Parity violation in low energy neutron deuteron scattering

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Abstract

Parity violating effects for low energy elastic neutron deuteron scattering are calculated for DDH and EFT-type of weak potentials in a Distorted Wave Born Approximation, using realistic hadronic strong interaction wave functions, obtained by solving three-body Faddeev equations in configuration space. The results of relation between physical observables and low energy constants can be used to fix low energy constants from experiments. Potential model dependencies of parity violating effects are discussed.

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I. INTRODUCTION

The study of parity violating (PV) effects in low energy physics is a very sensitive tool to test methods of calculations both of weak and strong interactions in the Standard model. This also can be a way to search for a possible manifestations of new physics resulted in deviations from unambiguous and precise calculations of PV effects and experimental measurements. However, to use this approach, it is crucial to prove that implemented theoretical techniques are sufficient to describe experimental data with high accuracy which exceeds experimental accuracy. There is a large amount of experimental data for different PV effects in nuclear physics, each of which in general agrees with theoretical predictions. However, in the last years it became clear (see, for example [1–4] and references therein) that the traditional DDH [5] method for calculation of PV effects cannot reliably describe the whole available set of experimental data within the same set of parameters. If this is not the manifestation of new physics, which is very unlikely for the current accuracy of experimental measurements and theoretical calculations, then this discrepancy could be blamed on systematic errors in experimental data, theoretical uncertainties in calculations of strong interactions at low energy, or it might be that DDH approach is not adequate for the description of the set of precise experimental data because it is based on a number of models and assumptions. To resolve this discrepancy and to eliminate nuclear model dependent factors in calculations, it is necessary to focus on the analysis of new and existing experimental data for different PV parameters in few-body systems, where calculations of nuclear related effects can be done with a high precision. Recently new approach, based on the effective field theory (EFT), has been introduced for a model independent parametrization of PV effects (see, papers [1, 4] and references therein), and some calculations for two-body systems have been done [6]. The power of the EFT approach for parametrization of all PV effects in terms of a small number of constants could be utilized if we can analyze a large enough number of PV effects to be able to constrain all free parameters of the theory which are usually called low energy constants (LEC). Thus, one can guarantee the adequate description (parametrization) of the strong interaction hadronic parts and weak interaction constants for symmetry violating observables. Unfortunately, the number of experimentally measured (and independent in terms of unknown LECs) PV effects in two body systems is not enough to constrain all LECs. In spite of the fact that five independent observable parameters in a two body system

could fix five unknown PV LECs [7–10], it is impossible to measure all of them using existing experimental techniques. Therefore, one has to include into analysis few-body systems and even heavier nuclei, the latter of which are actually preferable from the experimental point of view, because as a rule, the measured effects in nuclei are much larger than in nucleon-nucleon system due to nuclear enhancement factors [11–13].

The natural and unambiguous way to verify the applicability of the EFT for the calculation of symmetry violating effects in nuclear reactions requires a development of a regular and self consistent approach for calculation of PV amplitudes in three-body (few-body) systems [14], with a hope to extend the formalism for the description of many body systems. This systematic approach for the solution of three-body PV scattering problem in EFT framework [14] requires additional numerical efforts and will be presented elsewhere. As a first step for the clarification of the possible difference in contributions to PV effects from DDH and EFT-type potentials, one can use a "hybrid" method (similar to the method used in paper [15]) for the simplest process of neutron-deuteron scattering. We calculate three-body wave functions with realistic Hamiltonians of strong interaction using exact Faddeev equations in configuration space, and then, calculate PV effects in the first order of perturbation with DDH potential and potentials derived in EFT formalism. In the next section, we present our formalism for the calculation PV effects for elastic neutron-deuteron scattering with different set of nucleon weak potentials, with DDH and weak potentials obtained from pionless and pionful EFTs. Then, we present results of numerical calculations and discussions.

II. FORMALISM

We treat weak nucleon interactions as a perturbation and calculate three-body wave functions exactly using Faddeev equations with phenomenological potentials for strong interactions. Similar hybrid approach has been successfully applied to the weak and electromagnetic processes involving three-body and four-body hadronic systems [16–21]. We consider three types of parity violating potentials. The first one is the standard DDH potential which is based on meson exchange mechanism of nucleon-nucleon interactions. The second and third potentials are derived from pionless and pionful versions of effective field theory with parity violating hadronic interactions. Instead of calculating parity violating amplitudes by

summing PV diagrams in EFT, we use these potentials to calculate PV effects. This is a simplification, which we call a "hybrid" approach.

A. Observables

Since PV effects in neutron-deuteron system are very small, we consider only coherent processes which are related to the propagation of neutrons through unpolarized deuteron target and, therefore, do not have an additional suppression in low energy region. Then, two PV observable parameters are the angle ϕ of rotation of neutron polarization around neutron momentum and the relative difference of total cross sections $P = (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$ for neutrons with opposite helicities. The value of the angle of neutron spin rotation per unit length of the target sample can be expressed in terms of elastic scattering amplitudes at zero angle for opposite helicities f_+ and f_- as

$$\frac{d\phi}{dz} = -\frac{2\pi N}{p} \mathrm{Re}\left(f_+ - f_-\right), \tag{1}$$

where N is a number of target nuclei per unit volume and p is a relative neutron momentum. Using optical theorem, one can write the relative difference of total cross sections P in terms of these amplitudes as

$$P = \frac{\mathrm{Im}\left(f_+ - f_-\right)}{\mathrm{Im}\left(f_+ + f_-\right)}. \tag{2}$$

It is convenient to represent the amplitudes in terms of matrix $\hat{R}$ which is related to scattering matrix $\hat{S}$ as $\hat{R} = \hat{1} - \hat{S}$. With partial waves decomposition for the case of neutron-deuteron scattering

$$|\boldsymbol{p}, m_n, m_d\rangle = \sum_{l_y l_y^z} \sum_{SM, JJ^z} |p, (l_y \mathcal{S}) J J^z\rangle\langle J J^z | l_y l_y^z, SM\rangle\langle SM | \tfrac{1}{2} m_n, 1 m_d\rangle Y_{l_y l_y^z}^*(\hat{p}), \tag{3}$$

where l_y is an orbital angular momentum between neutron and deuteron, $\mathcal{S}$ is a sum of neutron spin and deuteron total angular momentum, and J is the total angular momentum of the neutron-deuteron system, the above equations can be written at low energies as

