# Spin polarization phenomena in dense neutron matter at a strong magnetic field

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Spin polarized states in neutron matter at strong magnetic fields up to $10^{18}$ G are considered in the model with the Skyrme effective interaction. Analyzing the self-consistent equations at zero temperature, it is shown that a thermodynamically stable branch of solutions for the spin polarization parameter as a function of density corresponds to the negative spin polarization when the majority of neutron spins are oriented oppositely to the direction of the magnetic field. Besides, it is found that in a strong magnetic field the state with the positive spin polarization can be realized as a metastable state at the high density region in neutron matter. At finite temperature, the entropy of the thermodynamically stable branch demonstrates the unusual behavior being larger than that for the nonpolarized state (at vanishing magnetic field) above certain critical density which is caused by the dependence of the entropy on the effective masses of neutrons in a spin polarized state.

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## I. INTRODUCTION. BASIC EQUATIONS

Neutron stars observed in nature are magnetized objects with the magnetic field strength at the surface in the range $10^9$-$10^{13}$ G [1]. For a special class of neutron stars such as soft gamma-ray repeaters and anomalous X-ray pulsars, the field strength can be much larger and is estimated to be about $10^{14}$-$10^{15}$ G [2]. These strongly magnetized objects are called magnetars [3] and comprise about 10% of the whole population of neutron stars [4]. However, in the interior of a magnetar the magnetic field strength may be even larger, reaching the values about $10^{18}$ G [5, 6]. Under such circumstances, the issue of interest is the behavior of a neutron star matter in a strong magnetic field [5–8]. Further we will approximate the neutron star matter by pure neutron matter as was done, e.g., in the recent study [8]. As a framework for consideration, we choose a Fermi liquid approach for description of nuclear matter [9, 10] and as a potential of nucleon-nucleon interaction, we utilize the Skyrme effective forces.

The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons $f_{\kappa_1\kappa_2} = \mathrm{Tr}\,\varrho a^+_{\kappa_2} a_{\kappa_1}$, where $\kappa \equiv (\mathbf{p}, \sigma)$, $\mathbf{p}$ is momentum, $\sigma$ is the projection of spin on the third axis, and $\varrho$ is the density matrix of the system [11–13]. Further it will be assumed that the third axis is directed along the external magnetic field $\mathbf{H}$. The self-consistent matrix equation for determining the distribution function $f$ follows from the minimum condition of the thermodynamic potential [9] and is

$$f = \{\exp(Y_0\varepsilon + Y_4) + 1\}^{-1} \equiv \{\exp(Y_0\xi) + 1\}^{-1} \quad (1)$$

Here the single particle energy $\varepsilon$ and the quantity $Y_4$ are matrices in the space of $\kappa$ variables, with $Y_{4\kappa_1\kappa_2} = Y_4\delta_{\kappa_1\kappa_2}$, $Y_0 = 1/T$, and $Y_4 = -\mu_0/T$ being the Lagrange multipliers, $\mu_0$ being the chemical potential of neutrons, and $T$ the temperature. Given the possibility for alignment of neutron spins along or oppositely to the magnetic field $\mathbf{H}$, the normal distribution function of neutrons and single particle energy can be expanded

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in the Pauli matrices $\sigma_i$ in spin space

$$f(\mathbf{p}) = f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3, \qquad (2)$$
$$\varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p})\sigma_0 + \varepsilon_3(\mathbf{p})\sigma_3.$$

Using Eqs. (1) and (2), one can express evidently the distribution functions $f_0, f_3$ in terms of the quantities $\varepsilon$:

$$f_0 = \frac{1}{2}\{n(\omega_+) + n(\omega_-)\}, \qquad (3)$$
$$f_3 = \frac{1}{2}\{n(\omega_+) - n(\omega_-)\}.$$

Here $n(\omega) = \{\exp(Y_0\omega) + 1\}^{-1}$ and

$$\omega_\pm = \xi_0 \pm \xi_3, \qquad (4)$$
$$\xi_0 = \varepsilon_0 - \mu_0,\ \xi_3 = \varepsilon_3.$$

