BNL-94342-2010-JA, KUNS-2310, NIKHEF 2010-040, RBRC-871

How Wide is the Transition to Deconfinement?

Adrian Dumitru[a,b], Yun Guo[c,d], Yoshimasa Hidaka[e], Christiaan P. Korthals Altes[f,g], and Robert D. Pisarski[h]
[a] *Department of Natural Sciences, Baruch College, 17 Lexington Avenue, New York, NY 10010, USA*
[b] *RIKEN/BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*
[c] *Department of Physics, Brandon University, Brandon, Manitoba, R7A 6A9 Canada*
[d] *Winnipeg Institute for Theoretical Physics, Winnipeg, Manitoba, Canada*
[e] *Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan*
[f] *Centre Physique Théorique au CNRS Case 907, Campus de Luminy F-13288 Marseille, France*
[g] *Nikhef Theory Group, Science Park 105, 1098 XG Amsterdam, The Netherlands*
[h] *Department of Physics, Brookhaven National Laboratory, Upton, NY 11973*

(Dated: November 18, 2010)

Pure $SU(3)$ glue theories exhibit a deconfining phase transition at a nonzero temperature, T_c. Using lattice measurements of the pressure, we develop a simple matrix model to describe the transition region, when $T \geq T_c$. This model, which involves three parameters, is used to compute the behavior of the 't Hooft loop. There is a Higgs phase in this region, where off diagonal color modes are heavy, and diagonal modes are light. Lattice measurements of the latter suggests that the transition region is narrow, extending only to about $\sim 1.2\,T_c$. This is in stark contrast to lattice measurements of the renormalized Polyakov loop, which indicates a much wider width. The possible implications for the differences in heavy ion collisions between RHIC and the LHC are discussed.

PACS numbers: 12.38.Mh,12.38.Gc,25.75.Nq

Heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) have demonstrated a rich variety of unexpected behavior [1]. Notably, in peripheral collisions the elliptical flow can only be described by nearly ideal hydrodynamics, with a very small ratio between the shear viscosity, η, and the entropy density, s. The differences between collisions at RHIC, and those which will soon be observed soon at the Large Hadron Collider (LHC), will be especially interesting: does the nearly ideal hydrodynamic behavior, observed at RHIC, persist at the much higher energies of the LHC?

One approach to deconfinement exploits the analogy to $\mathcal{N} = 4$ supersymmetric gauge theories: using the AdS/CFT correspondence, such theories are computable analytically in the limit of infinite coupling, for an infinite number of colors [2]. By introducing a potential for the dilation field, the behavior of the entropy density near the deconfining phase transition, at a temperature T_c, can be fit from measurements on the lattice [3–6]. While the entropy density, s, decreases strongly as $T \to T_c^+$ because it is related to Hawking radiation, in AdS/CFT models the ratio η/s remains completely independent of temperature. This suggests that like RHIC, that collisions at the LHC should also be described by nearly ideal hydrodynamics; see, also, Ref. [7].

In this work we consider a very different approach to the deconfining phase transition. It assumes that the coupling is moderate even down to the transition temperature, T_c [8]. We use an elementary matrix model, involving three parameters, to parametrize the behavior of the deconfining phase transition. A version of this model with one parameter was first proposed by Meisinger, Miller, and Ogilvie [9]. Similar models arise for theories in which one (or more) spatial directions are of femtoscale size [10–13].

The parameters of the model are fixed from lattice measurements of the pressure [14–17]. It then predicts how the 't Hooft loop [18–22] changes with temperature near T_c, which we compare to the results of lattice simulations [23]. Further, the model predicts that for a range of temperatures above T_c, there is a Higgs phase, where correlation functions of electric fields are a mixture of heavy and light modes, from fields which are off diagonal, and diagonal, in color, respectively. This may help to understand the results of lattice simulations [15, 24–26], which are otherwise somewhat puzzling.

The most direct prediction of our model is for the expectation value of the Polyakov loop. For the pure glue $SU(3)$ theory, lattice simulations find that the (renormalized) Polyakov loop vanishes below T_c, jumps to ~ 0.4 at T_c^+, and then rises with T, until it is approximately constant above $\sim 4.0\,T_c$ [27–29]. This represents confinement below T_c, a complete Quark Gluon Plasma (QGP) at high temperature, and a "semi"-QGP in between [30–

33]. Physically, there is no ionization of color in the confined phase, total ionization in the complete QGP, and only partial ionization in the semi-QGP [33]. (While we discuss a purely gluonic plasma, we adopt the common term QGP.)