$$\begin{aligned}
\frac{1}{N}\frac{d\phi}{dz} &= \frac{2\pi}{9p^2}\mathrm{Im}\left[R^{\frac{1}{2}}_{1\frac{1}{2},0\frac{1}{2}} + R^{\frac{1}{2}}_{0\frac{1}{2},1\frac{1}{2}} - 2\sqrt{2}R^{\frac{1}{2}}_{1\frac{3}{2},0\frac{1}{2}} - 2\sqrt{2}R^{\frac{1}{2}}_{0\frac{1}{2},1\frac{3}{2}} \right. \\
&\left. + 4R^{\frac{3}{2}}_{1\frac{1}{2},0\frac{3}{2}} + 4R^{\frac{3}{2}}_{0\frac{3}{2},1\frac{1}{2}} - 2\sqrt{5}R^{\frac{3}{2}}_{1\frac{3}{2},0\frac{3}{2}} - 2\sqrt{5}R^{\frac{3}{2}}_{0\frac{3}{2},1\frac{3}{2}} \right]
\end{aligned} \tag{4}$$

and

$$
\begin{aligned}
P = \frac{1}{3}\text{Re}\left[R^{\frac{1}{2}}_{1\frac{1}{2},0\frac{1}{2}} + R^{\frac{1}{2}}_{0\frac{1}{2},1\frac{1}{2}} - 2\sqrt{2}R^{\frac{1}{2}}_{1\frac{3}{2},0\frac{1}{2}} - 2\sqrt{2}R^{\frac{1}{2}}_{0\frac{1}{2},1\frac{3}{2}} + 4R^{\frac{3}{2}}_{1\frac{1}{2},0\frac{3}{2}} \right. \\
\left. + 4R^{\frac{3}{2}}_{0\frac{3}{2},1\frac{1}{2}} - 2\sqrt{5}R^{\frac{3}{2}}_{1\frac{3}{2},0\frac{3}{2}} - 2\sqrt{5}R^{\frac{3}{2}}_{0\frac{3}{2},1\frac{3}{2}} \right] / \text{Re}\left[R^{\frac{1}{2}}_{0\frac{1}{2},0\frac{1}{2}} + 2R^{\frac{3}{2}}_{0\frac{3}{2},0\frac{3}{2}} \right],
\end{aligned}
\tag{5}
$$

where $R^{J}_{l'\mathcal{S}',l\mathcal{S}} = \langle l'\mathcal{S}'|R^{J}|l\mathcal{S}\rangle$, unprimed and primed parameters correspond to initial and final states. Since we are interested in low energy neutron scattering, it would be sufficient to include only s- and p-waves contributions to parity violating amplitudes; for the total cross section (the denominator in the last equation), we keep only dominant contributions from s-wave neutrons. It should be noted that time-reversal invariance leads to the relation $\langle 1\mathcal{S}'|R^{J}|0\mathcal{S}\rangle = \langle 0\mathcal{S}'|R^{J}|1\mathcal{S}'\rangle$ between matrix elements, therefore, only half of parity violating amplitudes are independent.

Nucleon-nucleon interaction can be written as a sum $V = V_{pc} + V_{pv}$ of the parity conserving (V_{pc}) and weak parity violating (V_{pv}) terms. Due to the weakness of parity violating interaction, one can use Distorted Wave Born Approximation (DWBA) to calculate PV amplitudes with a high level of accuracy as

$$
R^{J}_{l'_y\mathcal{S}',l_y\mathcal{S}} \simeq 4i^{-l'_y+l_y+1}\mu p \,_{pc}^{(-)}\langle \Psi, (l'_y\mathcal{S}')JJ^{z}|V_{pv}|\Psi, (l_y\mathcal{S})JJ^{z}\rangle^{(+)}_{pc},
\tag{6}
$$

where μ is a neutron-deuteron reduced mass and $|\Psi, (l'_y\mathcal{S}')JJ^{z}\rangle^{(\pm)}_{pc}$ are solutions of 3-body Faddeev equations in configuration space for parity conserving strong interaction Hamiltonian, defined by V_{PC} and normalized as described in section II C. The factor $i^{-l'_y+l_y}$ in this expression is introduced to match the R-matrix definition in the modified spherical harmonics convention [22] with the wave functions which are calculated in this paper using spherical harmonics convention.

In the rest of the paper, we use only wave functions calculated for parity conserving potentials and, therefore, will omit subscript PC.

As will be explained in section II C, we use jj-coupling scheme (with a basis states $|l_y j_y\rangle$) when solving Faddeev equtions. One can transform jj-basis states into $l_y\mathcal{S}$-basis by means of

$$
|[l_y \otimes (s_k \otimes j_x)_{\mathcal{S}}]_{JJ_z}\rangle = \sum_{j_y} |[j_x \otimes (l_y \otimes s_k)_{j_y}]_{JJ_z}\rangle
$$

$$
\times (-1)^{j_x+j_y-J}(-1)^{l_y+s_k+j_x+J}[(2j_y+1)(2\mathcal{S}+1)]^{\frac{1}{2}} \begin{Bmatrix} l_y & s_k & j_y \\ j_x & J & \mathcal{S} \end{Bmatrix},
\tag{7}
$$

One interesting observation is that the neutron spin rotation, as well as parameter P, in $|l_y j_y\rangle$ basis involves potential matrix elements only between $j_y = \frac{1}{2}$ states.

It should be noted that at low energy the $\text{Im}(R^J_{l'_y \mathcal{S}', l_y \mathcal{S}}) \sim p^{l'_y + l_y + 1}$, and thus the expression eq.(4) for the angle ϕ of neutron spin rotation is finite and well defined in the zero energy limit of the n-d scattering. Numerically, it is calculated by evaluating expression $\text{Im}(R^J_{l'_y \mathcal{S}', l_y \mathcal{S}})/p^{l'_y + l_y + 1}$ at zero energy. On the other hand, $\text{Re}(R^J_{l'_y \mathcal{S}', l_y \mathcal{S}}) \sim p \cdot \text{Im}(R^J_{l'_y \mathcal{S}', l_y \mathcal{S}})$ at low energy, and thus the real part of this quantity vanishes in the zero energy limit. Therefore, the parameter P is calculated at 15 keV neutron kinetic energy in the laboratory system, where both imaginary and real parts of the R-matrix elements become comparable in magnitude and thus can be discerned numerically.

B. The parity violating potentials

To understand the possible difference in the description of parity violating effects by DDH and EFT-type for potentials, we compare calculations with the DDH potential[5] and two different choices of EFT potentials: the potential derived from pionless EFT lagrangian [1] and the potential derived from pionful EFT Lagrangian [1]. It was shown [15] that all these three potentials can be expanded in terms of a set of $O^{(n)}_{ij}$ operators as

$$v^\alpha_{ij} = \sum_n c^\alpha_n O^{(n)}_{ij}, \quad \alpha = \text{DDH or pionless EFT or pionful EFT} \tag{8}$$

with parameters c^α_n and operators $O^{(n)}_{ij}$ given in the Table I.

One can see that operators $O^{(n)}_{ij}$ are products of isospin, spin, and vector operators $\boldsymbol{X}^{(n)}_{ij,\pm}$ defined as

$$\boldsymbol{X}^{(n)}_{ij,+} \equiv [\boldsymbol{p}_{ij}, f_n(r_{ij})]_+,$$
$$\boldsymbol{X}^{(n)}_{ij,-} \equiv i[\boldsymbol{p}_{ij}, f_n(r_{ij})]_-, \tag{9}$$

where $\boldsymbol{p}_{ij} \equiv \frac{(\boldsymbol{p}_i - \boldsymbol{p}_j)}{2}$.

