Transport coefficients of gluonic fluid

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(Dated: November 23, 2010)

The shear (η) and bulk (ζ) viscous coefficients have been evaluated for a gluonic fluid. The elastic, $gg \rightarrow gg$ and the inelastic, number non-conserving, $gg \rightarrow ggg$ processes have been considered as the dominant perturbative processes in evaluating the viscous co-efficients to entropy density (s) ratios. Recently the processes: $gg \rightarrow ggg$ has been revisited and a correction to the widely used Gunion-Bertsch (GB) formula has been obtained. The η and ζ have been evaluated for gluonic fluid with the formula derived recently. The sensitivity of the quantity, η/s on the running coupling constant is also discussed. At $\alpha_s = 0.3$ we get $\eta/s = 0.24$ which is close to the value obtained from the analysis of the elliptic flow at RHIC experiments.

PACS numbers: 12.38.Mh,25.75.-q,24.85.+p,25.75.Nq

The nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) energies are aimed at creating a phase where the properties of the matter is governed by the quarks and gluons [1], such a phase, which is mainly composed of light quarks and gluons - is called quark gluon plasma (QGP). The weakly interacting picture of the QGP stems from the perception of the QCD asymptotic freedom at high temperatures and densities. However, the experimental data from RHIC, especially the measured elliptic flow [2] indicate that the matter produced at Au+Au collisions exhibit properties which are more like strongly interacting liquid than a weakly interacting gas. The shear viscosity or the internal friction of the fluid symbolizes the ability to transfer momentum over a distance of $\sim$mean free path. Therefore, in a system where the constituents interact strongly the transfer of momentum is performed easily - resulting in lower values of η. Consequently such a system may be characterized by a small value of η/s. The importance of viscosity also lies in the fact that it damps out the variation in the velocity and make the fluid flow laminar. A very small viscosity (large Reynold number) may make the flow turbulent.

On the other hand the bulk viscosity exhibit the exchange of energy between the translational and internal degrees of freedom. Although much emphasis has been given to the evaluation of the shear viscosity for a partonic system recently, the bulk viscosity is comparatively less discussed. Probably, because the bulk viscosity for a structureless point particles is zero both for relativistic and non-relativistic limits [3]. However, there are several reasons for which the bulk viscosity of a system formed in nuclear collisions at ultra-relativistic energies may be non-zero [4]. The trace anomaly in QCD will give rise to non-zero ζ, which will indicate the deviation of the system from the conformal invariance, because the ζ is defined as the correlation of the trace of the energy momentum tensor through Kubo's formula. The ζ for $SU(3)$ gauge theory has been evaluated in lattice QCD and its value is found to be quite large around the temperature domain for partons to hadrons transition (T_c)[5]. The divergence of ζ may be treated as a signal of critical point as it diverges near this point [6]. However, this point has been confronted in [7]. This indicate that both η and ζ can be used effectively to characterize QGP. Therefore, in the present work we would like to estimate both the ratios, η/s and ζ/s for a gluonic fluid by taking into account the perturbative QCD (pQCD) elastic process, $gg \rightarrow gg$ and the inelastic, number non-conserving process, $gg \rightarrow ggg$ [8]. While evaluating the transport coefficients we will use the newly obtained matrix element for $gg \rightarrow ggg$ process [9].

The transport coefficients for QCD matter has been evaluated in [10–12]. The calculation of the viscous coefficients within the ambit of diagrammatic approach of quantum field theory, along with its limitation has been discussed in [13]. Recently pQCD approaches [14–18] have been used to calculate η/s. Evaluation of η/s for a gluonic plasma by Xu and Greiner(XG) indicates that the contribution from gg $\rightarrow$ ggg is 7 times as large as that from gg $\rightarrow$ gg. This brings the value of η/s down to the AdS/CFT bound $\sim 1/4\pi$ [19], when the strong coupling constant, $\alpha_s = 0.6$. The GB formula [20](see also [21]) for the $gg \rightarrow ggg$ matrix element squared is used in [15]. However, we have shown recently that at the lower temperature domain the GB formula receives a significant correction. The ratio of matrix element squared with [9](henceforth will be denoted by the subscript DA) and without [20](henceforth denoted by the subscript GB) the correction term is given by:

