag{6}$$

Since we work in the $T = 0$ limit, the energy density is given by

$$\rho = \Omega + 3\mu n_B + \mathsf{B} = \frac{9\mu^4}{4\pi^2} - \frac{4B\mu^2}{\pi^2} + \mathsf{B} + \mathcal{O}(B^2), \tag{7}$$

whereas the pressure reads

$$P = -\Omega = \frac{3\mu^4}{4\pi^2} - \frac{2B\mu^2}{\pi^2} - \mathsf{B} + \mathcal{O}(B^2). \tag{8}$$

Note that we are not considering the anisotropy of pressures (González Felipe et al. 2008) because we are working in the limit of weak magnetic field, $\mu^2 \gg q_i B$. The relation between the total energy density (equation (7)) and the total pressure (equation (8)) determines the EoS of the system as

$$\rho = 3P + 4\mathsf{B} + \frac{2\mu^2 B}{\pi^2} + \mathcal{O}(B^2). \tag{9}$$

By a dimensional comparison between P_{mag} and $\mathsf{B} \approx (145\,\mathrm{MeV})^4 = 57\frac{\mathrm{MeV}}{\mathrm{fm}^3}$, we find that $P_{mag} \simeq 0.03\mathsf{B}$, when $B = 5\times 10^{18}$ G, typical of a magnetar. This guarantees the perturbative treatment method on the magnetic field.

2.3. Stability analysis: Strong Interactions

It is well known that SQM may be stable with respect to decay into nucleons at zero pressure and zero temperature if its energy per baryon $\frac{\rho}{n_B}$ is less than the energy per baryon of $^{56}\mathrm{Fe} = 930\,\mathrm{MeV}$ (Farhi & Jaffe 1984). The presence of a magnetic field changes somewhat this stability condition.

At $P = 0$ the chemical potential can be written as

$$\mu(B, \mathsf{B}) = \left[\frac{4B}{3} + \frac{2}{3}\left(4B^2 + 3\pi^2\mathsf{B}\right)^{1/2}\right]^{1/2},$$

which will be replaced in equations (6, 9) to estimate $\frac{\rho}{n_B}$. Contrary to previous results (Anand & Singh 1999; Chakrabarty 1999; González Felipe & Pérez Martínez 2009) we found that the energy per baryon increases with B (equation (9)).

There is a closely linked relationship between B and B: the B value determines an upper limit for the magnetic field to preserve the quark matter stability condition. Table 1 shows that as B increases the region for quark matter stability becomes more restricted. The variation of baryon density is quite small for $P = 0$ when increasing the magnetic field from 0 up to B_{max} (Table 1). Furthermore the baryon density becomes almost constant as B increases.

3. Mass-Radius Relationship by Variational Method

The energy variational method in general relativity is explained in detail in Harrison et al. (1965). Starting from an uniform density configuration in a spherically symmetric distribution the total mass M, the baryon number N_B and the radius R of the star are given by

$$\begin{aligned} M &= \frac{4}{3}\pi\rho R^3, \\ N_B &= 2\pi n_B a^3(\chi - \sin\chi\cos\chi), \\ R &= a\sin\chi \end{aligned}$$

where ρ is the mass-energy density and the angle χ comes from substituting $r = a\sin\chi$ where $a = [3/(8\pi\rho)]^{1/2}$ is the curvature radius in the metric inside the star which adopts the following form for the 3-geometry:

$$ds^2 = a^2\left[d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right].$$

Note that we are using natural units, $\hbar = c = G = 1$. The configuration of maximum density is achieved when $\chi = \pi/2$. By recognizing that $\sin^2\chi = 2M/R$, $\chi \sim 0$ corresponds to the Newtonian limit while $\chi = \pi/2$ corresponds to the Schwarzschild one.

To obtain the equilibrium condition is appropriate to treat χ as an independent variable. By doing $\partial M/\partial\chi = 0$ for fixed N_B, the equilibrium condition reads

$$w \equiv \frac{P}{\rho} = \zeta(\chi),$$

where ρ and P are given by equations (7, 8) and $\zeta(\chi)$ is a function independent of the EoS

$$\zeta(\chi) = 3\,\cos\chi\left(\frac{9}{2}\,\cos\chi - \frac{\sin^3\chi}{\chi - \sin\chi\cos\chi}\right)^{-1} - 1.$$

We get an approximate value of $\zeta(\chi)$ using a Taylor series, ζ_T, around $\chi = 0$. Truncation at eighth order gives

$$\zeta_T = \frac{1}{10}\chi^2 + \frac{113}{2100}\chi^4 + \frac{1747}{63000}\chi^6 + \frac{689687}{48510000}\chi^8.$$

A Padé approximant of order $(4,4)$ gives a better representation of this function than the Taylor series truncated at eighth order. Moreover, the advantage to apply Padé approximant is to obtain an approximate analytic continuation beyond the circle of convergence. Thus $\zeta(\chi)$ is given as a ratio of two polynomials as

$$\zeta_P = \left(-\frac{23}{6237}\chi^4 + \frac{1}{10}\chi^2\right)\left(1 - \frac{5123}{8910}\chi^2 + \frac{3002}{93555}\chi^4\right)^{-1}.$$

