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# GENERAL RELATIVISTIC EFFECTS ON NON-LINEAR POWER SPECTRA

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## ABSTRACT

Non-linear nature of Einstein equation introduces genuine relativistic higher order corrections to the usual Newtonian fluid equations describing the evolution of cosmological perturbations. We study the effect of such novel non-linearities on the next-to-leading order matter and velocity power spectra for the case of pressureless, irrotational fluid in a flat Friedmann background. We find that pure general relativistic corrections are negligibly small over all scales. Our result guarantees that, in the current paradigm of standard cosmology, one can safely use Newtonian cosmology even in non-linear regimes.
*Subject headings:* cosmology: theory —large-scale structure of universe

## 1. INTRODUCTION

Large scale structure (LSS) of the universe is a powerful probe to study the nature of cosmological density perturbations and to extract cosmological parameters (Peebles 1980). Combined with the anisotropy of the cosmic microwave background (CMB), most of the cosmological parameters are currently constrained within a few percent accuracy or even better (Komatsu et al. 2010). To continue our success in cosmology with CMB and LSS, it is crucial to predict the power spectra from theory accurately. While the temperature fluctuation in the CMB is as small as $\delta T/T \sim 10^{-5}$ (Smoot et al. 1992) so that linear perturbation theory is able to provide necessary accuracy, we have larger degree of non-linearities in LSS. We must take into account non-linearities of LSS properly to predict the power spectrum accurate enough for precision cosmology at a level similar to the CMB (Jeong & Komatsu 2006, 2009).

Most studies on LSS, however, have been based on Newtonian gravity, especially those including non-linear perturbations (Vishniac 1983; Goroff et al. 1986; Makino et al. 1992; Fry 1994; Bernardeau et al. 2002). This approach has to be justified a posteriori by comparing the result against fully general relativistic one. For example, in Noh & Hwang (2004), it is shown that the Newtonian hydrodynamic equations up to second order *coincide exactly* with the relativistic ones in the zero pressure case, after appropriately identifying hydrodynamical variables with gauge-invariant combinations of relativistic perturbation variables. Thus, compared to the Newtonian hydrodynamic equations which are closed at second order, any higher order contributions are originated from purely general relativistic effects (Hwang & Noh 2005b). A consistent expansion of density fluctuation tells us that the leading non-linear contributions to the power spectrum include third order perturbations (Noh & Hwang 2008; Noh et al. 2009). Thus non-linear density power spectrum naturally include pure general relativistic effects, which may have important implications as N-body simulations are becoming larger and larger to reach the horizon scale (Kim et al. 2009).

In this note, we examine the general relativistic effects on the power spectra of matter density fluctuations and peculiar velocity by including leading non-vanishing non-linear contributions. Our aim is to answer the question whether pure general relativistic effects can give rise to any cosmologically observable consequences. To our surprise, we find that the Newtonian terms in these power spectra are absolutely dominating over *all* relevant cosmological scales, even outside the horizon. Although the result sounds simple and pleasant, this is still a non-trivial result, because in the context of cosmology Newtonian gravity is *incomplete*: there is no concept of horizon, the propagation speed of an action at one point is infinite, and so on.

This note is outlined as follows. In Section 2, we present the formalism to set up the equations to solve, and give the solutions up to third order. In Section 3, we compute the matter and velocity power spectra including next-to-leading non-linear corrections which include genuine general relativistic effects. In Section 4 we conclude.

## 2. EQUATIONS AND SOLUTIONS

We consider Einstein-de Sitter universe, i.e. a flat universe dominated by pressureless, irrotational matter, and consider only the scalar perturbations. We work in the temporal co-

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moving gauge where $T^0{}_i = 0$ to all perturbation orders, with $i$ being a spatial index. As this temporal gauge condition, together with our unique spatial gauge condition (Bardeen 1988) $g_{ij} = a^2(1+2\varphi)\delta_{ij}$, fixes the gauge degrees of freedom completely, all the resulting perturbation variables can be equivalently regarded as fully gauge invariant, both spatially and temporally. This statement is valid in all perturbation orders (Noh & Hwang 2004).

