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The excited hadron spectrum in lattice QCD using a new variance reduction method

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Progress in calculating the spectrum of excited baryons and mesons in lattice QCD is described. Correlation matrices of sets of spatially-extended hadron operators have been studied and their effectiveness in facilitating the extraction of excited-state energies is demonstrated. First applications of the stochastic LapH method, a new method of estimating the low-lying effects of quark propagation, are presented.

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This talk is a progress report on our efforts to study the excited-state spectrum of QCD using the Monte Carlo method: results from our process of selecting optimal single-hadron operators are presented, and a new method of estimating slice-to-slice quark propagation is outlined.

The use of operators whose correlation functions $C(t)$ attain their asymptotic form as quickly as possible is crucial for reliably extracting excited hadron masses. Since excited hadrons are expected to be large objects, the use of spatially extended operators is a key ingredient in the operator design and implementation. A more detailed discussion of these issues can be found in Ref. [1]. First results using unquenched configurations were reported in Ref. [2].

A large effort has been undertaken to select optimal sets of baryon and meson operators in a large variety of isospin sectors. Low-statistics Monte Carlo computations were done to accomplish the operator selections using between 50 to 100 configurations on a $16^3\times 128$ anisotropic lattice for $N_f=2+1$ quark flavors with spacing $a_s\sim 0.12$ fm, $a_s/a_t\sim 3.5$, and quark masses such that the pion has mass around 380 MeV. The method described in Ref. [3] was used. Stationary-state energies using the finally selected operator sets are shown in Fig. 1. The nucleon, Δ, Ξ, Σ, and Λ baryons were studied, and light isovector and kaon mesons were investigated. Hundreds of operators were studied, and optimal sets containing eight or so operators in each symmetry channel were found. Future computations will focus solely on the operators in the optimal sets. See Ref. [4] for our most recent high-statistics study.

To study a particular eigenstate of interest in the Monte Carlo method, all eigenstates lying below that state must first be extracted, and as the pion gets lighter in lattice QCD simulations, more and more *multi-hadron* states will lie below the excited resonances. In the evaluation of the temporal correlations of the multi-hadron operators that we intend to use, it is not possible to completely remove all summations over the spatial sites on the source time-slice using translation invariance. Hence, the need for estimates of the quark propagators from all spatial sites on a time slice to all spatial sites on another time slice cannot be sidestepped. Some correlators will involve diagrams with quark lines originating at the sink time t and terminating at the same sink time t, so quark propagators involving a large number of starting times t must also be handled.

A new way of stochastically estimating such slice-to-slice quark propagators was introduced in Ref. [5]. The first ingredient in the method is the use of a new Laplacian Heaviside (LapH) quark-field smearing scheme defined by $\widetilde{\psi}(x)=\Theta\left(\sigma_s^2+\widetilde{\Delta}\right)\psi(x)$, where $\widetilde{\Delta}$ is the three-dimensional covariant Laplacian in terms of the stout-smeared gauge field and σ_s is the smearing cutoff parameter. The Heaviside function truncates the sum over Laplacian eigenmodes, restricting the summation to some number N_v of the lowest-lying $-\widetilde{\Delta}$ eigenmodes on each time slice. Quark propagation is then estimated using Z_N noise introduced in the LapH subspace. The noise vectors ρ have spin, time, and Laplacian eigenmode number as their indices. Variance reduction is achieved by *diluting* the noise vectors[6]. A given dilution scheme can be viewed as the application of a complete set of projection operators. Our dilution projectors are products of time dilution, spin dilution, and Laph eigenvector dilution projectors. For each type (time, spin, Laph eigenvector) of dilution, we studied four different dilution schemes: none, full, blocking, and interlacing. We use a triplet (T, S, L) to specify a given dilution scheme, where “T" denote time, “S" denotes spin, and “L" denotes Laph eigenvector dilution. The schemes are denoted by 1 for no dilution, F for full dilution, and BK and IK for block-K and interlace-K, respectively. For example, full time and spin dilution with interlace-8 Laph eigenvector dilution is denoted by (TF, SF, LI8). The volume dependence of this