For the DDH potential, radial functions $f_x(r)$, $x = \pi, \rho$, and ω are modified Yukawa functions,

$$f_x(r) = \frac{1}{4\pi r} \left\{ e^{-m_x r} - e^{-\Lambda_x r} \left[1 + \frac{\Lambda_x r}{2} \left(1 - \frac{m_x^2}{\Lambda_x^2} \right) \right] \right\}. \tag{10}$$

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TABLE I: Parameters and operators of parity violating potentials. πNN coupling $g_{\pi NN}$ can be represented by g_A by using Goldberger-Treiman relation, $g_\pi = g_A m_N/F_\pi$ with $F_\pi = 92.4$ MeV. $\mathcal{T}_{ij} \equiv (3\tau_i^z\tau_j^z - \tau_i\cdot\tau_j)$. Scalar function $\tilde{L}_\Lambda(r) \equiv 3L_\Lambda(r) - H_\Lambda(r)$.

n	c_n^{DDH}	$f_n^{DDH}(r)$	$c_n^{\not\pi}$	$f_n^{\not\pi}(r)$	c_n^{π}	$f_n^{\pi}(r)$	$O_{ij}^{(n)}$
1	$+\frac{g_\pi}{2\sqrt{2}m_N}h_\pi^1$	$f_\pi(r)$	$\frac{2\mu^2}{\Lambda_\chi^3}C_6^{\not\pi}$	$f_\mu^{\not\pi}(r)$	$+\frac{g_\pi}{2\sqrt{2}m_N}h_\pi^1$	$f_\pi(r)$	$(\tau_i\times\tau_j)^z(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(1)}$
2	$-\frac{g_\rho}{m_N}h_\rho^0$	$f_\rho(r)$	0	0	0	0	$(\tau_i\cdot\tau_j)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,+}^{(2)}$
3	$-\frac{g_\rho(1+\kappa_\rho)}{m_N}h_\rho^0$	$f_\rho(r)$	0	0	0	0	$(\tau_i\cdot\tau_j)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(3)}$
4	$-\frac{g_\rho}{2m_N}h_\rho^1$	$f_\rho(r)$	$\frac{\mu^2}{\Lambda_\chi^3}(C_2^{\not\pi}+C_4^{\not\pi})$	$f_\mu^{\not\pi}(r)$	$\frac{\Lambda^2}{\Lambda_\chi^3}(C_2^\pi+C_4^\pi)$	$f_\Lambda(r)$	$(\tau_i+\tau_j)^z(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,+}^{(4)}$
5	$-\frac{g_\rho(1+\kappa_\rho)}{2m_N}h_\rho^1$	$f_\rho(r)$	0	0	$\frac{2\sqrt{2}\pi g_A^3\Lambda^2}{\Lambda_\chi^3}h_\pi^1$	$L_\Lambda(r)$	$(\tau_i+\tau_j)^z(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(5)}$
6	$-\frac{g_\rho}{2\sqrt{6}m_N}h_\rho^2$	$f_\rho(r)$	$-\frac{2\mu^2}{\Lambda_\chi^3}C_5^{\not\pi}$	$f_\mu^{\not\pi}(r)$	$-\frac{2\Lambda^2}{\Lambda_\chi^3}C_5^\pi$	$f_\Lambda(r)$	$\mathcal{T}_{ij}(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,+}^{(6)}$
7	$-\frac{g_\rho(1+\kappa_\rho)}{2\sqrt{6}m_N}h_\rho^2$	$f_\rho(r)$	0	0	0	0	$\mathcal{T}_{ij}(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(7)}$
8	$-\frac{g_\omega}{m_N}h_\omega^0$	$f_\omega(r)$	$\frac{2\mu^2}{\Lambda_\chi^3}C_1^{\not\pi}$	$f_\mu^{\not\pi}(r)$	$\frac{2\Lambda^2}{\Lambda_\chi^3}C_1^\pi$	$f_\Lambda(r)$	$(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,+}^{(8)}$
9	$-\frac{g_\omega(1+\kappa_\omega)}{m_N}h_\omega^0$	$f_\omega(r)$	$\frac{2\mu^2}{\Lambda_\chi^3}\tilde{C}_1^{\not\pi}$	$f_\mu^{\not\pi}(r)$	$\frac{2\Lambda^2}{\Lambda_\chi^3}\tilde{C}_1^\pi$	$f_\Lambda(r)$	$(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(9)}$
10	$-\frac{g_\omega}{2m_N}h_\omega^1$	$f_\omega(r)$	0	0	0	0	$(\tau_i+\tau_j)^z(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,+}^{(10)}$
11	$-\frac{g_\omega(1+\kappa_\omega)}{2m_N}h_\omega^1$	$f_\omega(r)$	0	0	0	0	$(\tau_i+\tau_j)^z(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(11)}$
12	$-\frac{g_\omega h_\omega^1-g_\rho h_\rho^1}{2m_N}$	$f_\rho(r)$	0	0	0	0	$(\tau_i-\tau_j)^z(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,+}^{(12)}$
13	$-\frac{g_\rho}{2m_N}h_\rho'^1$	$f_\rho(r)$	0	0	$-\frac{\sqrt{2}\pi g_A\Lambda^2}{\Lambda_\chi^3}h_\pi^1$	$L_\Lambda(r)$	$(\tau_i\times\tau_j)^z(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(13)}$
14	0	0	0	0	$\frac{2\Lambda^2}{\Lambda_\chi^3}C_6^\pi$	$f_\Lambda(r)$	$(\tau_i\times\tau_j)^z(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(14)}$
15	0	0	0	0	$\frac{3\sqrt{2}\pi g_A^3\Lambda^2}{2\Lambda_\chi^3}h_\pi^1$	$\tilde{L}_\Lambda(r)$	$(\tau_i\times\tau_j)^z(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{X}_{ij,-}^{(15)}$

For pionless EFT ($\not\pi$EFT) one, $f_n(r)$ are described by single function $f_\mu(r)$,

$$f_\mu(r) = \frac{1}{4\pi r}e^{-\mu r}, \tag{11}$$

with $\mu \simeq m_\pi$.

