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Disconnected contributions to hadronic structure

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We present an update of an on-going project to determine the disconnected contributions to hadronic structure, specifically, the scalar matrix element, $\langle N|\bar{q}q|N\rangle$, and the quark contribution to the spin of the nucleon $\Delta q = \langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle/m_N$.

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1. Introduction

In the last few years there has been an upsurge in interest in calculating the scalar matrix element on the lattice [1 – 5] either directly, by calculating the corresponding connected and disconnected terms, or indirectly, via the Feynman-Hellman theorem. High statistics for dynamical simulations mean that reasonable signals can be obtained for disconnected terms and similarly the small statistical uncertainty on nucleon mass as a function of the quark masses enable reasonable fits to be made. Ideally, the results of both approaches should agree.

Such calculations have also become particularly timely since the advent of the LHC because the scalar coupling $f_{T_q} = m_q\langle N|\bar{q}q|N\rangle/m_N$ determines the fraction of the proton mass m_N that is carried by quarks of flavour q. The strength of the coupling of the Standard Model (SM) Higgs boson or of any similar scalar particle to the proton is mainly determined by $\sum_q f_{T_q}$ for $q \in \{u,d,s\}$. Therefore, an accurate calculation of these quantities will help to increase the precision of SM phenomenology and to shed light on non-SM processes.

The spin of the nucleon can be decomposed into a quark spin contribution $\Delta\Sigma = \Delta u + \Delta d + \Delta s + \ldots$, a quark angular momentum contribution L_q and a gluonic contribution (spin and angular momentum) ΔG:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G\,. \tag{1.1}$$

Experimentally, Δs is not well determined: HERMES obtained [6] $\Delta s = -0.085(13)(8)(9)$ in the $\overline{MS}$ scheme. However, the signal is dominated by contributions in the small x region where models are used to extrapolate from the experimental results obtained at larger x.

In these proceedings we present an update of an on-going project to calculate f_{T_q} and Δq. In particular, the following improvements have been implemented since Lattice 2009 [7]:

- Statistics on the $24^3 \times 48$ and $32^3 \times 64$ lattices have been significantly increased, by factors of roughly two and three, respectively, using the SFB/TR55 QPACE computers [8, 9] (details are given in the next section).
- An additional volume of $40^3 \times 64$ has been analyzed.

The analysis is not yet finalized and we plan to further increase the statistics for the larger two volumes. The results presented here are preliminary.

2. Simulation details

The simulations were performed on $n_{\rm f} = 2$ configurations of nonperturbatively improved clover fermions with Wilson gauge action at $\beta = 5.29$ and $\kappa_{\rm sea} = 0.13632$. Details of the volumes and the number of trajectories analyzed are given in table 1. The pseudoscalar mass corresponding to this $\kappa_{\rm sea}$ value is around 270 MeV, using an inverse lattice spacing of 2.59 GeV determined from $r_0(\beta,\kappa) = 0.467$ fm.

The scalar and axial-vector matrix elements we are interested in are extracted on the lattice from the three-point functions corresponding to the diagrams given in fig. 1. The axial-vector matrix element is related to Δq through,

$$\langle N,s|\bar{q}\gamma_\mu\gamma_5 q|N,s\rangle \;=\; 2m_N s_\mu \frac{\Delta q}{2}, \tag{2.1}$$

Figure 1: The connected (left) and disconnected (right) diagrams associated with the scalar ($\Gamma = \mathbb{1}$) and axial vector matrix elements ($\Gamma = \gamma_5\gamma_i$).

in Minkowski space notation, where m_N is the nucleon mass and s_μ its spin ($s_\mu^2 = -1$). The connected three-point functions were calculated separately by QCDSF, details of which can be found in refs. [10, 11]. The study presented here is only concerned with the disconnected terms. For Δs and $\langle N|\bar{s}s|N\rangle$ only these terms contribute.

We varied the quark mass of the current insertion (disconnected loop), as well as the mass of the valence quarks in the nucleon. In the following, we denote the κ value corresponding to the quark loop and the valence quarks in the nucleon as $\kappa_{\rm loop}$ and $\kappa_{\rm val}$, respectively. All combinations $\kappa_{\rm loop}, \kappa_{\rm val} \in \{0.13550, 0.13609, 0.13632\}$ were used. These values correspond to the pseudoscalar masses, $m_{\rm PS} \approx 690$, 440 and 270 MeV, respectively. The heaviest $\kappa_{\rm val} = 0.13550$ roughly corresponds to the strange quark mass.