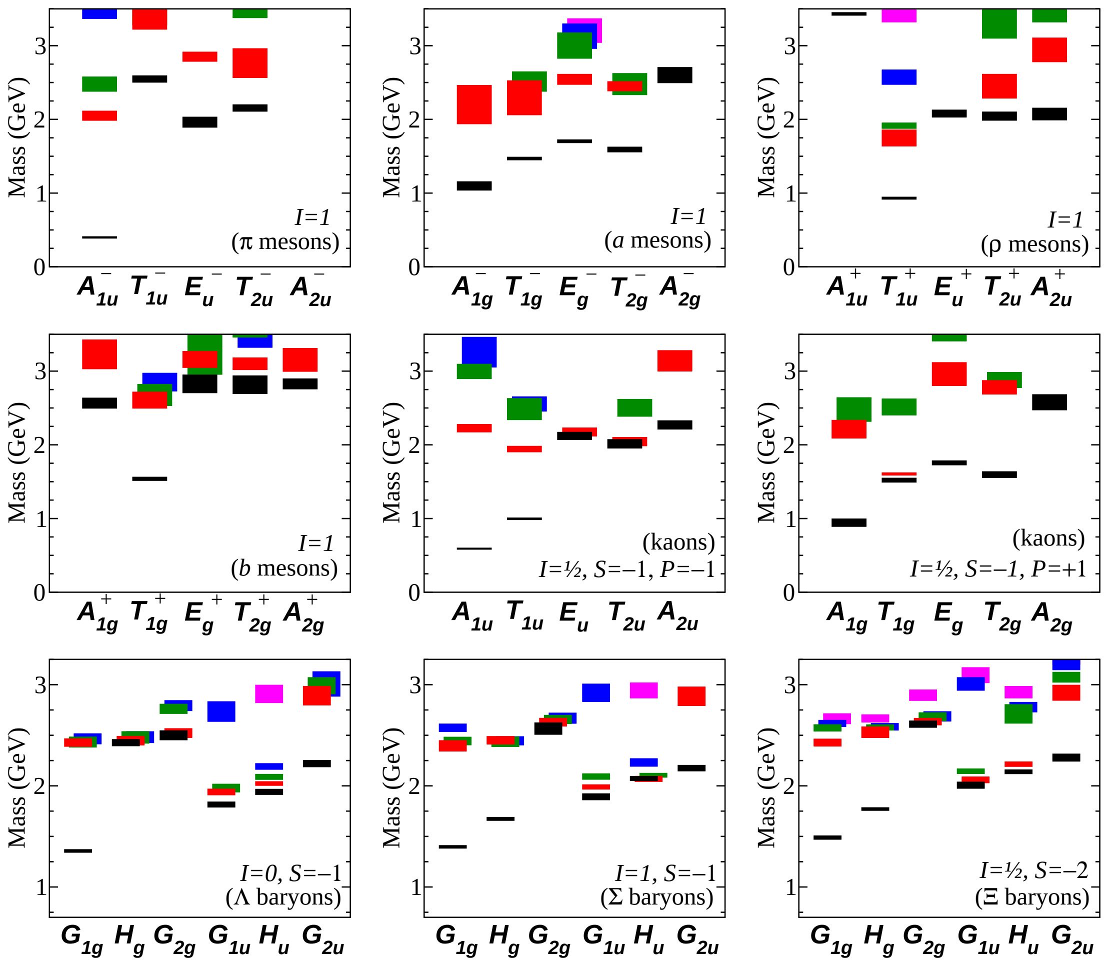

Figure 1: Hadron operator selection: low-statistics simulations have been performed to study the hundreds of single-hadron operators produced by our group-theoretical construction. A "pruning" procedure was followed in each channel to select good sets of between six to a dozen operators. The plots above show the stationary-state energies extracted to date from correlation matrices of the finally selected single-hadron operators. Results were obtained using between 50 to 100 configurations on a $16^3 \times 128$ anisotropic lattice for $N_f = 2+1$ quark flavors with spacing $a_s \sim 0.12$ fm, $a_s/a_t \sim 3.5$, and quark masses such that $m_\pi \sim 380$ MeV. Each box indicates the energy of one stationary state; the vertical height of each box indicates the statistical error.

new method was found to be very mild, making the method suitable for large volume calculations.

Results for three isoscalar mesons are shown in Fig. 2. Such mesons are notoriously difficult to study in lattice QCD, but the new method appears to produce estimates of their temporal correlations with unprecedented accuracy. These plots suggest that evaluating correlation functions involving our multi-hadron operators will be feasible with the stochastic LapH method.

