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Large-N reduction of SU(N) Yang-Mills theory with massive adjoint overlap fermions

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Abstract

We study four dimensional large-N SU(N) Yang-Mills theory coupled to adjoint overlap fermions on a single site lattice. Lattice simulations along with perturbation theory show that the bare quark mass has to be taken to zero as one takes the continuum limit in order to be in the physically relevant center-symmetric phase. But, it seems that it is possible to take the continuum limit with any renormalized quark mass and still be in the center-symmetric physics. We have also conducted a study of the correlations between Polyakov loop operators in different directions and obtained the range for the Wilson mass parameter that enters the overlap Dirac operator.

Keywords: 1/N Expansion, Eguchi-Kawai reduction, Adjoint fermions, Lattice Gauge Field Theories

The large N limit of gauge theories has many intriguing properties. One of these is continuum reduction [1]. It states that one obtains correct infinite volume zero temperature results by working on a finite volume lattice as long as the center symmetry is intact. In [2], it was proposed, that for a Yang-Mills theory with massless adjoint fermions with periodic boundary conditions, the volume can be reduced down to a single site as opposed to the pure gauge case [3], where weak coupling analysis shows all the center symmetries to be broken [4]. This has been confirmed both by lattice techniques and by perturbation theory [5, 6, 7, 8, 9, 10, 11, 12, 13].

The question we want to address in this paper is what occurs at the large N continuum limit when fermions have a mass. The large N continuum limit is taken by first extrapolating $N \to \infty$ and then $b \to \infty$, where b is the inverse 't Hooft coupling, $\frac{1}{g^2 N}$. It has been argued in [14] that for any finite mass, a center symmetry unbroken phase exists at sufficiently small volume. Lattice studies using Wilson fermions has shown a large range of masses at

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fixed lattice spacing where the center symmetry remains intact [10].

In this paper, we address the question of center symmetry both in the lattice and in the continuum using massive adjoint overlap fermions [16, 17]. We show that the critical bare quark mass μ_c, above which the center symmetry is broken, is zero at the continuum limit. However, on a lattice with a finite lattice spacing, $\mu_c > 0$. Values of masses, which are accessible to lattice simulations, depend on how μ_c scales as a function of the lattice spacing.

We study the problem with one Weyl fermion, $f = 0.5$, both by perturbation theory and lattice simulations. Using perturbation theory we show that center symmetry is broken even when quarks are given an arbitrarily small mass. We have performed lattice simulations with different b and N. The lattice results confirm with perturbation theory and we find a $\mu_c(b)$ that decreases as b increases. We do not see any evidence of scaling of $\mu_c(b)$ versus b. Our numerical results indicate that we can obtain the continuum limit with arbitrary physical mass for the adjoint quarks.

All details pertaining to the single site lattice model with adjoint overlap fermions are described in [9]. To study the continuum limit starting from the single site action, we use the weak coupling expansion and write the link matrices as

$$U_\mu = e^{ia_\mu} D_\mu e^{-ia_\mu}; \qquad D_\mu^{ij} = e^{i\theta_\mu^i}\delta_{ij}, \tag{1}$$

and perform an expansion in a_μ. The θ_μ^i are the eigenvalues of the Polyakov loop operator and they have to uniformly distributed in the range $[-\pi, \pi]$ and uncorrelated in all four directions in order to correctly reproduce infinite volume continuum perturbation theory. The leading order result is

$$S = \sum_{i\neq j} \left\{ \ln\left[\sum_\mu \sin^2\frac{1}{2}\left(\theta_\mu^i - \theta_\mu^j\right)\right] \right.$$
$$\left. -2f\ln\left[\frac{1+\mu^2}{2} + \frac{1-\mu^2}{2}\frac{2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m_w}{\sqrt{\left(2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m_w\right)^2 + \sum_\mu \sin^2(\theta_\mu^i - \theta_\mu^j)}}\right]\right\}, \tag{2}$$

where the first line of RHS is the contribution from gauge fields [4] and the second line is the contribution from f flavors of Dirac fermions [9]. The bare quark mass is $\frac{2m_w\mu}{\sqrt{1-\mu^2}}$ with $\mu \in [0, 1]$ and m_w is the Wilson mass parameter.