As follows from the structure of the distribution functions $f$, the quantities $\omega_\pm$ play the role of the quasiparticle spectrum and correspond to neutrons with spin up and spin down. The distribution functions $f$ should satisfy the normalization conditions

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \qquad (5)$$
$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho. \qquad (6)$$

Here $\varrho = \varrho_\uparrow + \varrho_\downarrow$ is the total density of neutron matter, $\varrho_\uparrow$ and $\varrho_\downarrow$ are the neutron number densities with spin up and spin down, respectively. The quantity $\Delta\varrho$ may be regarded as the neutron spin order parameter. It determines the magnetization of the system $M = \mu_n\Delta\varrho$, $\mu_n$ being the neutron magnetic moment. The magnetization may contribute to the internal magnetic field $B = H + 4\pi M$. However, we will assume, analogously to Refs. [6, 8], that the contribution of the magnetization to the magnetic field $B$ remains small for all relevant densities and magnetic field strengths, and, hence, $B \approx H$. This assumption holds true due to the tiny value of the neutron magnetic moment $\mu_n = -1.9130427(5)\mu_N \approx -6.031 \cdot 10^{-18}$ MeV/G [14] ($\mu_N$ being the nuclear magneton) and is confirmed numerically in a subsequent integration of the self-consistent equations.

In order to get the self–consistent equations for the components of the single particle energy, one has to set the energy functional of the system. In view of the above approximation, it reads [12]

$$E(f) = E_0(f,H) + E_{int}(f) + E_{field}, \qquad (7)$$
$$E_0(f,H) = 2\sum_{\mathbf{p}}\varepsilon_0(\mathbf{p})f_0(\mathbf{p}) - 2\mu_n H\sum_{\mathbf{p}} f_3(\mathbf{p}),$$
$$E_{int}(f) = \sum_{\mathbf{p}}\{\tilde\varepsilon_0(\mathbf{p})f_0(\mathbf{p}) + \tilde\varepsilon_3(\mathbf{p})f_3(\mathbf{p})\},$$
$$E_{field} = \frac{H^2}{8\pi}\mathcal{V},$$

where

$$\tilde\varepsilon_0(\mathbf{p}) = \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_0^n(\mathbf{k})f_0(\mathbf{q}),\ \mathbf{k} = \frac{\mathbf{p}-\mathbf{q}}{2}, \qquad (8)$$
$$\tilde\varepsilon_3(\mathbf{p}) = \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_1^n(\mathbf{k})f_3(\mathbf{q}).$$

Here $\varepsilon_0(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_0}$ is the free single particle spectrum, $m_0$ is the bare mass of a neutron, $U_0^n(\mathbf{k}), U_1^n(\mathbf{k})$ are the normal Fermi liquid (FL) amplitudes, and $\tilde\varepsilon_0, \tilde\varepsilon_3$ are the FL corrections to the free single particle spectrum. Note that in this study we will not be interested in the total energy density and pressure in the interior of a neutron star. By this reason, the field contribution $E_{field}$, being the energy of the magnetic field in the absence of matter, can be omitted. Using Eq. (7), one can get the self-consistent equations in the form [12]

$$\xi_0(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \tilde\varepsilon_0(\mathbf{p}) - \mu_0, \qquad (9)$$
$$\xi_3(\mathbf{p}) = -\mu_n H + \tilde\varepsilon_3(\mathbf{p}).$$

To obtain numerical results, we utilize the effective Skyrme interaction. The normal FL amplitudes can be expressed in terms of the Skyrme force parameters [9, 10]:

$$U_0^n(\mathbf{k}) = 2t_0(1-x_0) + \frac{t_3}{3}\varrho^\beta(1-x_3) \qquad (10)$$
$$+ \frac{2}{\hbar^2}[t_1(1-x_1) + 3t_2(1+x_2)]\mathbf{k}^2,$$
$$U_1^n(\mathbf{k}) = -2t_0(1-x_0) - \frac{t_3}{3}\varrho^\beta(1-x_3) + \frac{2}{\hbar^2}\cdot \qquad (11)$$
$$\cdot\,[t_2(1+x_2) - t_1(1-x_1)]\mathbf{k}^2 \equiv a_n + b_n\mathbf{k}^2.$$