The next steps in our spectrum project are to combine our moving single-hadron operators to form multi-hadron operators, then complete computations of QCD stationary-state energies using, for the first time, both single-hadron and multi-hadron operators. This work was supported by the U.S. National Science Foundation under awards PHY-0510020, PHY-0653315, PHY-0704171, and

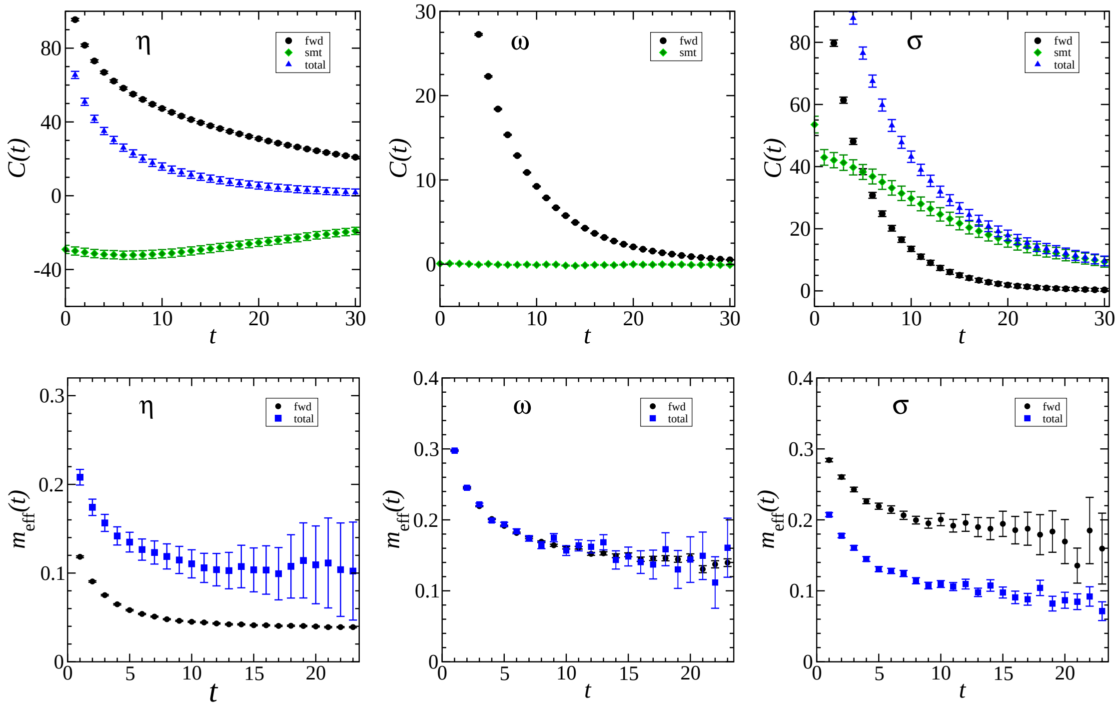

Figure 2: Correlators $C(t)$ and effective masses $m_{\text{eff}}(t)$ against temporal separation t for single-site operators which produce the isoscalar pseudoscalar η, vector ω, and scalar σ mesons. Results were obtained using 198 configurations with $N_f = 2+1$ flavors of quark loops on a $24^3 \times 128$ anisotropic lattice with spacing $a_s \sim 0.12$ fm and aspect ratio $a_s/a_t \sim 3.5$ for a pion mass $m_\pi \sim 220$ MeV. In the legends, “fwd" refers to contributions from the diagram containing only forward-time source-to-sink quark lines, “smt" refers to contributions from the diagram containing only quark lines that originate and terminate at the same time. For the σ channel, the “smt" contribution has a vacuum expectation value subtraction. Forward-time quark lines use dilution scheme (TF, SF, LI8) and same-time quark lines use (TI16, SF, LI8).

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References

[1] S. Basak, *et al.*, Phys. Rev. D**72**, 094506 (2005).

[2] J. Bulava, *et al.*, Phys. Rev. D **79**, 034505 (2009).

[3] M. Peardon, *et al.*, Phys. Rev. D **80**, 054506 (2009).

[4] J. Bulava, *et al.*, Phys. Rev. D **82**, 014507 (2010).

[5] C. Morningstar, *et al.*, AIP Conf. Proc. **1257**, 779 (2010) arXiv:1002.0818 [hep-lat].

[6] J. Foley, *et al.*, Comput. Phys. Commun. **172**, 145 (2005).

[7] R.G. Edwards and B. Joo, Nucl. Phys. B (Proc. Suppl.) **140**, 832 (2005).