The principal thrust of this paper is that from indirect measurements on the lattice, we suggest that the width of the semi-QGP is *much* narrower than indicated by present results for the renormalized Polyakov loop: not up to $\sim 4.0\,T_c$, but only $\sim 1.2\,T_c$. We do not understand this discrepancy in detail, but suggest a possible reason later. This discrepancy is the reason why, having fit the parameters of our model from the pressure, we compute both the 't Hooft loop and gluon masses.

While we treat the pure glue theory, our model can be extended to QCD, with dynamical quarks [17]. It is reasonable to assume that in QCD, the semi-QGP is like that of the pure glue theory, relatively narrow. We thus conclude by discussing the possible phenomenological implications of our results for heavy ion collisions.

How confinement arises in our model can be understood by analogy. Consider a bosonic field with a chemical potential μ; now take μ to be imaginary, defining $\mu = 2\pi T\, i\, q$. For a particle of energy E, the Bose-Einstein statistical distribution function is

$$n(E,q) = \frac{1}{\mathrm{e}^{E/T - 2\pi i q} - 1} \tag{1}$$

As a function of q, $n(E,q)$ is clearly a periodic function of q, invariant under $q \to q+1$. Thus we can choose to define q to lie within the range from $-\frac{1}{2}$ to $+\frac{1}{2}$.

Now *assume* that we integrate over q, with a distribution which is flat in q. Expand for large energy, so that the first term is the Boltzmann statistical distribution function. Given the assumed distribution in q, the integral of this term vanishes,

$$\mathrm{e}^{-E/T} \int_{-1/2}^{+1/2} \mathrm{e}^{2\pi i q}\, dq = 0\,. \tag{2}$$

Indeed, we can expand the Bose-Einstein distribution function term by term in powers of Boltzmann factors, $\mathrm{e}^{-E/T + 2\pi i q}$ [13]; doing so, the integral over each and every term obviously vanishes. The same is true for the Fermi-Dirac distribution function as well.

Thus a flat distribution in q represents the confined phase. To represent a phase with partial deconfinement, one integrates over a limited region, say $q : -q_0 \to +q_0$, with $q_0 < \frac{1}{2}$. Complete deconfinement occurs when one integrates over a distribution which is a delta-function in q.

This example appears somewhat artificial. For a given q, the statistical distribution functions are complex valued, and so, only integrals over q can possibly represent physical quantities. Indeed, the grand canonical ensemble is characterized by a fixed value for the chemical potential, and not by an integral over μ's.

Nevertheless, precisely this mechanism arises for the deconfining phase transition in a $SU(N)$ gauge theory. Consider the expansion about a background field for the time-like component of the vector potential,

$$\left(A_0^{cl}\right)_{ab} = \frac{2\pi T}{g}\, q_a\, \delta^{ab}\,; \tag{3}$$

a and b are colors indices, running from $1 \ldots N$. For nonzero q_a's, this background field acts like an imaginary chemical potential for the diagonal elements of the gauge group. Integration over the q_a's arises from imposing Gauss' law for those elements of the gauge group [19].

This background field generates a non-trivial expectation for the Polyakov loop, ℓ, which is the color trace of the thermal Wilson line, $\mathbf{L}$:

$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}\;\;;\;\; \mathbf{L} = \mathcal{P}\exp\left(ig\int_0^{1/T} A_0\, d\tau\right)\;; \tag{4}$$

$\mathcal{P}$ represents path ordering, T is the temperature, and τ the imaginary time, $\tau : 0 \to 1/T$.

Since the gauge potential A_0 is an element of $SU(N)$, $\sum_{a=1}^{N} q_a = 0$, modulo one, so there are $N-1$ independent q_a's. At infinite N, the q_a's form a continuum, and the example of Eq. (1) is exact; see, *e.g.*, computations on a femtosphere at $N = \infty$ [13]. For two colors, we can choose the eigenvalues to be $q_1 = -q_2$; for three, $q_1 = -q_2$, and $q_3 = 0$.

In the presence of the background field of Eq. (3), a potential for the q_a's is generated at one loop order [18–22],

$$\mathcal{V}_{pt}(q_a) \;=\; \frac{2\pi^2 T^4}{3} \sum_{a,b=1}^{N} q_{ab}^2\,(1 - |q_{ab}|)^2 - (N^2-1)\frac{\pi^2 T^4}{45}\,. \tag{5}$$

where $q_{ab} = q_a - q_b$, defined modulo one. The minimum is at $q_a = 0$, where $-\mathcal{V}_{pt}(0)$ is the pressure for an ideal gas of gluons.