The Arnowitt-Deser-Misner formulation (Arnowitt et al. 2008) is convenient in our case (Bardeen 1980). The comoving gauge condition imply the momentum density vanishes, i.e. $J_i \equiv NT^0{}_i = 0$, with $N$ being the lapse function. The pressureless condition implies $S_{ij} \equiv T_{ij} = 0$. Therefore the momentum conservation equation gives $N_{,i} = 0$, thus the lapse function $N$ is uniform. The energy and momentum conservation equations and the trace part of the propagation equation then become (Bardeen 1980)

$$E_{,0} - N^i E_{,i} = NKE\,, \tag{1}$$

$$\overline{K}^j{}_{i|j} - \frac{2}{3}K_{|i} = 0\,, \tag{2}$$

$$K_{,0} - N^i K_{,i} = N\left(\frac{1}{3}K^2 + \overline{K}^i{}_j\overline{K}^j{}_i + 4\pi GE - \Lambda\right)\,, \tag{3}$$

where $N_i$ is the shift vector, $K \equiv K^i{}_i$ is the trace of the extrinsic curvature tensor $K_{ij}$, $\Lambda$ is the cosmological constant, $E \equiv N^2 T^{00}$ is the energy density, an overbar denotes the traceless part, and a vertical bar denotes a covariant derivative with respect to $\gamma_{ij}$. These are the complete equations we need in our non-linear perturbations, and are valid in fully non-linear situation.

We introduce the density and the velocity fluctuations as $E \equiv \rho(t) + \delta\rho(t,\boldsymbol{x})$ and $K \equiv 3H - \theta(t,\boldsymbol{x})$ with $\theta(t,\boldsymbol{x}) \equiv a^{-1}\nabla\cdot\boldsymbol{u}(t,\boldsymbol{x})$, with $a$ being the cosmic scale factor. We can identify $\delta\rho(t,\boldsymbol{x})$ and $\boldsymbol{u}(t,\boldsymbol{x})$ as the Newtonian density and velocity perturbation variables respectively, because the relativistic equations coincide exactly with the corresponding Newtonian hydrodynamic equations up to second order.

From the above equations we can derive the hydrodynamic equations of density fluctuation $\delta(t,\boldsymbol{x}) \equiv \delta\rho(t,\boldsymbol{x})/\rho(t)$ and velocity gradient $\theta(t,\boldsymbol{x})$ to the third order (Hwang & Noh 2005b). The relativistic continuity and Euler equations are found to be

$$\frac{\partial\delta}{\partial t} + \frac{1}{a}\nabla\cdot\boldsymbol{u} = -\frac{1}{a}\nabla\cdot(\delta\boldsymbol{u}) + \frac{1}{a}\left[2\varphi\boldsymbol{u} - \nabla\left(\Delta^{-1}X_2\right)\right]\cdot(\nabla\delta)\,, \tag{4}$$

$$\begin{aligned}\frac{1}{a}\nabla\cdot\left(\frac{\partial\boldsymbol{u}}{\partial t} + H\boldsymbol{u}\right) + 4\pi G\rho\delta = &-\frac{1}{a^2}\nabla\cdot[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]\\ &+\frac{4}{a^2}\nabla\cdot\left\{\varphi\left[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \frac{1}{3}(\nabla\cdot\boldsymbol{u})\boldsymbol{u}\right]\right\} - \frac{2}{3a^2}\varphi(\boldsymbol{u}\cdot\nabla)(\nabla\cdot\boldsymbol{u})\\ &-\frac{1}{a^2}\Delta\left[(\boldsymbol{u}\cdot\nabla)\Delta^{-1}X_2\right] + \frac{1}{a^2}(\boldsymbol{u}\cdot\nabla)X_2 + \frac{2}{3a^2}X_2(\nabla\cdot\boldsymbol{u})\,,\end{aligned} \tag{5}$$

where $\varphi$ and $X_2$ are the linear and the second order quantity respectively, and are defined as

$$\frac{\Delta}{a^2}\varphi = \frac{1}{c^2}\left(-4\pi G\rho\delta + \frac{H}{a}\nabla\cdot\boldsymbol{u}\right)\,, \tag{6}$$

$$X_2 = 2\varphi\nabla\cdot\boldsymbol{u} - (\boldsymbol{u}\cdot\nabla)\varphi + \frac{3}{2}\Delta^{-1}\nabla\cdot[\boldsymbol{u}\Delta\varphi + \boldsymbol{u}\cdot\nabla(\nabla\varphi)]\,. \tag{7}$$

In relativistic perturbation theory, the dimensionless quantity $\varphi$ is proportional to the spatial curvature perturbation in the comoving gauge. All the perturbation variables $\delta$, $\boldsymbol{u}$ and $\varphi$ can be regarded as equivalently gauge-invariant combinations to non-linear order. Proper choice of variables and gauge conditions are important to have these equations. Note that the relativistic continuity and Euler equations coincide with those from Newtonian fluid approximation up to the second order in perturbations (Peebles 1980; Noh & Hwang 2004). Therefore, the perturbative solutions are also the same up to the second order, and pure general relativistic effects appear from third order. We emphasize that the above equations are valid in the presence of the cosmological constant in the background world model.