$$R_c = \frac{|M_{gg\rightarrow ggg}|^2_{\mathrm{DA}}}{|M_{gg\rightarrow ggg}|^2_{\mathrm{GB}}} = 1 + \frac{(q_\perp^2 + m_D^2)^2}{s^2} \qquad (1)$$

where

$$|M_{gg\to ggg}|^2_{\rm DA} = \left(\frac{4g^4N_c^2}{N_c^2-1}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right) \left(\frac{4g^2N_cq_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) + \frac{16g^6N_c^3}{N_c^2-1}\frac{q_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]} \quad (2)$$

and

$$|M_{gg\to ggg}|^2_{\rm GB} = \left(\frac{4g^4N_c^2}{N_c^2-1}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right) \left(\frac{4g^2N_cq_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) \quad (3)$$

where g is the colour charge, N_c is the number of the colour, m_D is the Debye mass, $k_\perp$ is the transverse momentum of the emitted gluon and $q_\perp$ is the transverse momentum of the exchanged gluon. Following our previous work [9] we depict the magnitude of the correction, R_c in Fig. 1. The values of $q_\perp$ and s are same as the values taken in Ref. [9]. It is observed that for large values of α_s the corrections to the GB matrix element is significant. Therefore, it is expected that the values of energy loss, η/s and ζ/s will also be affected by the correction term in the lower temperature (higher coupling) domain.

Before discussing the bulk and shear viscosities we estimate the effects of the correction term to the radiative energy loss mechanism of partons propagating through QGP which is measured experimentally thorough the nuclear suppression factor [22] in heavy ion collision. To evaluate the radiative energy loss we start with the soft gluon distribution, which can written as [9]

$$\frac{dn_g}{d\eta dk_\perp^2} = \frac{C_A\alpha_s}{\pi^2}\left(\frac{q_\perp^2}{k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) + \frac{C_A\alpha_s}{\pi^2}\left(\frac{q_\perp^2(q_\perp^2+m_D^2)^2}{s^2k_\perp^2[(k_\perp-q_\perp)^2+m_D^2]}\right) \quad (4)$$

where $k=(k_0,k_\perp,k_3)$ is the four momenta of the emitted gluon, $q=(q_0,q_\perp,q_3)$ is the four momenta of the exchanged gluon and $C_A=3$ is the Casimir invariant of the adjoint representation. The m_D in Eq. 3 is the Debye mass required to shield the infra-red divergence. We use the above spectrum of the soft gluon to evaluate the radiative energy loss, dE/dx of gluons. The Landau-Pomeronchuk-Migdal (LPM) effects has been taken into account by the procedure outlined in [23]. The variation of α_s (and hence $m_D^2 \sim \alpha_s(T)T^2$) with T has been taken from Ref. [24]. The dE/dx is evaluated with temperature dependent α_s.

In Fig. 2 the variation of radiative energy loss with T has been depicted for the process $gg\to ggg$.

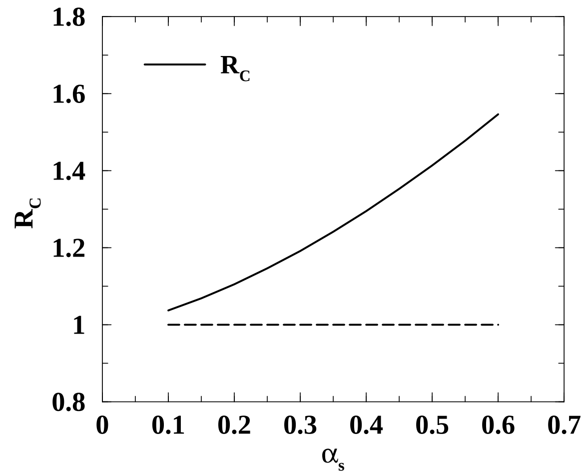

FIG. 1: The variation of the quantity, R_c (Eq. 1) with coupling constant.