For the case of pionful EFT model (πEFT), there are long range interactions from one pion exchange($V_{-1,LR}$) and from their corrections ($V_{1,LR}$), middle range interactions due to two pion exchange ($V_{1,MR}$), and short range interactions ($V_{1,SR}$) due to nucleon contact terms. The radial part of the leading term of long range one pion exchange, $V_{-1,LR}$, is described by the function $f_\pi(r)$. Since one-pion exchange contribution is dominated by long range part, we do not use a regulator for it, i.e. we assume that the long range interactions have the same radial functions $f_\pi(r)$ as DDH potential with infinite cutoff. The short range

interaction $V_{1,SR}$ in pionful theory has the same structure as for pionless EFT; however, in spite of the structural similarity, their meanings are rather different. One can ignore the higher order corrections of long range interactions, $V_{1,LR}$, because they can either be absorbed by renormalization of low energy constants [6] or suppressed. The middle range interactions $V_{1,MR}$ are described by functions $L(q)$ and $H(q)$ in momentum space

$$L(q) \equiv \frac{\sqrt{4m_\pi^2 + \boldsymbol{q}^2}}{|\boldsymbol{q}|} \ln \left(\frac{\sqrt{4m_\pi^2 + \boldsymbol{q}^2} + |\boldsymbol{q}|}{2m_\pi} \right), \quad H(q) \equiv \frac{4m_\pi^2}{4m_\pi^2 + \boldsymbol{q}^2} L(q), \tag{12}$$

where, $q^\mu = (q^0, \boldsymbol{q}) = p_1^\mu - p_1'^\mu = p_2'^\mu - p_2^\mu$. To calculate two pion exchange functions (divergent at large q) in spacial representation , we use regulators $\frac{(\Lambda^2 - 4m_\pi^2)^2}{(\Lambda^2 + \boldsymbol{q}^2)^2}$. For the sake of simplicity, we use only one cutoff parameter with the same regulator, both for middle range and for short range interactions. Then, one can write

$$\{L_\Lambda(r), H_\Lambda(r), f_\Lambda(r)\} = \frac{1}{\Lambda^2} \int \frac{d^3q}{(2\pi)^3} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{(\Lambda^2 - 4m_\pi^2)^2}{(\Lambda^2 + \boldsymbol{q}^2)^2} \{L(q), H(q), 1\}. \tag{13}$$

In the given representation, coefficients c_n^α have fm dimension and scalar functions $f_n^\alpha(r)$ have fm^{-1} dimension. One can see that only the new operator structure, which is not included in DDH or pionless EFT, is due to $V_{1,LR}^{PV}$. Therefore, pionful EFT does not introduce new operator structure, provided we neglect $V_{1,LR}^{PV}$ term [6].

To see a sensitivity to the choice of cutoffs for parity violating potentials, we used two set of cutoff parameters for each models, which are listed in the Table II.

Using the discussed above three potentials, one can represent parity violating amplitudes as a linear expansion in terms of given set of matrix elements for corresponding operators $O_{ij}^{(n)}$. Thus, the angle of neutron spin rotation can be written as

$$\frac{1}{N} \frac{d\phi}{dz} = \sum_{n=1}^{13} c_n^\alpha I_n^\alpha, \tag{14}$$

and the parameter P as

$$P = \sum_{n=1}^{13} c_n^\alpha \tilde{I}_n^\alpha, \tag{15}$$

in terms of coefficients I_n^α and $\tilde{I}_n^\alpha$ with $\alpha = $ DDH-I,II, $\pi\!\!\!/$EFT-I,II, πEFT-I,II for different potentials and cutoff parameters.

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TABLE II: Parameter of parity violating potentials in GeV units. We used masses of mesons m_π, m_ρ, and m_ω, respectively, as 0.138, 0.771, and 0.783 in DDH potential.

	Λ_π	Λ_ρ	Λ_ω		μ		Λ
DDH-I	1.72	1.31	1.50	$\not{\pi}$EFT-I	0.138	πEFT-I	0.8
DDH-II	∞	∞	∞	$\not{\pi}$EFT-II	1.0	πEFT-II	1.0

C. Faddeev wave function

To obtain 3-body wave functions for neutron-deuteron scattering with parity conserving interactions, we solve Faddeev equations (also often called Kowalski-Noyes equations) in configuration space [23, 24]. For isospin invariant interactions (with nucleon masses fixed to $\hbar^2/m = 41.471$ MeV·fm), three Faddeev equations become formally identical, having the form

$$(E - H_0 - V_{ij})\,\psi_k = V_{ij}(\psi_i + \psi_j), \tag{16}$$

where (ijk) are particle indices, H_0 is kinetic energy operator, V_{ij} is two body force between particles i, and j, $\psi_k = \psi_{ij,k}$ is Faddeev component.

The wave function in Faddeev formalism is the sum of three Faddeev components,

$$\Psi(\boldsymbol{x}, \boldsymbol{y}) = \psi_1(\boldsymbol{x}_1, \boldsymbol{y}_1) + \psi_2(\boldsymbol{x}_2, \boldsymbol{y}_2) + \psi_3(\boldsymbol{x}_3, \boldsymbol{y}_3). \tag{17}$$

Using relative Jacobi coordinates $\boldsymbol{x}_k = (\boldsymbol{r}_j - \boldsymbol{r}_i)$ and $\boldsymbol{y}_k = \frac{2}{\sqrt{3}}(\boldsymbol{r}_k - \frac{\boldsymbol{r}_i + \boldsymbol{r}_j}{2})$, one can expand these Faddeev components in bipolar harmonic basis:

$$\psi_k = \sum_\alpha \frac{F_\alpha(x_k, y_k)}{x_k y_k} \left| \left(l_x\,(s_i s_j)_{s_x}\right)_{j_x} (l_y s_k)_{j_y} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_x} t_k \right\rangle_{TT_z}, \tag{18}$$

where index α represents all allowed combinations of the quantum numbers presented in the brackets: l_x and l_y are the partial angular momenta associated with respective Jacobi coordinates, s_i and t_i are the spins and isospins of the individual particles. Functions $F_\alpha(x_k, y_k)$ are called partial Faddeev amplitudes. It should be noted that the total angular momentum J as well as its projection M are conserved, but the total isospin T of the system is not conserved due to the presence of charge dependent terms in nuclear interactions.

Boundary conditions for Eq. (16) can be written in the Dirichlet form. Thus, Faddeev amplitudes satisfy the regularity conditions:

$$F_\alpha(0, y_k) = F_\alpha(x_k, 0) = 0. \tag{19}$$

For neutron-deuteron scattering with energies below the break-up threshold, Faddeev components vanish for $\mathbf{x}_k \to \infty$. If $\mathbf{y}_k \to \infty$, then interactions between the particle k and the cluster ij are negligible, and Faddeev components ψ_i and ψ_j vanish. Then, for the component ψ_k, which describes the plane wave of the particle k with respect to the bound particle pair ij,

$$
\lim_{y_k \to \infty} \psi_k(\mathbf{x}_k, \mathbf{y}_k)_{l_n j_n} = \frac{1}{\sqrt{3}} \sum_{j'_n l'_n} \left| \{\phi_d(\mathbf{x}_k)\}_{j_d} \otimes \{Y_{l'_n}(\hat{\mathbf{y}}_k) \otimes s_k\}_{j'_n} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_d} t_k \right\rangle_{\frac{1}{2}, -\frac{1}{2}}
$$
$$
\times \frac{i}{2} \left[\delta_{l'_n j'_n, l_n j_n} h_{l'_n}^-(p r_{nd}) - S_{l'_n j'_n, l_n j_n} h_{l'_n}^+(p r_{nd}) \right], \tag{20}
$$

where deuteron, being formed from nucleons i and j, has quantum numbers $s_d = 1$, $j_d = 1$, and $t_d = 0$, and its wave function $\phi_d(\mathbf{x}_k)$ is normalized to unity. Here, $r_{nd} = (\sqrt{3}/2)y_k$ is relative distance between neutron and deuteron target, and $h_{l_n}^\pm$ are the spherical Hankel functions. The expression (20) is normalized to satisfy a condition of unit flux for nd scattering wave function.