An examination of the third order terms in Equations (4) and (5) shows that the pure third order terms are simple convolutions of the linear order $\varphi$ with the second-order combinations of fluid variables $\delta$ and $\boldsymbol{u}$. Note that to the linear order, $\varphi$ is a well-known conserved quantity whose amplitude of the growing mode solution is conserved on the super-horizon scales, independent of the changing equation of state or even changing underlying gravity theories (Hwang & Noh 2005a). In a flat background without cosmological constant the amplitude of $\varphi$ near horizon scale is directly related to the amplitude of relative temperature fluctuations of the CMB as $\delta T/T = \varphi/5$.

The linear solutions of Equations (4) and (5) are easily found to be

$$\delta_1(\boldsymbol{k},t) = D(t)\delta_1(\boldsymbol{k},t_0)\,, \tag{8}$$

$$\theta_1(\boldsymbol{k},t) = -aHD(t)\delta_1(\boldsymbol{k},t_0)\,, \tag{9}$$

where $D(t)$ is the linear growth factor so that $\delta_1(\boldsymbol{k},t_0)$ is the present linear density fluctuation. With these linear solutions, we can perturbatively expand the density contrast $\delta(\boldsymbol{k},t) = \delta_1 + \delta_2 + \delta_3 + \cdots$, where $\delta_n$ is a $n$-th order quantity in linear density contrast $\delta_1(\boldsymbol{k},t_0)$, and similarly for $\theta(\boldsymbol{k},t)$. With this expansion, we can find the full non-linear solutions of Equations (4) and (5) by using momentum dependent symmetric kernels as

$$\delta(\boldsymbol{k},t) = \sum_{n=1}^{\infty} D^n(t)\int\frac{d^3q_1\cdots d^3q_n}{(2\pi)^{3(n-1)}}\delta^{(3)}\left(\boldsymbol{k} - \sum_{i=1}^{n}\boldsymbol{q}_i\right)\times F_n^{(s)}(\boldsymbol{q}_1,\cdots\boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n)\,, \tag{10}$$

$$\theta(\boldsymbol{k},t) = -aH\sum_{n=1}^{\infty} D^n(t)\int\frac{d^3q_1\cdots d^3q_n}{(2\pi)^{3(n-1)}}\delta^{(3)}\left(\boldsymbol{k} - \sum_{i=1}^{n}\boldsymbol{q}_i\right)\times G_n^{(s)}(\boldsymbol{q}_1,\cdots\boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n)\,. \tag{11}$$

Then, Equations (4) and (5) become simple differential equations of $F_n^{(s)}$ and $G_n^{(s)}$. Especially, the general relativistic terms which explicitly include $k_H \equiv aH$, the comoving wavenumber corresponding to the comoving horizon, are reduced to the algebraic equations

$$2F_{3,\rm Einstein}-G_{3,\rm Einstein}=-\frac{5}{2}k_H^2\left\{2\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_3}{q_1^2q_2^2}\right.$$
$$\left.+\frac{\boldsymbol{q}_{12}\cdot\boldsymbol{q}_3}{q_{12}^2}\left[-\frac{2}{q_2^2}+\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{q_1^2q_2^2}-\frac{3}{2}\frac{\boldsymbol{q}_{12}\cdot\boldsymbol{q}_1}{q_{12}^2q_1^2}-\frac{3}{2}\frac{\boldsymbol{q}_{12}\cdot\boldsymbol{q}_2}{q_{12}^2}\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{q_1^2q_2^2}\right]\right\}, \tag{12}$$