By doing $\zeta_P = w$ we obtain the only physical solution, always possitive and fulfilling the condition $\lim_{w\to 0}\chi = 0$, for χ given by:

$$\chi = \frac{\sqrt{3}}{2}\,\frac{\sqrt{\left(35861\,w + 6237 - \sqrt{786718681\,w^2 + 389949714\,w + 38900169}\right)}}{\sqrt{3002\,w + 345}}.$$

Hence we get an analytical expression of the HMQS mass and radius as a function of the baryonic chemical potential. This allows us to obtain the M-R relationship for different Bag constant and magnetic field values. In particular in Figure 1 we show the $\mathsf{B} = 75\,\mathrm{MeV/\,fm^3}$ case for two different magnetic field values. Furthermore, in Table 2 we present the results for different values of B.

3.1. Dynamical stability

In our model the condition for stable equilibrium is given by $\partial^2 M/\partial^2\chi > 0$. For a given EoS, it is possible determine the quark densities and pressures where quark stars are stable against gravitational collapse from the condition

$$\Gamma > \Gamma_c,$$

where the adiabatic index for SQM, Γ, is given by:

$$\Gamma = \frac{n_B}{P}\frac{dP}{dn_B} = \frac{4\mu^4}{3\mu^4 - 4\pi^2\mathsf{B}} + \frac{16B\mu^2(4\pi^2\mathsf{B} - \mu^4)}{(4\pi^2\mathsf{B} - 3\mu^4)^2} + \mathcal{O}(B^2),$$

and the critical adiabatic index, Γ_c, for a cold star in general relativity is

$$\Gamma_c = (1+w)\left[1 + \frac{(3w+1)}{2}\left[\frac{(w+1)}{6w}\tan^2\chi - 1\right]\right].$$

To get dynamical stability the condition $\Gamma > \frac{4}{3}$ must be satisfied. The intersection between Γ and Γ_c determines when quark star becomes gravitationally unstable. Table 3 shows the critical values for $\mathsf{B} = 75\,\mathrm{MeV/\,fm^3}$. The values of w_c correspond to the maximum mass values of Table 2.

Another quantity that is related with the stability of the star is the speed of sound c_s. To satisfy the causality of quark matter,

$$\frac{dP}{d\rho} = c_s^2 \leq 1.$$

For the extremely relativistic systems, the speed of sound is $1/\sqrt{3}$, and in general it will be less than $1/\sqrt{3}$. We find

$$c_s = \frac{1}{\sqrt{3}} - \frac{2B}{9\mu^2\sqrt{3}} + \mathcal{O}(B^2).$$

4. Summary and conclusions

In this article we have furnished an analytical treatment to study a HMQS in the framework of the MIT Bag model. We have analyzed the stability of quark matter with respect to strong interactions and we have found a restriction in the stability condition: there is a maximum value for the magnetic field beyond which quark matter becomes unstable. In the limit of "weak" magnetic field that we have studied, quark magnetic moments are aligned in the same direction of the field and this situation leads to such restriction. This could mean that if the magnetic field strength exceeds that critical value, then quark or hybrid stars should not be considered as magnetars.

We have also found an analytical approximate solution for the M-R relationship. Even tough we used very simple physics, our results are in good agreement with the confidence contours of available observational data. Furthermore, comparison with numerical results (Pérez Martínez et. al. 2010) indicate a reasonable agreement: the difference in the maximum mass is $\sim 15\%$ while in the maximum radius value is $\sim 10\%$. This discrepancies are similar to the ones obtained when considering the same composition of the compact object but changing some parameters in the equation of state. Although the uniform energy density regime is a good approximation for quark stars, deviations in the determination of M-R relationship may also occur because in the limit of high densities such approximation is no longer valid.

Finally we calculate the adiabatic index and the speed of sound. The critical value for the adiabatic index, which correspond to the collapse of the star is in agreement with that of (Naurenberg & Chapline 1977), a pioneering work about quark stars. On the other hand, the speed of sound is consistent with the expected values for quark stars.