Further we do not take into account the effective tensor forces, which lead to coupling of the momentum and spin degrees of freedom, and, correspondingly, to anisotropy in the momentum dependence of FL amplitudes with respect to the spin quantization axis. Then

$$\xi_0 = \frac{p^2}{2m_n} - \mu, \tag{12}$$

$$\xi_3 = -\mu_n H + (a_n + b_n \frac{\mathbf{p}^2}{4})\frac{\Delta\varrho}{4} + \frac{b_n}{16}\langle \mathbf{q}^2\rangle_3, \tag{13}$$

where the effective neutron mass $m_n$ reads

$$\frac{\hbar^2}{2m_n} = \frac{\hbar^2}{2m_0} + \frac{\varrho}{8}[t_1(1-x_1) + 3t_2(1+x_2)], \tag{14}$$

and the renormalized chemical potential $\mu$ should be determined from Eq. (5). The quantity $\langle \mathbf{q}^2\rangle_3$ in Eq. (13) is the second order moment of the distribution function $f_3$:

$$\langle \mathbf{q}^2\rangle_3 = \frac{2}{V}\sum_{\mathbf{q}} \mathbf{q}^2 f_3(\mathbf{q}). \tag{15}$$

In view of Eqs. (12), (13), the branches $\omega_\pm \equiv \omega_\sigma$ of the quasiparticle spectrum in Eq. (4) read

$$\omega_\sigma = \frac{p^2}{2m_\sigma} - \mu + \sigma\big(-\mu_n H + \frac{a_n\Delta\varrho}{4} + \frac{b_n}{16}\langle \mathbf{q}^2\rangle_3\big), \tag{16}$$

where $m_\sigma$ is the effective mass of a neutron with spin up ($\sigma=+1$) and spin down ($\sigma=-1$)

$$\frac{\hbar^2}{2m_\sigma} = \frac{\hbar^2}{2m_0} + \frac{\varrho_\sigma}{2}t_2(1+x_2) + \frac{\varrho_{-\sigma}}{4}\cdot \\ \cdot [t_1(1-x_1) + t_2(1+x_2)],\ \varrho_{+(-)} \equiv \varrho_{\uparrow(\downarrow)}. \tag{17}$$

Thus, with account of expressions (3) for the distribution functions $f$, we obtain the self–consistent equations (5), (6), and (15) for the effective chemical potential $\mu$, spin order parameter $\Delta\varrho$, and second order moment $\langle \mathbf{q}^2\rangle_3$. To check the thermodynamic stability of different solutions of the self-consistent equations, it is necessary to compare the corresponding free energies $F = E - TS$, where the entropy reads

$$S = -\sum_{\mathbf{p}}\sum_{\sigma=\uparrow,\downarrow}\{n(\omega_\sigma)\ln n(\omega_\sigma) \\ + \bar n(\omega_\sigma)\ln \bar n(\omega_\sigma)\},\ \bar n(\omega) = 1 - n(\omega). \tag{18}$$

## II. ANALYSIS OF THE SELF-CONSISTENT EQUATIONS

In solving numerically the self-consistent equations, we utilize SLy7 Skyrme force [15], constrained originally to reproduce the results of microscopic neutron matter calculations. We consider magnetic fields up to the values allowed by the scalar virial theorem. For a neutron star with the mass $M$ and radius $R$, equating the magnetic field energy $E_H \sim (4\pi R^3/3)(H^2/8\pi)$ with the gravitational binding energy $E_G \sim GM^2/R$, one gets the estimate $H_{max} \sim \frac{M}{R^2}(6G)^{1/2}$. For a typical neutron star with $M = 1.5M_\odot$ and $R = 10^{-5}R_\odot$, this yields for the maximum value of the magnetic field strength $H_{max} \sim 10^{18}$ G. This magnitude can be expected in the interior of a magnetar while recent observations report the surface values up to $H \sim 10^{15}$ G [16].