$\beta = 5.29$, $n_{\rm f} = 2$, $\kappa_{\rm sea} = 0.13632$			
Volume	$24^3 \times 48$	$32^3 \times 64$	$40^3 \times 64$
# traj.	≈ 2000	≈ 1600	≈ 730
$m_{\rm PS}L$	2.6	3.4	4.3

Table 1: Details of the configurations used.

The disconnected contributions to Δq and $\langle N|\bar{q}q|N\rangle$ were extracted from the ratios of three-point functions to two-point functions (at zero momentum),

$$R^{\rm dis}(t_{\rm f}, t, t_{\rm i}) = -\frac{\mathrm{Re}\left\langle \Gamma_{\rm 2pt}^{\alpha\beta} C_{\rm 2pt}^{\beta\alpha}(t_{\rm f}, t_{\rm i}) \sum_{\mathbf{x}} \mathrm{Tr}\left(M^{-1}(\mathbf{x}, t; \mathbf{x}, t)\Gamma_{\rm loop}\right)\right\rangle}{\left\langle \Gamma_{\rm unpol}^{\alpha\beta} C_{\rm 2pt}^{\beta\alpha}(t_{\rm f}, t_{\rm i})\right\rangle}, \tag{2.2}$$

where the nucleon source and sink are at $t_{\rm i}$ and $t_{\rm f}$ respectively, and the current is inserted at t. The three-point function is simply the combination of the nucleon two-point function, $C_{\rm 2pt}(t_{\rm f}, t_{\rm i})$, and the disconnected loop, $\sum_{\mathbf{x}} \mathrm{Tr}[M^{-1}(\mathbf{x}, t; \mathbf{x}, t)\Gamma_{\rm loop}]$. For the scalar matrix element we used, $\Gamma_{\rm 2pt} = \Gamma_{\rm unpol} := (\mathbb{1} + \gamma_4)/2$ and $\Gamma_{\rm loop} = \mathbb{1}$. For Δq we calculated the difference between two polarizations: $\Gamma_{\rm 2pt} = \gamma_j\gamma_5\Gamma_{\rm unpol}$ and $\Gamma_{\rm loop} = \gamma_j\gamma_5$, where we average over all three possible j-orientations.

In the limit of large times, $t_{\rm f} \gg t \gg t_{\rm i}$, depending on the Γ-combination used, $R^{\rm dis}$ will either approach the disconnected axial matrix element $\Delta q^{\rm dis}$ or the disconnected scalar matrix element $\langle N|\bar{q}q|N\rangle^{\rm dis}$ (once the vacuum contribution is subtracted). However, the statistical noise increases rapidly with increasing $t - t_{\rm i}$ and this time difference needs to be minimized, using smeared sources and sinks for the nucleon, in order to obtain a reasonable signal. A smearing study on a limited number of configurations indicated that the nucleon plateaued around $t \geq 4a \approx 0.3$ fm and we chose to insert the current at this timeslice ($t_{\rm i} = 0$). However, the higher statistics now available mean the ground state dominates around $t \geq 6a$ only. At zero momentum, the excited state contribution to $R^{\rm dis}$ is governed by the time difference, $t_{\rm f} - t_{\rm i}$, and we must be careful to choose $t_{\rm f}$ large enough.

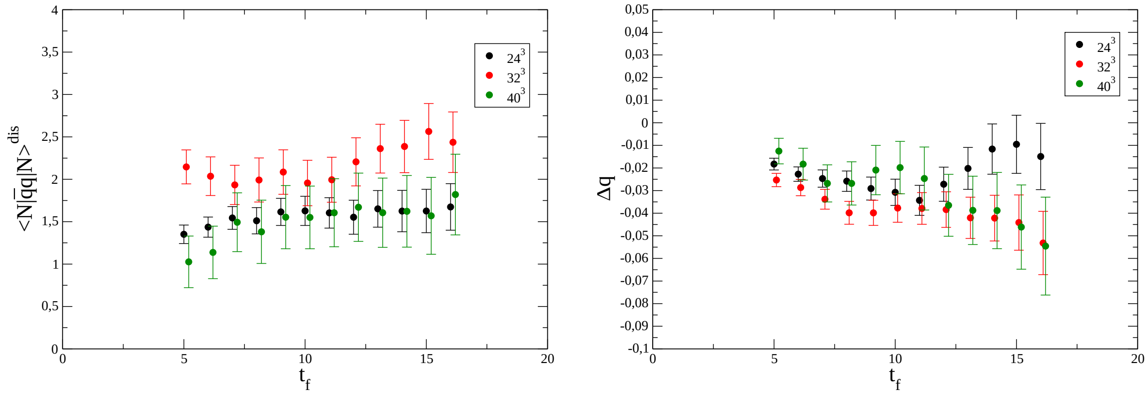

Figure 2: The results for $\langle N|\bar{q}q|N\rangle^{\rm dis}$ and $\Delta q^{\rm dis}$ extracted from the corresponding $R^{\rm dis}$ as a function of the sink timeslice, $t_{\rm f}$, for all three volumes used and the heaviest $\kappa_{\rm val} = \kappa_{\rm loop} = 0.13550$ combination.