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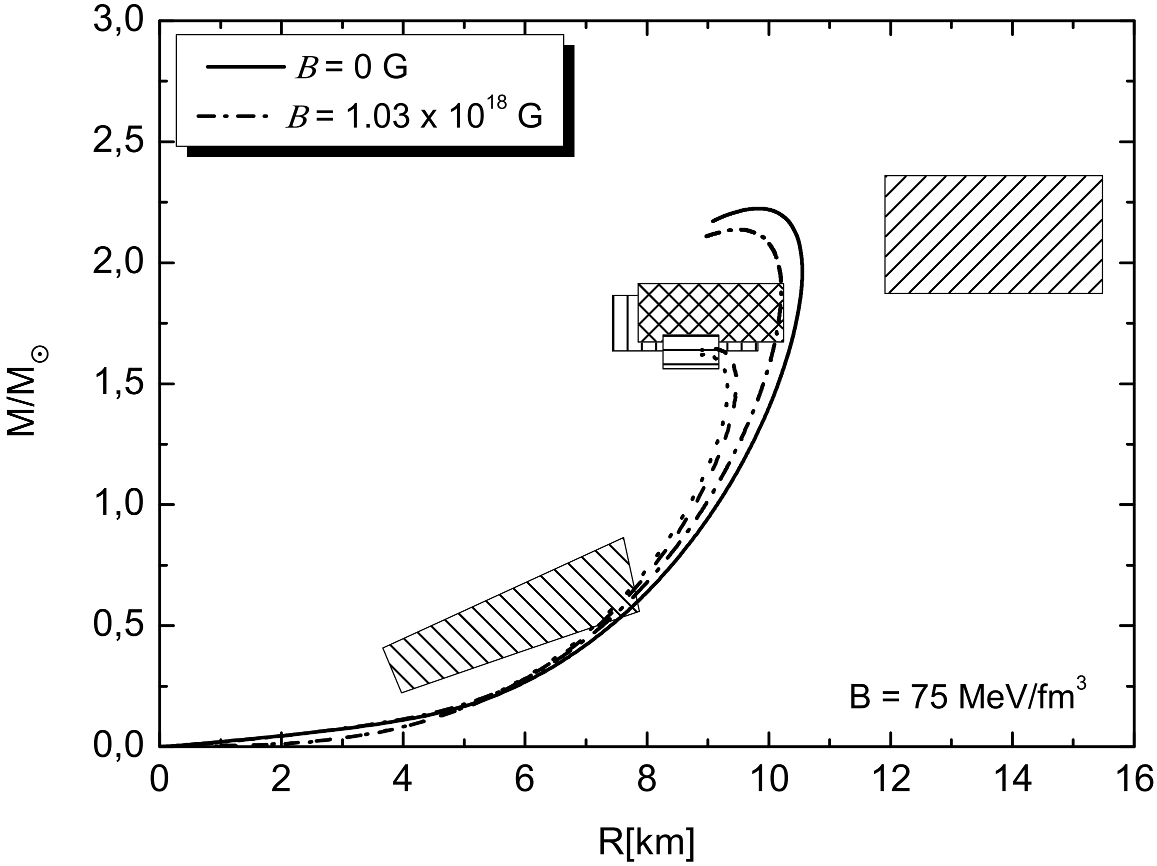

Fig. 1.— Mass-Radius relationship for $\mathsf{B} = 75\,\mathrm{MeV/fm^3}$. Solid line and Dash-dotted line correspond to our model while dashed and dotted line correspond to (González Felipe & Pérez Martínez 2009). The rectangle with diagonal pattern corresponds to EXO 0748-676, interpreted as a hadronic star. Rectangles with crossed, vertical and horizontal patterns correspond to quarks or hybrid stars (Drago & Lavagno 2010). The polygon could be a low-mass strange star as sugested in (Zhang et al. 2007).

Table 1: Bag constant, baryon density and magnetic field upper limit to preserve quark matter stability condition.

B[MeV/fm^3]	n_B/n_0	B_{max} [Gauss]
57	1.74 ($\pm 4 \times 10^{-2}$)	2.16×10^{18}
60	1.81 ($\pm 3 \times 10^{-2}$)	1.98×10^{18}
75	2.12 ($\pm 8 \times 10^{-3}$)	1.03×10^{18}
80	2.23 ($\pm 4 \times 10^{-3}$)	7.12×10^{17}
85	2.33 ($\pm 1 \times 10^{-3}$)	4.00×10^{17}
90	2.43 ($\pm 5 \times 10^{-5}$)	8.40×10^{16}

Table 2: Maximum mass, maximum radius and baryonic number for different bag constants.

B $[MeV/fm^3]$	B $[G]$	R_{max} $[km]$	$M_{max}/M_{\odot}$	$N_B/N_{\odot}$
57	0	12.10	2.55	4.30
	2.16×10^{18}	11.11	2.31	3.51
60	0	11.80	2.49	4.14
	1.98×10^{18}	10.94	2.27	3.45
75	0	10.55	2.22	3.50
	1.03×10^{18}	10.20	2.14	3.22
80	0	10.21	2.15	3.34
	7.12×10^{17}	9.99	2.10	3.16
85	0	9.91	2.09	3.19
	4.00×10^{17}	9.79	2.06	3.09
90	0	9.63	2.03	3.05
	8.40×10^{16}	9.60	2.02	3.04

Table 3: Adiabatic index and p/ρ critical value for $\mathsf{B} = 75\,[MeV]/\,[fm]^3$.

Γ_c	w_c	B_{max} [Gauss]
2.26	0.173	0
2.17	0.160	1.03×10^{18}