Fig. 1 shows the neutron spin polarization parameter $\Pi = \Delta\varrho/\varrho$ as a function of density for a set of fixed values of the magnetic field at zero temperature. At $H = 0$, the self-consistent equations are invariant with respect to the global flip of neutron spins and we have two branches of solutions for the spin polarization parameter, $\Pi_0^+(\varrho)$ (upper) and $\Pi_0^-(\varrho)$ (lower) which differ only by sign, $\Pi_0^+(\varrho) = -\Pi_0^-(\varrho)$. At $H \neq 0$, the self-consistent equations lose the invariance with respect to the global flip of the spins and, as a consequence, the branches of spontaneous polarization are modified differently by the magnetic field. The lower branch $\Pi_1(\varrho)$, corresponding to the negative spin polarization, extends down to the very low densities. There are three characteristic density domains for this branch. At low densities $\varrho \lesssim 0.5\varrho_0$, the absolute value of the spin polarization parameter increases with decreasing density. At intermediate densities $0.5\varrho_0 \lesssim \varrho \lesssim 3\varrho_0$, there is a plateau in the $\Pi_1(\varrho)$ dependence, whose characteristic value depends on $H$, e.g., $\Pi_1 \approx -0.08$ at $H = 10^{18}$ G. At densities $\varrho \gtrsim 3\varrho_0$, the magnitude of the spin polarization parameter increases with density, and neutrons become totally polarized at $\varrho \approx 6\varrho_0$.

It is seen also from Fig. 1 that beginning from some threshold density the self-consistent equations at a given density have two positive solutions for the spin polarization parameter (apart from one negative solution). These solutions be-

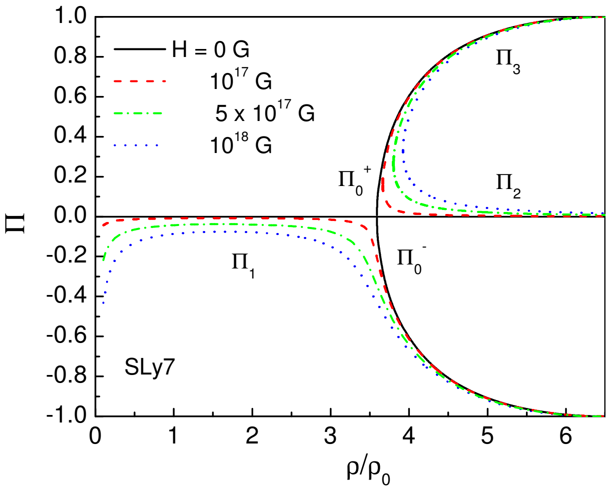


FIG. 1: (Color online) Neutron spin polarization parameter as a function of density at $T = 0$ and different magnetic field strengths for SLy7 interaction. The branches of spontaneous polarization $\Pi_0^-, \Pi_0^+$ are shown by solid curves.

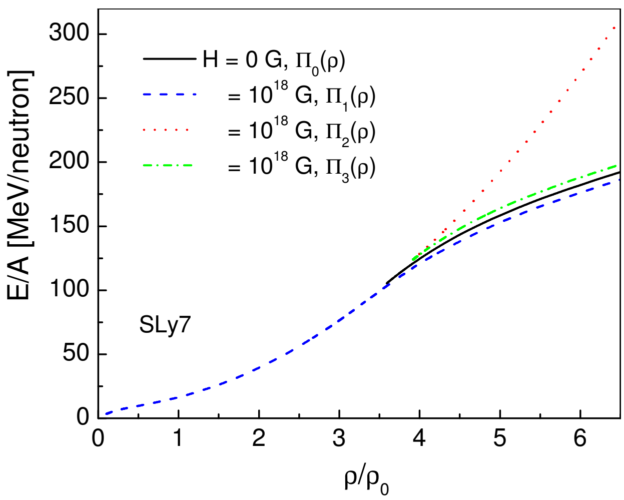


FIG. 2: (Color online) Energy per neutron as a function of density at $T = 0$ for different branches $\Pi_1(\varrho)$-$\Pi_3(\varrho)$ of solutions of the self-consistent equations at $H = 10^{18}$ G, including a spontaneously polarized state.