In fig. 2 we show the results for $R^{\rm dis}$ as a function of $t_{\rm f} > t$ for all three volumes studied and the heaviest $\kappa_{\rm val} = \kappa_{\rm loop} = 0.13550$ combination. So far we have chosen $t_{\rm f} = 8a$ or $9a$ based on the quality of the plateau within given statistical errors. This depends on the observable and lattice volume. Once final statistics are reached we will fit the three- and two-point functions within $R^{\rm dis}$ of eq. (2.2) separately, as functions of $t_{\rm f}$, in order to extract the asymptotic values.

The disconnected loop, $\sum_{\mathbf{x}} {\rm Tr}\,[M^{-1}(\mathbf{x},t;\mathbf{x},t)\Gamma_{\rm loop}]$, was calculated using stochastic estimates, together with several noise reduction techniques:

- Partitioning [12, 13]: the stochastic source has support on eight timeslices. Additional two-point functions were generated for four time separated source points on each configuration. The forward and backward propagation from these four source points was combined with the loop to give us eight measurements of $R^{\rm dis}$ per configuration.

- Hopping parameter expansion [14]: for the clover action the first two terms in the expansion of the disconnected loop,

$$ {\rm Tr}(M^{-1}\Gamma_{\rm loop}) = 2\kappa {\rm Tr}[(\mathbb{1}-\kappa \not{D})^{-1}\Gamma_{\rm loop}] = {\rm Tr}[(2\kappa\mathbb{1} + 2\kappa^2 \not{D} + \kappa^2 \not{D}^2 M^{-1})\Gamma_{\rm loop}]\,, \qquad (2.3) $$

  vanish and hence only contribute to the noise. This means ${\rm Tr}\,[\kappa^2 \not{D}^2 M^{-1}(\mathbf{x},t;\mathbf{x},t)\Gamma_{\rm loop}]$ can be used as an improved estimate of the loop. (In the case of $\Gamma_{\rm loop} = \mathbb{1}$ the non-vanishing first term $\sum_{\mathbf{x}} 2\kappa {\rm Tr}\,\mathbb{1} = 24\kappa L^3$ can easily be corrected for.)

- Truncated solver method [15]: 730 conjugate gradient solves were used, where the solver was truncated after 40 iterations. 50 BiCGStab solves running to full convergence were generated to correct for the truncation error.

The noise reduction techniques other than time partitioning are only necessary for determining Δq; for the scalar matrix element the gauge noise dominates.

3. The scalar matrix element: f_{T_s} and $m_q\langle N|\bar{q}q|N\rangle^{\rm dis}$

The results for f_{T_s} are presented in fig. 3 as functions of $m^2_{\rm PS}$ corresponding to the mass of the (valence) quarks in the nucleon. No renormalization is required as the combination, $m_q\langle N|\bar{q}q|N\rangle$,