The gauge action has its minimum, $-\infty$, when all the angles θ_μ^i are equal. With one massless Weyl fermion ($f = 0.5$) the fermionic part cancels out the infinity and renders the action finite. In [9] we used Monte Carlo techniques to find out the actual minimum. Namely, we consider the Hamiltonian

$$H = \frac{1}{2}\sum_{\mu,i} \left(\pi^i_\mu\right)^2 + \beta S. \tag{3}$$

For large β, the Boltzmann measure e^{-H} is dominated by the minimum. Hence, this minimum can be found by performing a HMC update for the π, θ system.

To reduce rounding errors in equations of motions, we introduce a regulator Δ to the gauge field action

$$S_g \to \sum_{i\neq j} \ln\left[\sum_\mu \sin^2 \frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right) + \Delta\right]. \tag{4}$$

In the computations we choose $\Delta = 10^{-4}$, which is much smaller than the average difference between angles $2\pi/N$ when $N < 200$.

A choice for the order parameters associated with the Z_N^4 symmetries is [4]

$$P_\mu = \frac{1}{2}\left(1 - \frac{1}{N^2}|\mathrm{Tr}U_\mu|^2\right) = \frac{1}{N^2}\sum_{i,j}\sin^2\frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right). \tag{5}$$

If $P_\mu = \frac{1}{2}$, then the Z_N symmetry in that direction is unbroken.

In Fig. 1 we reproduce the results of [9] with a large N scaling. To better observe the center symmetry breaking, the measurements P_i are ordered for each configuration s.t. $P_1 < P_2 < P_3 < P_4$. This indicates that center symmetry is probably restored when Wilson mass is in the range $3.0 < m_w < 10.0$. The range of m_w with broken symmetry does not depend on N.

It is possible that center symmetry is broken in a subtle manner in the range $3.0 < m_w < 10.0$. For example, the eigenvalues of the individual Polyakov loop operator might be uniformly distributed but they might show correlations in different directions[1]. In order to have a correct sum over all *momenta* as one would have in a infinite lattice, we need to ensure that the traces of Polyakov loops vanish and there are no correlations between different directions on the single site model. Let us assume that the eigenvalues are uniformly distributed and choose $\frac{2\pi j}{N}$, $j = 1, \cdots, N$ as the N eigenvalues. Let π_j, $j = 1\cdots, N$ denote a permutation of $j = 1\cdots, N$. We compute the correlated action, S_c, with $\theta^j_\mu = \frac{2\pi j}{N}$ and compare it to the uncorrelated action, S_u, with $\theta^j_\mu = \frac{2\pi\pi^\mu_j}{N}$ where π^μ are different permutations for different μ.

Fig. 2 shows the difference, $S_u - S_c$, as a function of Wilson mass m_w with different N. The value for S_u is obtained by averaging over several different

[1]This is the problem with quenched Equchi-Kawai model [15].

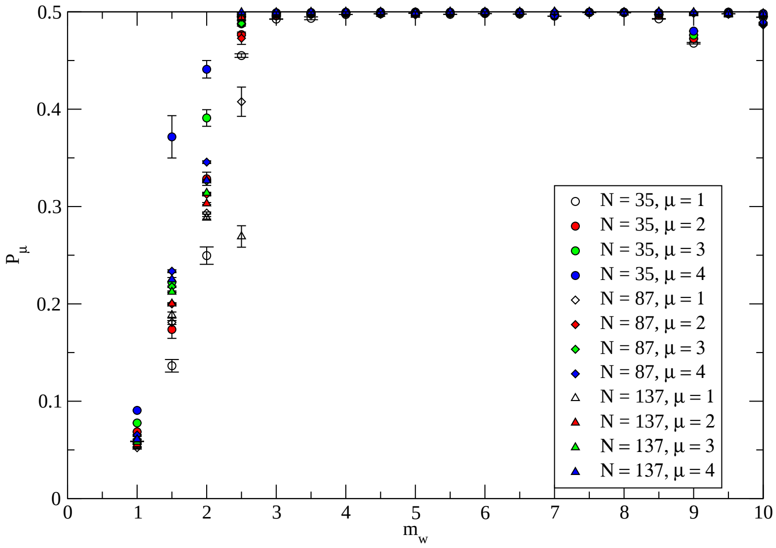

Figure 1: Plot of P_μ as a function of Wilson mass.