$$\frac{3}{2}F_{3,\rm Einstein}-\frac{5}{2}G_{3,\rm Einstein}=-\frac{5}{2}k_H^2\left\{\left[\frac{2}{3}+\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_{23}}{q_1^2}\left(1-\frac{k^2}{q_{23}^2}\right)\right]\right.$$
$$\times\left[-\frac{2}{q_3^2}+\frac{\boldsymbol{q}_2\cdot\boldsymbol{q}_3}{q_2^2q_3^2}-\frac{3}{2}\frac{\boldsymbol{q}_{23}\cdot\boldsymbol{q}_2}{q_{23}^2q_2^2}-\frac{3}{2}\frac{\boldsymbol{q}_{23}\cdot\boldsymbol{q}_3}{q_{23}^2}\frac{\boldsymbol{q}_2\cdot\boldsymbol{q}_3}{q_2^2q_3^2}\right]$$
$$\left.+\frac{1}{q_3^2}\left[\frac{2}{3}\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{q_2^2}-4\left(\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{q_2^2}-\frac{1}{3}\right)\frac{\boldsymbol{k}\cdot\boldsymbol{q}_1}{q_1^2}\right]\right\}, \tag{13}$$

where we have introduced $\boldsymbol{q}_{12\cdots n}=\sum_{i=1}^{n}\boldsymbol{q}_i$. The second and third order Newtonian kernels can be found in e.g. Equations [(2.32), (2.33)] and [(2.34), (2.35)] in Jeong (2010), respectively.

## 3. MATTER AND VELOCITY POWER SPECTRA

From Equation (10), we can find the non-linear power spectrum, which is defined as

$$\langle\delta(\boldsymbol{k}_1,t)\delta(\boldsymbol{k}_2,t)\rangle\equiv(2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)P(k_1,t)\,. \tag{14}$$

If we assume perfect Gaussianity of $\delta_1$, which is a very good approximation consistent with current observations, any higher order correlation function beyond the linear power spectrum $P_{11}(k)$ disappears and $P_{11}(k)$ is all that we need to specify the statistics of density fluctuation $\delta$: as we will see shortly, all the non-linear corrections to the power spectrum can be written in terms of $P_{11}$. Then, from Equation (14) we can write, beyond the linear density power spectrum $P_{11}$,

$$P=P_{11}+P_{22}+P_{13}+\cdots\,, \tag{15}$$

with $\langle\delta_i(\boldsymbol{k}_1)\delta_j(\boldsymbol{k}_2)\rangle\equiv(2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)s_{ij}P_{ij}(k_1)$. Here, $s_{ij}$ is a symmetric factor which is 1 for $i=j$ and 1/2 otherwise. The leading non-linear correction $P_{12}$ includes bispectrum and thus disappears according to our assumption of Gaussianity of $\delta_1$. $P_{22}+P_{13}$ denotes the next-to-leading order non-linear correction to the power spectrum. As mentioned above, $P_{13}$ includes general relativistic terms.

The density power spectrum up to next-to-leading order non-linear corrections is

$$P(k,t)=P_{11}(k,t)+\frac{1}{98}\frac{k^3}{(2\pi)^2}\int_0^\infty drP_{11}(kr,t)$$
$$\times\int_{-1}^{1}dxP_{11}\left(k\sqrt{1+r^2-2rx},t\right)\frac{\left(3r+7x-10rx^2\right)^2}{\left(1+r^2-2rx\right)^2}$$
$$+\frac{1}{252}\frac{k^3}{(2\pi)^2}P_{11}(k,t)\int_0^\infty drP_{11}(kr,t)$$
$$\times\left[-42r^4+100r^2-158+\frac{12}{r^2}+\frac{3}{r^3}\left(r^2-1\right)^3\left(7r^2+2\right)\log\left|\frac{1+r}{1-r}\right|\right]$$
$$+\frac{5}{56}\left(\frac{k_H}{k}\right)^2\frac{k^3}{(2\pi)^2}P_{11}(k,t)\int_0^\infty drP_{11}(kr,t)$$
$$\times\left[86r^2-130-\frac{72}{r^2}+\frac{1}{r^3}\left(36+53r^2-46r^4-43r^6\right)\log\left|\frac{1+r}{1-r}\right|\right]$$
$$\equiv P_{11}+P_{22}+P_{13,\rm Newton}+P_{13,\rm Einstein}\,. \tag{16}$$

where $r$ and $x$ are the magnitude of dummy integration momentum $\boldsymbol{q}$ and the cosine between $\boldsymbol{q}$ and $\boldsymbol{k}$, respectively, introduced as $q\equiv rk$ $(0\le r\le\infty)$ and $\boldsymbol{k}\cdot\boldsymbol{q}\equiv k^2rx$ $(-1\le x\le 1)$. We have divided $P_{13}$ into the Newtonian part $P_{13,\rm Newton}$ and the general relativistic contribution $P_{13,\rm Einstein}$. Compared with $P_{13,\rm Newton}$ the general relativistic contribution $P_{13,\rm Einstein}$ is multiplied by a factor $(k_H/k)^2$, where $k_H/k$ is the ratio between a scale of interest and the horizon scale, and is thus highly suppressed far inside the horizon.