The solid line (dotted line) represents the energy loss when DA (GB) gluon multiplicity distributions are used. To emphasize the importance of the corrections to GB formula we display the ratio,

$$R_{EL} = \frac{{\rm DA}_{EL}}{{\rm GB}_{EL}} \quad (5)$$

in the inset of Fig. 2. It is observed that the correction to the gluon spectrum, which leads to the energy loss is appreciable for lower temperature domain. This may affect the suppression of high p_T partons in QGP and the elliptic flow of the matter formed at RHIC [2] and LHC [25] energies.

We calculate the quantity, η/s for a gluonic system for the pQCD processes: $gg\to gg$, $gg\to ggg$ and $ggg\to gg$ The η/s is evaluated using the following equation [15]:

$$\frac{\eta}{s} = \left(5\frac{R_{gg\to gg}}{T} + \frac{25}{3}\frac{R_{gg\to ggg}}{T}\right)^{-1} \quad (6)$$

where R is the reaction rate for the relevant reactions. The phase space part or the kinematics for the reaction rate of the process $gg\to ggg$ is same as that of [17], however, the matrix element is different [9]. For the process: $gg\to ggg$ we use the matrix element obtained recently in [9] in contrast to [15] where GB matrix element is used. The other notable difference with [15] is that the phase space part is treated in the present work as in [17]. The variation of η/s with α_s obtained in the present work is depicted in Fig. 3. The η/s is quite large at low α_s because for weakly interacting system the momentum transfer between the constituents become strenuous which give rise to large η. However, with the increase in the coupling strength the momentum

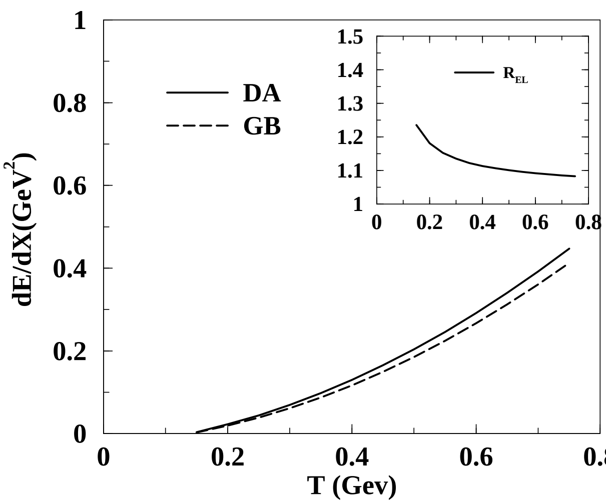

FIG. 2: The variation of energy loss with temperature for the process: $gg \to ggg$. Inset: The Variation of R_{EL} (Eq. 5) with temperature. The results depicted here is obtained with temperature dependent α_s.

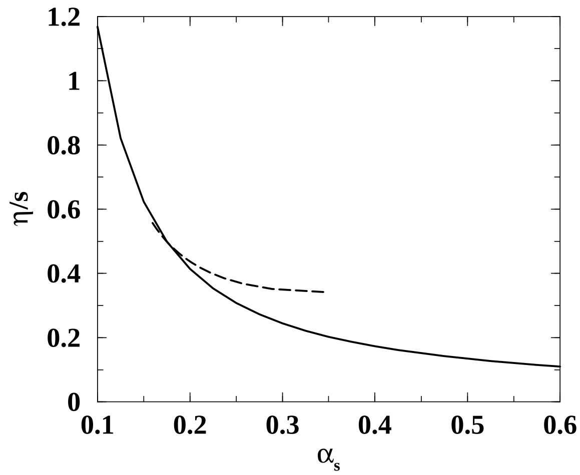

FIG. 3: Variation of η/s with coupling constant. The dotted line indicates the results when the temperature variation of α_s is taken into account and the solid line represents result when the α_s is changed by hand.