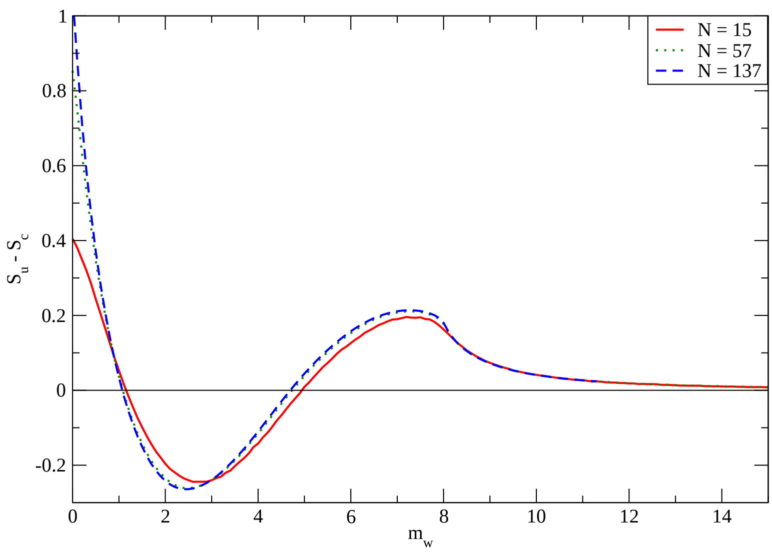

Figure 2: The difference between actions with correlated S_c and uncorrelated eigenvalues to each direction.

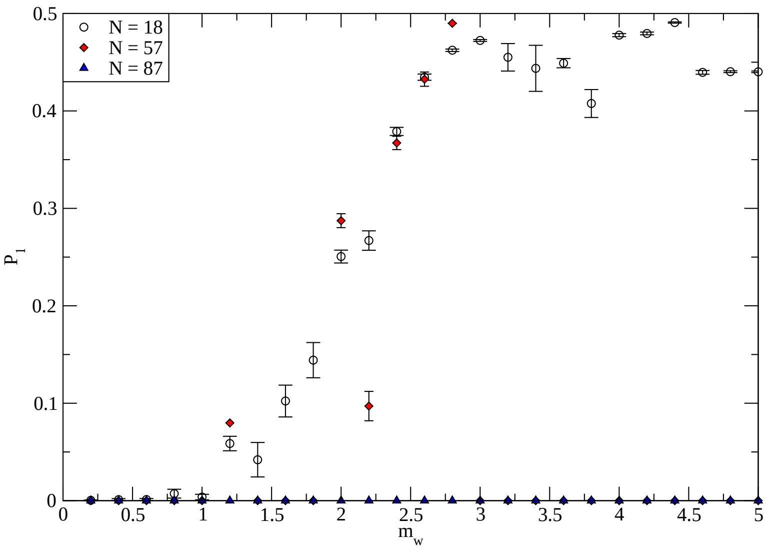

Figure 3: Plot of P_μ as a function of m_w with massive quarks.

random permutations but the fluctuations get smaller as N increases and it is sufficient to consider just one random permutation as $N \to \infty$. The uncorrelated minimum is preferred when $1 < m_w < 5$. There is again virtually no dependence on N. This combined with the restriction of center symmetry restoration gives

$$3 < m_w < 5 \tag{6}$$

as the range for Wilson mass.

One might wonder why the region of allowed m_w does not include zero. In a typical free field analysis of overlap fermions [18, 19], one shows that it correctly represents a single Dirac flavor in a region around zero momentum as long as $0 < m_w < 2$. Momentum in our case is replaced by $(\theta^i_\mu - \theta^j_\mu)$ and we want to cover the whole range of allowed momenta. If this does not occur, we will not have proper reduction or a correct realization of the center symmetric phase. Because the range of allowed momenta (volume of the Brillouin zone) in the conventional free field analysis increases as m_w increases, we see why m_w close to zero is not appropriate. Since we do not have a concept of doublers on a single site lattice, we do not require $m_w < 2$ [9]. Therefore, to reach momenta close to π and have proper sampling of all momenta as per the infinite lattice, we find a range of allowed m_w than includes $m_w > 2$. One can also understand why m_w cannot be