In Figure 1 we present the total power spectrum of Equation (16) along with its components $P_{11}$, $P_{22}$, $P_{13,\rm Newton}$ and $P_{13,\rm Einstein}$ when our Universe is dominated by matter, at $z=6$. The linear power spectrum is calculated by CAMB (Lewis et al. 2000) code with the maximum likelihood cosmological parameters given in the Table 1 of Komatsu et al. (2009) ("WMAP+BAO+SN"). Figure 1 shows that the general relativistic contribution $P_{13,\rm Einstein}$ is smaller than the linear power spectrum $P_{11}$ on *all* cosmological scales.

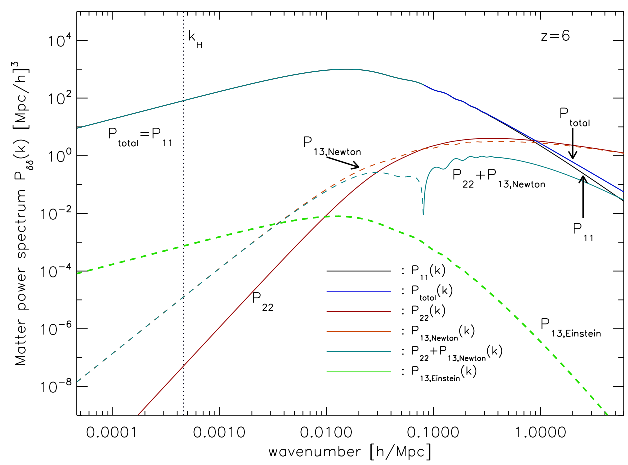


**Figure 1.** Non-linear matter power spectrum (solid blue line) and the contribution from each component of Equation (16) at $z=6$. The black, red, and orange lines show the contributions from the Newtonian perturbation theory: $P_{11}$, $P_{22}$, and $P_{13,\rm Newton}$, respectively. The green line shows the general relativistic effect, $P_{13,\rm Einstein}(k)$. Note that we take the absolute value for negative terms, and show with dashed lines: $P_{22}$ and $P_{13}$ are positive and negative,respectively, in all scales. Vertical dotted line shows the wavenumber correspond to the comoving horizon $k_H$ at $z=6$. Over all scales, the general relativistic term $P_{13,\rm Einstein}$ (green) is negligibly small compare to the linear power spectrum $P_{11}$ (black).

Let us examine $P_{13,\rm Einstein}$ more closely. For notational simplicity, we shall abbreviate the integration in $P_{13,\rm Einstein}$ as $\int drP_{11}(kr,t)f(r)$. Then, scale dependence of $P_{13,\rm Einstein}$ can be understood as follows. First, setting $kr=q$, we find that $P_{13,\rm Einstein}\sim P_{11}(k)f(q/k)$. On small scales ($k\gg 0.01h$/Mpc), $q/k$ is also small, and by using Taylor expansion of $f(r)=-(656/15)r^2+\mathcal{O}(r^4)$ we find $P_{13,\rm Einstein}\sim k^{-2}P_{11}$. On the other hands, in large scale limit ($k\ll 0.01h$/Mpc) where $q/k$ takes larger value, $f(r)=-752/3+\mathcal{O}(r^{-2})$ and $P_{13,\rm Einstein}$ has a scale dependence $P_{13,\rm Einstein}\sim P_{11}$. Numerical calculation reveals that $P_{13,\rm Einstein}$ is smaller than $P_{11}$ by a factor $10^{-5}$ on large scales. Our result shows that the leading order non-linear power spectrum is finite in both infrared and ultraviolet regions[2].

[2] The previous result reporting infrared divergence in $P_{13,\rm Einstein}$ (Noh et al.