long to two branches, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, characterized by the different dependence from density. For the branch $\Pi_2(\varrho)$, the spin polarization parameter decreases with density and tends to zero value while for the branch $\Pi_3(\varrho)$ it increases with density and is saturated. These branches appear step-wise at the same threshold density $\varrho_{\rm th}$ dependent on the magnetic field and being larger than the critical density of spontaneous spin instability in neutron matter. For example, for SLy7 interaction, $\varrho_{\rm th} \approx 3.80\,\varrho_0$ at $H = 5\cdot 10^{17}$ G, and $\varrho_{\rm th} \approx 3.92\,\varrho_0$ at $H = 10^{18}$ G. The magnetic field, due to the negative value of the neutron magnetic moment, tends to orient the neutron spins oppositely to the magnetic field direction. As a result, the spin polarization parameter for the branches $\Pi_2(\varrho)$, $\Pi_3(\varrho)$ with the positive spin polarization is smaller than that for the branch of spontaneous polarization $\Pi_0^+$, and, vice versa, the magnitude of the spin polarization parameter for the branch $\Pi_1(\varrho)$ with the negative spin polarization is larger than the corresponding value for the branch of spontaneous polarization $\Pi_0^-$. Note that the impact of even such strong magnetic field as $H = 10^{17}$ G is small: The spin polarization parameter for all three branches $\Pi_1(\varrho)$-$\Pi_3(\varrho)$ is either close to zero, or close to its value in the state with spontaneous polarization, which is governed by the spin-dependent medium correlations.

Thus, at densities larger than $\varrho_{\rm th}$, we have three branches of solutions: one of them, $\Pi_1(\varrho)$, with the negative spin polarization and two others, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, with the positive polarization. In order to clarify, which branch is thermodynamically preferable, one should compare the corresponding free energies. Fig. 2 shows the energy per neutron as a function of density at $T = 0$ and $H = 10^{18}$ G for these three branches, compared with the energy per neutron for a spontaneously polarized state [the branches $\Pi_0^\pm(\varrho)$]. It is seen that the state with the majority of neutron spins oriented oppositely to the direction of the magnetic field [the branch $\Pi_1(\varrho)$] has a lowest energy. However, the state, described by the branch $\Pi_3(\varrho)$ with the positive spin polarization, has the energy very close to that of the thermodynamically stable state. This means that despite the presence of a strong magnetic field $H \sim 10^{18}$ G, the state with the majority of neutron spins directed along the magnetic field can be realized as a metastable state in the dense core of a neutron star in the model consideration with the Skyrme effective interaction. Note here that in the study [8] of neutron matter at a strong magnetic field only thermodynamically stable branch of solutions for the spin polarization parameter was found in the model with the SLy7 Skyrme

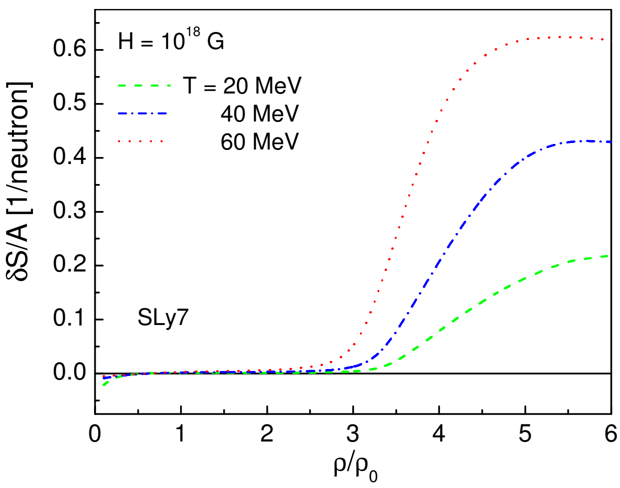


FIG. 3: (Color online) The entropy per neutron for the $\Pi_1$ branch, measured from its value in the nonpolarized state (at $H=0$), as a function of density at $H=10^{18}$ G and different temperatures.

interaction.