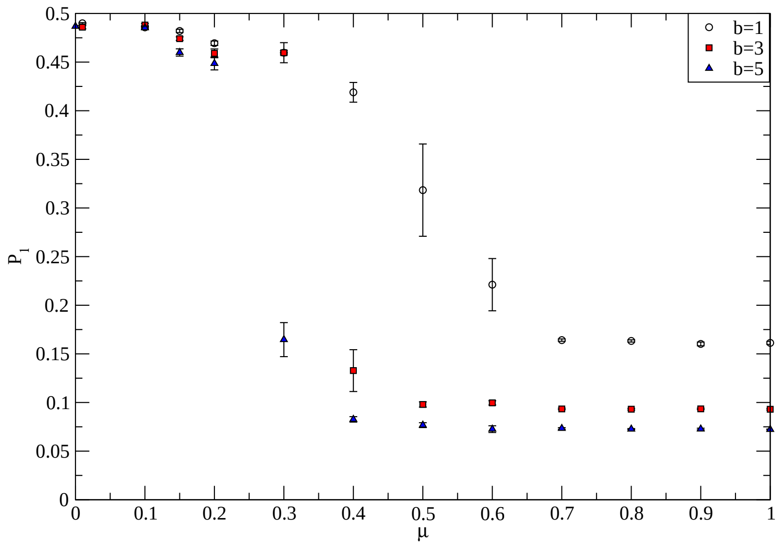

Figure 4: Plot of P_1 as a function of mass for three different b with $N = 15$

arbitrarily large since we would be approaching the limit of naïve fermions which does not have a center symmetric phase on a single site lattice [9].

Once fermions have a non-zero mass, the fermionic contribution to (2) is always finite. Then the minimum of the perturbative action is dominated by the pure gauge part and occurs when all the eigenvalues are the same. The effect of finite N is is demonstrated in Fig. 3 for $\mu = 0.1$. We have only plotted the component P_1, since it determines the center symmetry breaking point. The symmetry breaking is evident as $N \to \infty$.

For the actual lattice simulation, we used HMC-algorithm described in [9]. All the simulations were performed with $f = 0.5$, Wilson mass $m_w = 5$, [2] and they consist of about 100 independent measurements. Thermalization is fast and requires only about ten iterations. Most of the simulations were performed with $N = 15$, but to study $1/N$ effects we did also simulations with $N = 11$ and $N = 18$. The purpose of the simulations are to find out the critical mass μ_c for center symmetry breaking as a function of N and b.

In Fig. 4 we have plotted P_1 as a function of mass with $N = 15$ for

[2] This value is slightly high, since it is on the high end of (6). This is because the argument presented with regard to Fig. 2 was realized after we obtained the numerical results presented in this section.

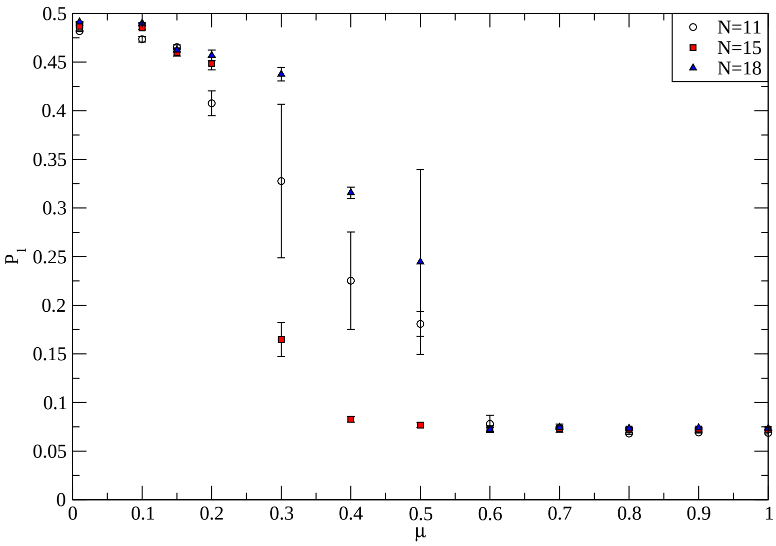

Figure 5: Plot of P_1 as a function of mass for three different N with $b = 5$

$b = 1, 3$, and 5. The data shows that $\mu_c(b)$ does decrease with increasing b but the decrease is clearly slower than scaling would dictate. The range of center symmetry breaking is between 0.1 and 0.3 for b in the range $[1, 5]$. Ignoring wave function renormalization, the dominant part of the scaling dictates that we need to keep $\mu e^{\frac{8\pi^2}{3} b}$ fixed as we take $b \to \infty$ in order to take the continuum limit at a fixed physical mass. Our data for $\mu_c(b)$, therefore, clearly indicates that we can take the continuum limit of a massive adjoint fermion coupled to a large N gauge field without any restriction on its physical mass.

To understand the effects of finite N, we performed simulations with $b = 5$ also at $N = 11$ and $N = 18$. The $1/N$ effects are rather small except in the region of the phase transitions as can be seen in Fig. 5. The critical value at $b = 5$ is about 0.1.

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