transfer gets easier as a result the shear viscosity reduces. The dashed line in Fig. 3 is obtained with the temperature dependent $\alpha_s(T)$ for the temperature range $T = 0.15 - 1$ GeV. The solid line is obtained when α_s is varied by hand (not dictated by temperature variation). The results indicate that for large α_s the quantity, η/s approaches the AdS/CFT limit. However, in such a scenario the validity of the pQCD calculations remains to be addressed. For the values of $\alpha_s(T)$ [24] corresponding to T accessible at RHIC and LHC energies the η/s does not attain the AdS/CFT bound for a strongly coupled system, indicating the inadequacy of the perturbative processes to achieve such bound. This can be verified by performing a lattice QCD based calculations (which include the non-perturbative effects) for pure SU(3) gauge theory. In fact, such calculation of η/s has been done in [26] and it is found that the value is close to AdS/CFT bound. With temperature dependent α_s, for T accessible at RHIC/LHC collision energies the value of η/s approaches a value between 0.3 to 0.4 with pQCD processes.

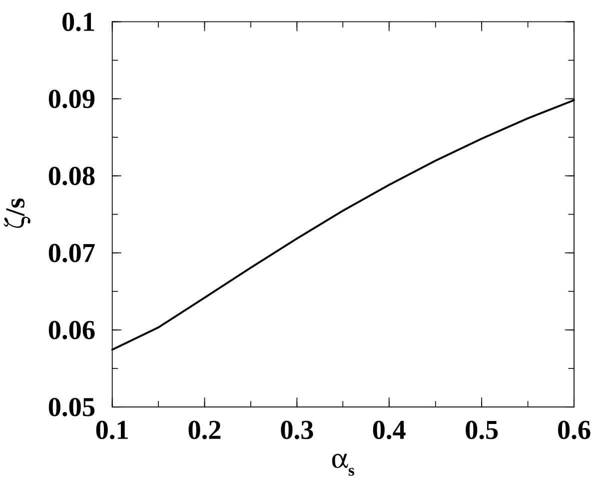

FIG. 4: The variation of ζ/s with the strong coupling constant.

As mentioned before the bulk viscosity, which is connected with the trace of the energy momentum tensor through Kubo's formula will be non-zero for a system where the conformal symmetry is broken. Lattice QCD calculations indicates non-zero ζ for a gluonic plasma [5] due to purely quantum effects (trace anomaly) (see also [27, 28] for QGP and [29] for pions) for temperatures around T_c. Physically, the bulk viscosity appears in the processes which are accompanied by a change in the volume(i.e. in density) of the fluid. In compression or expansion, as in any rapid change of state, the fluid ceases to be in thermodynamic equilibrium, and internal processes are set up in it which tend to restore the equilibrium. But the processes which drives the system toward equilibrium are irreversible associated with the increase in entropy and therefore involve energy dissipation. Hence, if the relaxation time of these processes is long, a considerable dissipation of energy occurs when the fluid is compressed or expanded and this dissipation must be determined by the bulk viscosity [30]. We evaluate the bulk viscous coefficient with the following formula (see Refs. [31–35]):

$$\zeta = \frac{\mathrm{d}eg}{T}\int \frac{p^2 dp}{2\pi^2}\frac{1}{\Gamma(p)} f_p(1+f_p)\left[\delta c_s^2 E - \frac{m_g^2}{3E}\right]^2 \quad (7)$$

where deg is the statistical degeneracy for the gluons, f_p is the Bose-Einstein distribution for the gluons, m_g is the thermal gluon mass, Γ^{-1} is the relaxation time, c_s^2 is the velocity of sound and $\delta c_s^2 = (1/3 - c_s^2)$. The value of velocity of sound, c_s for a massless system in equilibrium is $1/\sqrt{3}$, therefore, the results indicate that the bulk viscosity vanishes for a massless system in equilibrium. Eq. 7 also imply that the bulk viscosity will play an important role in the absence of chemical equilibrium. As δc_s^2 is a measure of the deviation from conformal symmetry (for massless system) the ζ increases with δc_s^2.