For the cases where Urbana type three-nucleon interaction (TNI) is included, we modify the Faddeev equation (16) into

$$
(E - H_0 - V_{ij}) \psi_k = V_{ij}(\psi_i + \psi_j) + \frac{1}{2}(V_{jk}^i + V_{ki}^j)\Psi \tag{21}
$$

by noting that the TNI among particles ijk can be written as sum of three terms: $V_{ijk} = V_{ij}^k + V_{jk}^i + V_{ki}^j$.

D. Evaluation of matrix elements

Due to anti-symmetry of the total wave function in isospin basis, one has $\langle \Psi | V_{12} + V_{23} + V_{31} | \Psi \rangle = 3 \langle \Psi | V_{ij} | \Psi \rangle$ for any pair $i \neq j$.

Using decomposition of momentum $\boldsymbol{p}$,

$$
\boldsymbol{p} = -i \nabla_x = -i \left(\hat{x} \frac{\partial}{\partial x} + \frac{1}{x} \hat{\nabla}_\Omega \right), \tag{22}
$$

we can represent general matrix elements of local two-body parity violating potential operators as

$$
{}^{(-)}\langle \Psi_f | O | \Psi_i \rangle^{(+)} = (\frac{\sqrt{3}}{2})^3 \sum_{\alpha\beta} \left[\int dx x^2 dy y^2 \left(\frac{\widetilde{F}_{f,\alpha}^{(+)}(x,y)}{xy} \right) \hat{X}(x) \left(\frac{\widetilde{F}_{i,\beta}^{(+)}(x,y)}{xy} \right) \right] \langle \alpha | \hat{O}(\hat{x}) | \beta \rangle \tag{23}
$$

where ($\pm$) means outgoing and incoming boundary conditions and $\hat{X}(x)$ is derivative of scalar function or derivative of wave function with respect to x. (Note that we have used the fact that $(\widetilde{F}^{(-)})^* = \widetilde{F}^{(+)}$.) The partial amplitudes $\widetilde{F}_{i(f),\alpha}(x,y)$ represent the total systems wave function in one selected basis set among three possible angular momentum coupling sequences for three particle angular momenta:

$$\Psi_{i(f)}(x,y) = \sum_{\alpha} \frac{\widetilde{F}_{i(f),\alpha}(x,y)}{xy} \left| \left(l_x \left(s_i s_j \right)_{s_x} \right)_{j_x} \left(l_y s_k \right)_{j_y} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_x} t_k \right\rangle_{TT_z}. \tag{24}$$

The "angular" part of the matrix element is

$$\langle \alpha | \hat{O}(\hat{x}) | \beta \rangle \equiv \int d\hat{x} \int d\hat{y} \mathcal{Y}_{\alpha}^{\dagger}(\hat{x}, \hat{y}) \hat{O}(\hat{x}) \mathcal{Y}_{\beta}(\hat{x}, \hat{y}), \tag{25}$$

where $\mathcal{Y}_{\alpha}(\hat{x}, \hat{y})$ is a tensor bipolar spherical harmonic with a quantum number α. One can see that operators for "angular" matrix elements have the following structure:

$$\hat{O}(\hat{x}) = (\tau_2 \odot \tau_3)(\boldsymbol{\sigma}_2 \circledcirc \boldsymbol{\sigma}_3) \cdot \hat{x}, \text{ or } (\tau_2 \odot \tau_3)(\boldsymbol{\sigma}_2 \circledcirc \boldsymbol{\sigma}_3) \cdot \nabla_{\Omega}, \tag{26}$$

where $\odot, \circledcirc = \pm, \times$. The explicit values of these matrix elements are summarized in the appendix.

III. RESULTS AND DISCUSSIONS

As it was mentioned in the previous section, because of low energy property of $R_{\alpha'\alpha}^{J}$, it is convenient to present results for elements R_{l_y',l_y}^{J} in terms of a ratio,

$$\frac{R_{\alpha'\alpha}^{J}(p)}{4\mu i^{-l_y'+l_y+1} p^{l_y'+l_y+1}} = \frac{1}{p^{l_y'+l_y}} {}^{(-)}\langle \Psi, (l_y'\mathcal{S}')JJ_z | V_n^{PV} | \Psi, (l_y\mathcal{S})JJ_z \rangle^{(+)} \tag{27}$$

For the case of parity violation, we fix $l_y' = 1$ and $l_y = 0$. To obtain the observable parameters when neutron energies are larger than thermal ones (which correspond to zero energy limit for neutron spin rotation), one can use a simple extrapolation based on the above representation with a good accuracy up to hundreds keV.

The contributions to parity violating matrix elements $\frac{2}{\pi} \frac{1}{c_n} \text{Im} \left[\frac{R_{\alpha'\alpha}^{J}(p)}{4\mu p^2} \right]$ from different terms of parity violating potentials (see Table I) are presented in the Table III. These matrix elements were calculated using strong AV18+UIX and weak DDH-II parity violating potentials for the case of low neutron energies (up to thermal ones). From this table, one can see that

TABLE III: Contributions to $\frac{2}{\pi}\mathrm{Im}\left[\frac{R^J_{1S',0J}(p)}{4\mu p^2}\right]$ at very low energy in fm^2 units. We chose AV18+UIX as strong potential and DDH-II as parity violating potential. Matrix elements of $n = 6, 7$ are zero due to of isospin structure.

n	$\mathcal{S}' = \frac{1}{2}, J = \frac{1}{2}$	$\mathcal{S}' = \frac{3}{2}, J = \frac{1}{2}$	$\mathcal{S}' = \frac{1}{2}, J = \frac{3}{2}$	$\mathcal{S}' = \frac{3}{2}, J = \frac{3}{2}$
1	0.229D+00	0.119D+00	-0.137D-01	-0.474D+00
2	-0.165D-01	-0.949D-02	0.800D-02	0.436D-03
3	0.308D-02	0.210D-01	-0.388D-02	-0.258D-03
4	0.371D-02	-0.140D-01	0.200D-03	0.723D-04
5	0.431D-02	-0.161D-01	0.284D-03	0.602D-04
8	0.172D-02	0.163D-01	-0.273D-02	-0.207D-03
9	-0.509D-02	0.871D-02	0.968D-03	0.252D-04
10	0.352D-02	-0.133D-01	0.189D-03	0.685D-04
11	0.411D-02	-0.153D-01	0.270D-03	0.573D-04
12	0.410D-02	0.150D-03	-0.202D-03	-0.953D-02
13	0.657D-02	0.103D-02	-0.342D-03	-0.159D-01

the main contribution to PV effects comes from $J = 3/2$ channel for the "best values" of DDH coupling constants.