One can consider also finite temperature effects on spin polarized states in neutron matter at a strong magnetic field. Calculations show that the influence of finite temperatures on spin polarization remains moderate in the Skyrme model, at least, for temperatures relevant for protoneutron stars (up to 60 MeV). An unexpected moment appears when we consider the behavior of the entropy of spin polarized state as a function of density. Fig. 3 shows the density dependence of the difference between the entropies per neutron of the polarized (the $\Pi_1$ branch) and nonpolarized (at $H=0$) states at different fixed temperatures. It is seen that with increasing density the difference of the entropies becomes positive. It looks like the polarized state in a strong magnetic field beginning from some critical density $\varrho_s$ is less ordered than the nonpolarized state. Such unusual behavior of the entropy was found also in the earlier works for spontaneously polarized states in neutron [17] and nuclear [18, 19] matter with the Skyrme and Gogny effective forces, respectively. Providing the low temperature expansion for the entropy in Eq. (18), one can get the condition for the difference between the entropies per neutron of the polarized and nonpolarized states to be negative in the form

$$\frac{m_\uparrow}{m_n}(1+\Pi)^{\frac{1}{3}}+\frac{m_\downarrow}{m_n}(1-\Pi)^{\frac{1}{3}}-2<0. \qquad (19)$$

For low temperatures, it can be checked numerically that this condition is violated for the $\Pi_1$ branch of the spin polarization parameter above the critical density $\varrho_s$ being weakly dependent on temperature.

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[1] A. Lyne, and F. Graham-Smith, *Pulsar Astronomy* (Cambridge Univ. Press, Cambridge, 2005).
[2] C. Thompson, and R.C. Duncan, Astrophys. J. **473**, 322 (1996).
[3] R.C. Duncan, and C. Thompson, Astrophys. J. **392**, L9 (1992).
[4] C. Kouveliotou, et al., Nature, **393**, 235 (1998).
[5] S. Chakrabarty, D. Bandyopadhyay, and S. Pal, Phys. Rev. Lett. **78**, 2898 (1997).
[6] A. Broderick, M. Prakash, and J. M. Lattimer, Astrophys. J. **537**, 351 (2000).
[7] C. Cardall, M. Prakash, and J. M. Lattimer, Astrophys. J. **554**, 322 (2001).
[8] M. A. Perez-Garcia, Phys. Rev. C **77**, 065806 (2008).
[9] A. I. Akhiezer, A. A. Isayev, S. V. Peletminsky, A. P. Rekalo, and A. A. Yatsenko, JETP **85**, 1 (1997).
[10] A.A. Isayev, and J. Yang, in *Progress in Ferromagnetism Research*, edited by V.N. Murray (Nova Science Publishers, New York, 2006), p. 325 [arXiv:nucl-th/0403059].
[11] A.A. Isayev, JETP Letters **77**, 251 (2003).
[12] A.A. Isayev, and J. Yang, Phys. Rev. C **69**, 025801 (2004).
[13] A.A. Isayev, Phys. Rev. C **74**, 057301 (2006).
[14] C. Amsler et al. (Particle Data Group), Phys. Lett. **B667**, 1 (2008).
[15] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. **A635**, 231 (1998).
[16] A. I. Ibrahim, S. Safi-Harb, J. H. Swank, et al., Astrophys. J. **574**, L51 (2002).
[17] A. Rios, A. Polls, and I. Vidaña, Phys. Rev. C **71**, 055802 (2005).
[18] A.A. Isayev, Phys. Rev. C **72**, 014313 (2005).
[19] A.A. Isayev, Phys. Rev. C **76**, 047305 (2007).