The potential $\mathcal{V}_{pt}(q_a)$ enters in computations of the 't Hooft loop. It is useful to consider deconfinement as a type of spin system. A pure $SU(N)$ gauge theory has N degenerate vacua, where the thermal Wilson line $\mathbf{L}$ equals one of the N roots of unity,

$$\mathbf{L} = \mathrm{e}^{2\pi i j/N}\,\mathbf{1}\,, \tag{6}$$

$j = 0 \ldots (N-1)$. The usual vacuum, with $j = 0$ and $\mathbf{L} = \mathbf{1}$, corresponds to all $q_a = 0$. A $Z(N)$ vacua with $j = 1$ and $\mathbf{L} = \mathrm{e}^{2\pi i/N}\mathbf{1}$ corresponds to $N-1$ q_a's $= 1/N$, and the remaining element $= -1 + 1/N$.

At high temperature in the complete QGP, the theory lies in one spin state, which we can choose to be $j = 0$. One can compute tunneling between two degenerate vacua by constructing a box which is long in one spatial direction, with $j = 0$ at one end, and $j = 1$ at the other. An interface between the two ordered states forms in the center of the box, with the interface tension between the two computable semi-classically, using the potential $\mathcal{V}_{pt}(q_a)$ [18–22]. This interface is equivalent to

a 't Hooft loop which wraps around the center of the box [20].

As the temperature decreases and T approaches T_c, domains with $j \neq 0$ form and grow in size. They become increasingly probable, until at T_c^- and below, as a spin system the vacuum is completely disordered, a sum over many spin domains.

We want to add terms to the effective potential which model the transition to deconfinement. We could add perturbative corrections to $\mathcal{V}_{pt}(q_a)$, which have been computed to $\sim g^3$ [21], but invariably they give $q_a = 0$ (or a $Z(N)$ equivalent state) as the vacuum. With a complete theory, such as the monopole model of Liao and Shuryak [7], this potential could be computed directly.

We adopt a more modest approach, attempting to guess the form of the non-perturbative potential. We fit the coefficients which enter to lattice results for the pressure, and then use it to compute other quantities. The advantage of our approach is that we can compute quantities not just in, but near thermal equilibrium. Such quantities, like the shear viscosity [33], are much harder to extract on the lattice.

Since the Polyakov loop is an order parameter for deconfinement, a natural guess is that the non-perturbative potential involves $Z(N)$ invariant elements of the Lie group. The first such term is the adjoint loop [12, 30–35]. Instead, following the authors of Ref. [9], and computations of the 't Hooft loop [18–22], we write a potential which is a polynomial in the q_a's.

There are several symmetries which any potential of the q_a's must satisfy. It must be periodic in each q_a, with $q_a \to q_a + 1$. It must also be invariant under $Z(N)$ transformations, where $N-1$ of the q_a's shift by $1/N$, and the last element, by $-1 + 1/N$. Lastly, if we interchange the ordering of the q_a's, we can change $q_{ab} \to q_{ba} = -q_{ab}$. These symmetries can be satisfied by constructing a potential as a function of $q_{ab}(1 - q_{ab})$.

We can still form an infinite number of terms by tying together the color indices in different ways; see, *e.g.*, the examples at two [21] and three [13] loop order. We adopt the simplest approach, and take terms like those which arise at one loop order, Eq. (5), which involve a sum over one q_{ab}:

$$\mathcal{V}_{non}(q_a) = T^2\, T_c^2 \sum_{a,b=1}^{N} \left(c_1\, |q_{ab}|(1 - |q_{ab}|) \right.$$

$$\left. + c_2\, q_{ab}^2\, (1 - |q_{ab}|)^2 + c_3 \right) . \qquad (7)$$

The potential of Ref. [9] involves one term, $\sim c_1$. We introduce two more: that $\sim c_2$, which is proportional to the perturbative term in Eq. (5), and c_3, which is just a constant. We take all of the non-perturbative terms to be $\sim T^2$, since lattice simulations indicate that in the pure glue theory, the leading corrections to terms $\sim T^4$ are $\sim T^2$ [9, 10, 32, 36].

When the q_a's develop an expectation value, this represents symmetry breaking for an adjoint scalar field, A_0, coupled to an $SU(N)$ gauge field, the A_i's [32]. As an adjoint scalar, though, there is no strict order parameter which distinguishes between the symmetric and broken phases. Thus there need not be a phase transition in going from the symmetric phase, the complete QGP, to the "broken" phase, the semi-QGP.