Our results for the angle of neutron spin rotation for DDH, pionless EFT, and pionful EFT weak interaction potentials with different sets of parameters are summarized in Tables IV,V, and VI. For these calculations, we used two types of strong interacting potentials: Argonne two nucleon interaction AV18 and inclusion of Urbana IX three nucleon interaction, AV18+UIX. One can see that these results practically do not depend on a choice of the strong interaction potential. Also, it is clear that the matrix element related to pion-exchange ($n = 1$) is dominant for DDH potential, slightly enhanced for pionfull potential, and about equal to other ones for pionless potential.

The neutron spin asymmetry P was calculated for laboratory neutron energy $E = 15\ KeV$. The results are summarized in tables VII, VIII, and IX for DDH, pionless EFT, and pionful EFT weak interaction potentials with different sets of parameters, correspondingly. These results provide a pattern similar to that of the results for the angle of neutron

TABLE IV: Coefficients I_n^{DDH} for AV18 and AV18+UIX strong potentials, and DDH-I and DDH-II parameter sets for parity violating potentials. $I_{6,7}^{DDH} = 0$.

n	DDH-I/AV18	DDH-I/AV18+UIX	DDH-II/AV18	DDH-II/AV18+UIX
1	0.555D+02	0.541D+02	0.558D+02	0.544D+02
2	0.604D+00	0.659D+00	0.103D+01	0.112D+01
3	-0.118D+01	-0.120D+01	-0.192D+01	-0.196D+01
4	0.827D+00	0.848D+00	0.119D+01	0.122D+01
5	0.889D+00	0.900D+00	0.139D+01	0.141D+01
8	-0.114D+01	-0.118D+01	-0.145D+01	-0.151D+01
9	-0.558D+00	-0.564D+00	-0.713D+00	-0.723D+00
10	0.906D+00	0.929D+00	0.112D+01	0.115D+01
11	0.101D+01	0.102D+01	0.133D+01	0.135D+01
12	0.899D+00	0.892D+00	0.127D+01	0.127D+01
13	0.131D+01	0.130D+01	0.205D+01	0.204D+01

TABLE V: Coefficients $I_n^{\not{\pi}}$ for AV18 and AV18+UIX strong potentials, and $\not{\pi}$EFT-I and $\not{\pi}$EFT-II parameter sets for parity violating potentials. $I_{2,3,5,6,7,10,11,12,13}^{\not{\pi}} = 0$.

n	$\not{\pi}$EFT-I/AV18	$\not{\pi}$EFT-I/AV18+UIX	$\not{\pi}$EFT-II/AV18	$\not{\pi}$EFT-II/AV18+UIX
1	0.558D+02	0.544D+02	0.879D+00	0.879D+00
4	0.550D+02	0.534D+02	0.453D+00	0.467D+00
8	-0.690D+02	-0.686D+02	-0.614D+00	-0.643D+00
9	-0.858D+01	-0.601D+01	-0.309D+00	-0.315D+00

spin rotation. The parameter J_n in these tables is defined as

$$J_n \equiv \frac{1}{c_n} \frac{2}{\pi} \mathrm{Re} \left[\frac{1}{4\mu p^2} \left(R_{1\frac{1}{2},0\frac{1}{2}}^{\frac{1}{2}} - 2\sqrt{2} R_{1\frac{3}{2},0\frac{1}{2}}^{\frac{1}{2}} + 4 R_{1\frac{1}{2},0\frac{3}{2}}^{\frac{3}{2}} - 2\sqrt{5} R_{1\frac{3}{2},0\frac{3}{2}}^{\frac{3}{2}} \right) \right], \tag{28}$$

and is related to the parameter $\tilde{I}_n$ in the expression $P = \sum c_n \tilde{I}_n$ by

$$\tilde{I}_n = \frac{\frac{2}{3}(2\pi\mu p^2) J_n}{\mathrm{Re} \left[R_{0\frac{1}{2},0\frac{1}{2}}^{\frac{1}{2}} + 2 R_{0\frac{3}{2},0\frac{3}{2}}^{\frac{3}{2}} \right]} = \frac{8\pi^2 \mu}{9} \frac{J_n}{\sigma_{tot}}, \tag{29}$$

TABLE VI: Coefficients I_n^π for AV18 and AV18+UIX strong potentials, and πEFT-I and πEFT-II parameter sets for parity violating potentials. $I_{2,3,6,7,10,11,12}^\pi = 0$.

n	πEFT-I/AV18	πEFT-I/AV18+UIX	πEFT-II/AV18	πEFT-II/AV18+UIX
1	0.558D+02	0.544D+02	0.558D+02	0.544D+02
4	0.138D+01	0.129D+01	0.498D+00	0.443D+00
5	0.395D+01	0.168D+01	0.112D+01	0.600D-01
8	-0.167D+01	-0.162D+01	-0.710D+00	-0.679D+00
9	-0.744D+00	-0.663D+00	-0.308D+00	-0.261D+00
13	0.205D+02	0.198D+02	0.880D+01	0.849D+01
14	0.308D+01	0.302D+01	0.160D+01	0.158D+01
15	0.594D+02	0.572D+02	0.248D+02	0.239D+02

where σ_{tot} is the total $n-d$ cross section. The total cross section σ_{tot} can be calculated, or one can use its known experimental value.

From the presented data, one can see that the results of our calculations are only slightly different for the cases when we use AV18 and AV18+UIX strong Hamiltonians. This indicates stability of the results with respect to the three nucleon forces. Indeed, by analyzing the DDH one-pion exchange matrix element (see Table III), one can see that for DDH-I with potentials AV18 and AV18+UIX, the contributions to the $I_{n=1}$ are -0.163D+01 and -0.302D+01 for doublet channel ($J = 1/2$), and for the quartet channel ($J = 3/2$) they are 0.571D+02 and 0.571D+02, correspondingly. The quartet channel is dominated by the repulsive and long-range part of the strong interactions, but the doublet channel is defined by attractive part. Therefore the quartet channel is less sensitive to the off-energy shell structure of the strong interactions compared to the doublet channel. Then, due to the dominant contribution from the quartet channel, the net result turns to be rather independent on the contribution from three nucleon forces. This fact demonstrates the independence of our results on models of strong interactions. However, further investigations with different strong interaction potentials are desirable.