Within the ambit of the current formalism the bulk viscosity has been evaluated. In Fig. 4 the variation of ζ/s with α_s is displayed. We have taken $s = 16 \times \frac{2\pi^2}{45}T^3$. The c_s^2 as a function of T is taken from [36]. The most striking observation one can make here is the completely different kind of variation of η/s and ζ/s with α_s. While ζ/s increases with α_s [18], the η/s reduces with it [14]. At very high T where the α_s is very small due to asymptotic freedom the bulk viscosity will be negligibly small.

In Fig. 5, the ratio of ζ/η is depicted as a function of strong coupling. For values of α_s, from 0.1 to 0.6 the bulk viscosity is smaller than the shear viscosity, however, not negligible, in fact for large α_s the values are comparable. The values of c_s^2 at $T = 170, 360$ and 630 MeV are 0.17, 0.29 and 0.31 [36] respectively. Subsequently this gives the values of ζ/η as 0.4, 0.028 and 0.008 according to the relation $\zeta/\eta = 15(1/3 - c_s^2)^2$. The results currently obtained show qualitatively similar trend.

We have evaluated the shear and bulk viscosities for a gluonic system including the pQCD processes: $gg \to gg$ and $gg \to ggg$. The matrix element for the later processes is taken from [9]. We find that the value of η/s does not reach the AdS/CFT bound within the ambit of the perturbative processes [37] considered in the present work. However, the value of $\eta/s = 0.24$ at $\alpha_s = 0.3$ is within the limit extracted from the analysis of elliptic flow of matter formed in nuclear collisions at RHIC energy [38]. The value of ζ is non-zero but remains always less than η within the range of α_s considered here.

At $\alpha_s = 0.1$ we obtain $\eta/s \sim 1.16$ in contrast to the value of this quantity accomplished in previous works 0.5 in [15], 1.0 in [17] and 2.7 in [14] for the same value of the coupling strength. Our value is close to that of [17]. For higher values of $\alpha_s = 0.6$ where the corrections to GB formula is large, we get $\eta/s = 0.11$ whereas in earlier works these values are 0.076 in [15] and 0.15 in [17]. At higher coupling the η/s in the present work is less than the value obtained in [17] because of the significant contributions from the corrections to the GB formula. In QCD, the α_s increases with decrease in T, therefore, the coupling strength of the matter formed at RHIC and LHC energies may be determined by the temperature range accessible at these collision energies. With the values of α_s within the expected temperature interval (attainable at RHIC and LHC collision energies) we obtain $\eta/s \sim 0.3 - 0.4$.

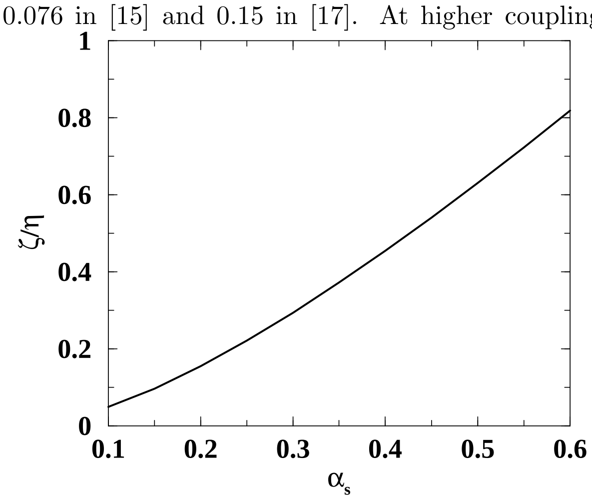

FIG. 5: The variation of ζ/η with the strong coupling constant.

Acknowledgment: We are grateful to Andrey Khvorostukhin, Qun Wang and Purnendu Chakrabarty for useful comments. This work is supported by DAE-BRNS project Sanction No. 2005/21/5-BRNS/2455.

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