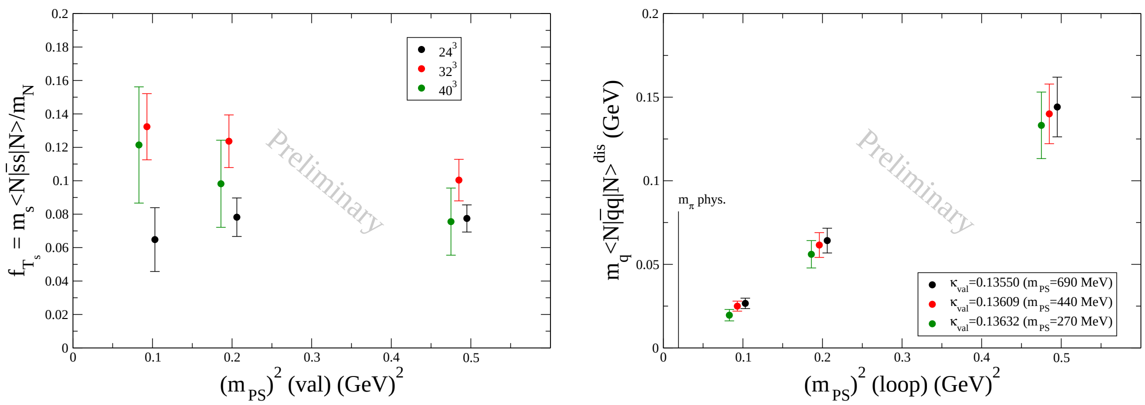

Figure 3: Results for the scalar matrix element: (left) f_{T_s} as a function of $m_{\rm PS}^2$ for the valence quark mass (i.e. the mass of the quarks in the nucleon) for the three volumes studied. (right) $m_q\langle N|\bar{q}q|N\rangle^{\rm dis}$ as a function of $m_{\rm PS}^2$ for the loop quark mass on the $32^3\times 64$ volume, for the three valence quark masses.

is scale and scheme independent. There is consistency between the values obtained on the different volumes for the heaviest valence quark mass, however, the spread between the results increases when the quark mass is reduced. Whether this is an indication of finite size effects will be clarified once the statistics for the 40^3 volume is increased.

In fig. 3 we also display the results for the disconnected scalar matrix element, $m_q\langle N|\bar{q}q|N\rangle$, for the 32^3 volume as a function of $m_{\rm PS}^2$ corresponding to the loop quark mass. This combination is relevant for extracting the sigma term,

$$\sigma_N = m_q\langle N|\bar{u}u+\bar{d}d|N\rangle\,. \tag{3.1}$$

We found $2m_q\langle N|\bar{u}u|N\rangle^{\rm dis}$ for $\kappa_{\rm val}=\kappa_{\rm loop}=\kappa_{\rm sea}=0.13632$ to amount to roughly 40% of the connected contribution to σ_N for this volume. However, a more sophisticated method of extracting the matrix element from $R^{\rm dis}$ is required in order to make a firm comparison.

4. The spin contribution: Δs and Δq

The results for Δs on all three volumes are shown in fig. 4. No significant dependence on the valence quark mass nor on the lattice size is seen in the data. Neither is there any significant variation in the results if the loop quark mass is reduced. These numbers will have to be multiplied by a renormalization constant of approximately 0.8 for the $\overline{MS}$ scheme [16]. In contrast to the scalar case, the disconnected contributions are much smaller than the connected terms, at around 10% for Δd and 5% for Δu.

5. Outlook

In the short term, the aim is to reach our target statistics of 2000 trajectories for each volume. We then plan to begin an analysis close to the physical sea quark mass. The nonperturbative renormalization for Δq needs to be calculated while for the scalar strangeness matrix element mixing with the light flavours needs to be considered.

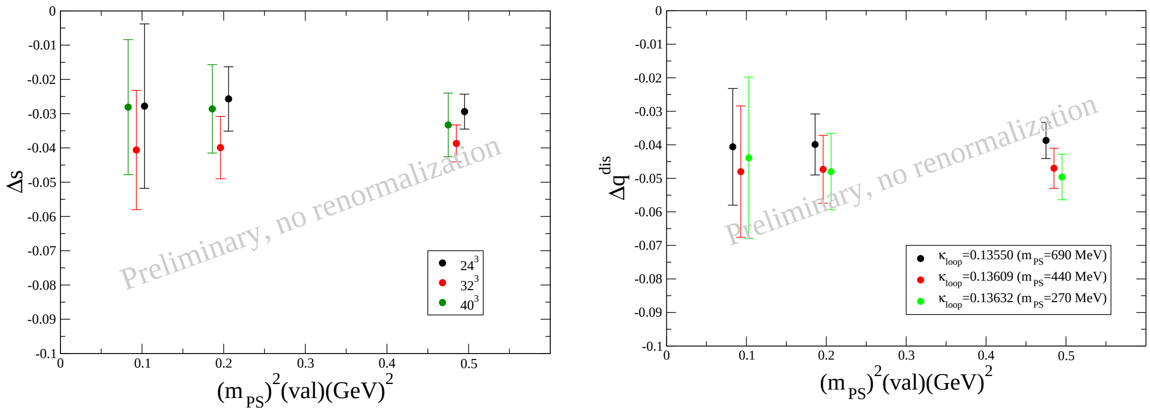

Figure 4: Results for Δq: (left) Δs as a function of $m^2_{\rm PS}$ for the valence quark mass (i.e. the mass of the quarks in the proton) for the three volumes studied. (right) $\Delta q^{\rm dis}$ from the $32^3\times 64$ volume for different loop quark masses, again as a function of $m^2_{\rm PS}$ for the valence quark mass.

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