It should be noted, that the dependence on cutoff parameters for the contributions from potentials with short and middle range interactions, even though it appears large, does not lead to cutoff dependence for the observable parameters. Indeed, the renormalization of

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TABLE VII: Coefficients J_n^{DDH} for AV18 and AV18+UIX strong potentials, and DDH-I and DDH-II parameter sets for parity violating potentials at $E = 15$ KeV in the laboratory frame. $J_{6,7}^{DDH} = 0$.

n	DDH-I/AV18	DDH-I/AV18+UIX	DDH-II/AV18	DDH-II/AV18+UIX
1	0.226D+00	0.227D+00	0.228D+00	0.228D+00
2	0.220D-02	0.219D-02	0.349D-02	0.344D-02
3	-0.171D-02	-0.132D-02	-0.281D-02	-0.218D-02
4	0.693D-03	0.354D-03	0.993D-03	0.508D-03
5	0.762D-03	0.398D-03	0.119D-02	0.621D-03
8	-0.158D-02	-0.120D-02	-0.205D-02	-0.157D-02
9	-0.214D-03	0.489D-04	-0.237D-03	0.103D-03
10	0.759D-03	0.388D-03	0.941D-03	0.481D-03
11	0.862D-03	0.450D-03	0.113D-02	0.592D-03
12	0.335D-02	0.331D-02	0.473D-02	0.468D-02
13	0.504D-02	0.501D-02	0.783D-02	0.778D-02

TABLE VIII: Coefficients $J_n^{\not{\pi}}$ for AV18 and AV18+UIX strong potentials, and $\not{\pi}$EFT-I and $\not{\pi}$EFT-II parameter sets for parity violating potentials. $J_{2,3,5,6,7,10,11,12,13}^{\not{\pi}} = 0$.

n	$\not{\pi}$EFT-I/AV18	$\not{\pi}$EFT-I/AV18+UIX	$\not{\pi}$EFT-II/AV18	$\not{\pi}$EFT-II/AV18+UIX
1	0.228D+00	0.228D+00	0.333D-02	0.330D-02
4	0.454D-01	0.216D-01	0.379D-03	0.194D-03
8	-0.100D+00	-0.766D-01	-0.884D-03	-0.684D-03
9	-0.221D-02	0.301D-02	-0.829D-04	0.661D-04

low energy constants would cancel those cutoff dependencies by the cutoff dependencies of LECs. Therefore, as a result, calculated PV observables are practically cutoff independent.

All these tables present information about contributions of different PV operators to PV effects, provided we know corresponding weak coupling constants. Then, to calculate parity violating effects, we can use either DDH potential or one of the considered EFT potentials. However, for the case of EFT potentials, we need to know a set of LECs which cannot be calculated in the given theoretical framework but must be obtained from a number of inde-

TABLE IX: Coefficients J_n^π for AV18 and AV18+UIX strong potentials, and πEFT-I and πEFT-II parameter sets for parity violating potentials. $J_{2,3,6,7,10,11,12}^\pi = 0$.

n	πEFT-I/AV18	πEFT-I/AV18+UIX	πEFT-II/AV18	πEFT-II/AV18+UIX
1	0.228D+00	0.228D+00	0.228D+00	0.228D+00
4	0.954D-03	0.319D-03	0.279D-03	0.311D-04
5	0.665D-02	0.459D-02	0.262D-02	0.198D-02
8	-0.248D-02	-0.190D-02	-0.114D-02	-0.900D-03
9	-0.137D-03	0.268D-03	-0.269D-04	0.149D-03
13	0.874D-01	0.878D-01	0.377D-01	0.379D-01
14	0.123D-01	0.122D-01	0.639D-02	0.638D-02
15	0.253D+00	0.255D+00	0.107D+00	0.107D+00

pendent experiments. Unfortunately, currently available experimental data are not enough to define the LECs with required precision. Even for pionless EFT, the estimated LECs [1] have large uncertainties preventing us from predicting the values of PV effects. For the pionful EFT, the situation with determination of LECs is even worse. Therefore, it is impossible to make reliable predictions for PV effects using EFT-type potentials at this time, and the only reasonable way to estimate magnitudes of PV effects is to use the DDH potential. Taking into account the difficulty of the systematic description of PV effects using "standard" DDH potentials (see discussions in the introduction), we estimate PV effects using the DDH potential for different sets of weak coupling constants: both for the "best value" coupling constants and for two possible sets of the values of the coupling constants recently obtained by Bowman [25] from the fit of reliable existing experimental data (see Table X). The results for these three sets of weak coupling constants are summarized in Tables XI and XII for the angle of spin rotation and for neutron spin asymmetry, correspondingly. One can see that in contrast to the fact that the one-pion exchange dominates in the DDH-"best" coupling parameter set, the rho meson exchange dominates in the case of Bowman's coupling parameter set. One can see that the angle of neutron spin rotation has almost the same magnitude for all three sets of parameters, but it has opposite signs for the "best value" set and for the Bowman's fits. The neutron spin asymmetry does not only have opposite signs but also essentially different values for these two choices of parameters. This allows

TABLE X: DDH PV coupling constants in units of 10^{-7}. Strong couplings are $\frac{g_\pi^2}{4\pi} = 13.9$, $\frac{g_\rho^2}{4\pi} = 0.84$, $\frac{g_\omega^2}{4\pi} = 20$, $\kappa_\rho = 3.7$, and $\kappa_\omega = 0$, h'_ρ contribution is neglected. 4-paramter fir and 3-parameter fit uses the same h_ρ^1 and h_ω^1 with DDH 'best'.

DDH Coupling	DDH 'best'	4-parameter fit[25]	3-parameter fit[25]
h_π^1	+4.56	−0.456	−0.5
h_ρ^0	−11.4	−43.3	−33
h_ρ^2	−9.5	37.1	41
h_ω^0	−1.9	13.7	0
h_ρ^1	−0.19	−0.19	−0.19
h_ω^1	−1.14	−1.14	−1.14

TABLE XI: Neutron spin rotation in 10^{-7} rad-cm^{-1} for the case of DDH-II potential with AV18+UIX strong potential for a liquid deuteron density $N = 0.4 \times 10^{23}$ atoms per cm^3.

	DDH 'best'	4-parameter fit[25]	3-parameter fit[25]
1	0.976E-01	-0.976E-02	-0.107E-01
2	0.350E-02	0.133E-01	0.101E-01
3	-0.288E-01	-0.109E+00	-0.833E-01
4	0.316E-04	0.316E-04	0.316E-04
5	0.136E-03	0.136E-03	0.136E-03
8	-0.383E-02	0.276E-01	0.000E+00
9	-0.183E-02	0.132E-01	0.000E+00
10	0.878E-03	0.878E-03	0.878E-03
11	0.103E-02	0.103E-02	0.103E-02
12	0.930E-03	0.930E-03	0.930E-03
total	0.697E-01	-0.619E-01	-0.809E-01

one to choose between two possible sets of DDH parameters and, as a consequence, to test the dominance of pion-meson contribution in PV effects in $n - d$ scattering.