We can proceed in the same way to compute the power spectrum of the peculiar velocity. As Equation (14), we can define

$$\langle\theta(\boldsymbol{k}_1,t)\theta(\boldsymbol{k}_2,t)\rangle \equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)P_{\theta\theta}(k_1,t)\,, \tag{17}$$

and we can find

$$\begin{aligned}
k_H^{-2}P_{\theta\theta}(k,t) &= P_{11}(k,t)+\frac{1}{98}\frac{k^3}{(2\pi)^2}\int_0^\infty drP_{11}(kr,t)\\
&\times\int_{-1}^{1}dxP_{11}\left(k\sqrt{1+r^2-2rx},t\right)\frac{(r-7x+6rx^2)^2}{(1+r^2-2rx)^2}\\
&+\frac{1}{84}\frac{k^3}{(2\pi)^2}P_{11}(k,t)\int_0^\infty drP_{11}(kr,t)\\
&\times\left[-6r^4+4r^2-82+\frac{12}{r^2}+\frac{3}{r^3}\left(r^2-1\right)^3\left(r^2+2\right)\log\left|\frac{1+r}{1-r}\right|\right]\\
&+\frac{5}{56}\left(\frac{k_H}{k}\right)^2\frac{k^3}{(2\pi)^2}\int_0^\infty drP_{11}(kr,t)\\
&\times\left[46r^2-50-\frac{144}{r^2}+\frac{1}{r^3}\left(-23r^6-50r^4+r^2+72\right)\log\left|\frac{1+r}{1-r}\right|\right].
\end{aligned} \tag{18}$$

Figure 2 shows the non-linear velocity power spectrum of Equation (18) for exactly the same cosmology as Figure 1. As in the case of the total matter power spectrum, the non-linear general relativistic correction is negligibly small for all scales. It is because the third order kernel for velocity $G_{3,\rm Einstein}$ behaves in the same way as that for the matter density $F_{4,\rm Einstein}$ in both large ($r\to 0$) and small ($r\to\infty$) scale limit: $\lim_{r\to 0} g(r) = -(368/15)r^2+\mathcal{O}(r^4)$ and $\lim_{r\to\infty} g(r) = -496/3+\mathcal{O}(r^{-2})$ when denoting the last integration in Equation (18) as $\int drP_{11}(kr)g(r)$.

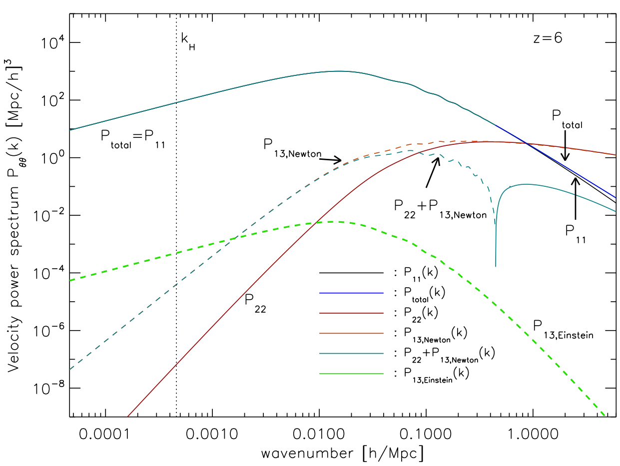


**Figure 2.** Same as Figure 1, but for the velocity power spectrum $P_{\theta\theta}(k)$.

## 4. CONCLUSION

To conclude, in this note we have examined the general relativistic non-linear contributions to the density and velocity power spectra. We have found that, with pleasant surprise, the pure general relativistic effects are completely negligible on *all* cosmologically relevant scales, even outside the horizon. It is interesting to see that the linear power spectrum is totally dominating even outside the horizon. Our conclusion has the following important implication. As the general relativistic effect is very small, Newtonian theory can be safely applied to the non-linear evolution of cosmic structure on all cosmologically relevant scales. In the literature it has been common to use Newtonian gravity to study the non-linear clustering properties of large scale structure without justifying that approach. The result we present in this note provides a confirmation of using Newtonian gravity to handle non-linear clustering in cosmology.

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2009) turns out to be due to an incorrect calculation of the power spectrum: the third order general relativistic kernels $F_{3,\rm Einstein}$ has not been fully symmetrized, thus causing logarithmic infrared divergence in $P_{13,\rm Einstein}$.