Finally, we would like to mention that our results are quite different from the results obtained in paper [15]. For example, in paper [15], the values of I_n for $J = \frac{1}{2}$ and $J = \frac{3}{2}$ have

TABLE XII: Neutron spin asymmetry for the case of DDH-II potential with AV18+UIX strong potential (the total cross section $\sigma_{tot} = 3.35$ b at $E = 15 \ keV$).

	DDH 'best'	4-parameter fit[25]	3-parameter fit[25]
1	0.848D-08	-0.848D-09	-0.930D-09
2	0.222D-09	0.845D-09	0.644D-09
3	-0.664D-09	-0.252D-08	-0.192D-08
4	0.274D-12	0.274D-12	0.274D-12
5	0.124D-11	0.124D-11	0.124D-11
8	-0.828D-10	0.597D-09	0.000D+00
9	0.543D-11	-0.391D-10	0.000D+00
10	0.760D-11	0.760D-11	0.760D-11
11	0.936D-11	0.936D-11	0.936D-11
12	0.714D-10	0.714D-10	0.714D-10
total	0.805D-08	-0.188D-08	-0.212D-08

the same signs for operator with $n = 1$, but our results show opposite signs for these matrix elements. Another discrepancy is related to the systematic difference between the values of matrix elements calculated [15] for AV18 and AV18+UIX potentials, which indicates a large wave function difference for AV18 and AV18+UIX potentials. Contrary to those, our results show that these matrix elements are insensitive to the presence of the three nucleon force. [1]

IV. CONCLUSION

We have calculated parity violating angle of neutron spin rotation and asymmetry in transmission of neutrons with opposite helicities for low energy neutron deuteron scattering. Using Distorted Wave Born Approximation for weak interactions with realistic three-nucleon wave functions from Faddeev equations in configuration space, we have parameterized PV observables in terms of matrix elements presented in the DDH weak potential and in weak potentials derived from pionless and pionful EFTs. It is shown that our results practically

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[1] We thanks R. Schiavilla and M. Viviani for discussions which clarified that the reason for theses discrepancies is related to numerical errors in paper [15].

do not depend on the choice for strong interaction potentials and on cutoff parameters.

Based on the given analysis, one can see that for DDH potential, the dominant contribution to observable PV effects comes from the pion-exchange matrix element with $n = 1$. However, for pionless EFT potential, all types of matrix elements contribute almost equally, and for pionful EFT potential the pion-exchange matrix element is sightly enhanced as compared to the other ones. Therefore, it would be interesting to compare the estimation of observable PV effects using appropriate LECs and coupling constants for DDH. Unfortunately, due to insufficient data for LECs this is impossible at this time. However, a comparison of PV effects for two different sets of coupling constants shows that $n-d$ scattering experimental results can be used to distinguish between different sets of DDH coupling constants and to help in clarification of the issue about the importance of the contribution of pion-exchange weak potential.

Appendix: Explicit results of angular part of Matrix elements

Explicit values of matrix elements of iso-spin operators for two-body states are

$$\langle T'T_z'|\tau_1 \cdot \tau_2|TT_z\rangle = \delta_{T_z',T_z}\delta_{T',T}[1\delta_{T,1} - 3\delta_{T,0}],$$

$$\langle T'T_z'|(\tau_1 + \tau_2)^z|TT_z\rangle = \delta_{T'1}\delta_{T1}\delta_{T_z'T_z}[2T_z],$$

$$\langle T'T_z'|(\tau_1 - \tau_2)^z|TT_z\rangle = \delta_{T',T\pm1}\delta_{T_z,T_z'}\delta_{T_z,0}[2],$$

$$\langle T'T_z'|i(\tau_1 \times \tau_2)^z|TT_z\rangle = \delta_{T_z',T_z}\delta_{T_z,0}\delta_{T',T\pm1}[\pm2],$$

$$\langle T'T_z'|\mathcal{T}_{12}^z|TT_z\rangle = \delta_{T',1}\delta_{T,1}\delta_{T_z',T_z}[2\delta_{T_z,1} - 4\delta_{T_z,0} + 2\delta_{T_z,-1}], \tag{A.1}$$

and matrix elements of orbital and spin operators for two-body states $|(l_x s_x)j_x j_x^z\rangle$ are

$$\langle(j_x \pm 1, 1)j_x j_x^z|(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{x}|(j_x, 1)j_x j_x^z\rangle$$

$$= \langle(j_x, 1)j_x j_x^z|(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{x}|(j_x \pm 1, 1)j_x j_x^z\rangle$$

$$= -2\sqrt{\frac{j_x + 1/2 \mp 1/2}{2j_x + 1}} \tag{A.2}$$

$$\langle(j_x \pm 1, 1)j_x j_x^z|(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{x}|(j_x, 0)j_x j_x^z\rangle$$

$$= \langle(j_x, 0)j_x j_x^z|(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{x}|(j_x \pm 1, 1)j_x j_x^z\rangle$$

$$= \mp2\sqrt{\frac{j_x + 1/2 \pm 1/2}{2j_x + 1}} \tag{A.3}$$

$$\langle (j_x, 0)j_x j_x^z | i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \hat{x} | (j_x \pm 1, 1)j_x j_x^z \rangle$$
$$= (-)\langle (j_x \pm 1, 1)j_x j_x^z | i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \hat{x} | (j_x, 0)j_x j_x^z \rangle$$
$$= \pm 2\sqrt{\frac{j_x + 1/2 \pm 1/2}{2j_x + 1}} \tag{A.4}$$

$$\langle (j_x \pm 11)j_x j_x^z | (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\nabla}_{\Omega_x} | (j_x 1)j_x j_x^z \rangle = \pm 2\frac{(j_x + 1/2 \mp 1/2)\sqrt{j_x + 1/2 \mp 1/2}}{\sqrt{2j_x + 1}}$$
$$\langle (j_x 1)j_x j_x^z | (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\nabla}_{\Omega_x} | (j_x \pm 11)j_x j_x^z \rangle = \mp 2\frac{(j_x + 1/2 \pm 3/2)\sqrt{j_x + 1/2 \mp 1/2}}{\sqrt{2j_x + 1}} \tag{A.5}$$

$$\langle (j_x \pm 11)j_x j_x^z | (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{\nabla}_{\Omega_x} | (j_x 0)j_x j_x^z \rangle = 2\frac{(j_x + 1/2 \mp 1/2)\sqrt{j_x + 1/2 \pm 1/2}}{\sqrt{2j_x + 1}}$$
$$\langle (j_x 0)j_x j_x^z | (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{\nabla}_{\Omega_x} | (j_x \pm 11)j_x j_x^z \rangle = -2\frac{(j_x + 1/2 \pm 3/2)\sqrt{j_x + 1/2 \pm 1/2}}{\sqrt{2j_x + 1}} \tag{A.6}$$

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