If there were such a phase transition, it would represent a second transition, above T_c, separate from that for deconfinement. While possible, in a pure $SU(N)$ gauge theory lattice simulations only find evidence for one phase transition, at T_c [14–17]. To avoid a phase transition between the complete and semi-QGP, it is essential that the non-perturbative potential has a term $\sim c_1$. Assume that the effective potential only involved terms such as $\sim q_{ab}^n (1 - q_{ab})^n$ for $n \geq 2$. For small q_a, these are of quadratic or higher order in the q_a's, and of necessity, there would then be a phase transition when the q_a's developed a nonzero expectation value, of either first or second order. For small q_a, the term $\sim c_1$ is *linear* in the q_a's, and ensures that there is no such phase transition. Instead, even for high temperature, there is always a small but non-zero expectation value for the q_a's, $< q_a > \sim 1/T^2$.

(We remark that effective theories on the lattice exhibit phases with broken symmetry [37]. The necessity of such a broken phase near T_c does not seem to have been appreciated previously, though.)

To determine the parameters of the model we compare to lattice measurements of the pressure. For three colors, this is illustrated in Fig. (1); for two colors, in Fig. (2). If $p(T)$ is the pressure, and $e(T)$ the energy density, then a more sensitive test of the fit is also to plot the interaction measure, $\Delta = e - 3p$. Thus in each figure we plot p/T^4, e/T^4, and Δ/T^4, both from the lattice, from Ref. ([14]) for two colors, and from Ref. ([15]) for three colors.

The parameters of the fit are

$$c_1 = -.41488\,;\; c_2 = -5.45957\,;\; c_3 = 0.21954\,. \qquad (8)$$

for three colors, and

$$c_1 = -0.30267\,;\; c_2 = - \; -5.97440\,;\; c_3 = 0.18341\,. \qquad (9)$$

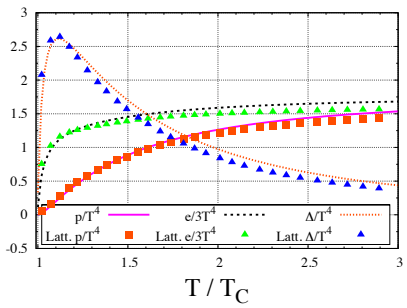

FIG. 1: Comparison of lattice results for $SU(3)$ pure gauge to the model, for the pressure, energy density, and interaction measure.

for two colors.

Our model appears to involve three parameters, but this is misleading. One parameter fixes the critical temperature, T_c. A second is chosen so that the pressure vanishes at T_c. Thus we really have only one free parameter, which is tuned to fit the interaction measure. The model of Ref. ([9]) involves one parameter, c_1, which then defines T_c; the pressure in that model is negative at T_c. In our model, for two colors the energy density is negative within $\sim 1\%$ of T_c; this could be ameliorated by adding an additional parameter.

Given the effective Lagrangian, it is then straightforward to compute the 't Hooft loop. In the complete QGP, the potential includes only the perturbative potential, $\mathcal{V}_{pt}(q_a)$, Eq. (5); in the semi-QGP, it is a sum of this and the non-perturbative potential, $\mathcal{V}_{non}(q_a)$.

For two colors, as $q_2 = -q_1$ there is only one independent direction, and it is direct to compute the tunneling path, and its associated action, analytically. The result for the 't Hooft loop is

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6g^2(T)}}\,\xi(g^2)\,\frac{(1-(T_c/T)^2)^{3/2}}{1-0.908\,(T_c/T)^2}\;, \qquad (10)$$

where

$$\xi(g^2) = 1 - 0.16459\,g^2(T)\;.$$

The factors involving T_c/T are special to the semi-QGP, so that as $T \gg T_c$, the result reduces to that in the complete QGP [18]. The function $\xi(g^2)$ is the correction $\sim g^2$ in the complete QGP; in plotting, we take $g^2(2\pi T)$ [8]. The 't Hooft loop vanishes at T_c, as expected for a second order phase transition.

For three colors, in the semi-QGP the vacua is along λ_3 (using the Gell-Mann notation), while the path for the 't Hooft loop depends upon a change in λ_8. The path was determined numerically, and lies along both λ_3 and λ_8. The action of the tunneling path was also determined numerically, and the result for the 't Hooft loop for three colors is illustrated in Fig. (3). (For $N = 2$, we take $T_c/\Lambda_{\bar{M}S} = 1.31$; for $N = 3$, 1.14. For the same value of $T_c/\Lambda_{\bar{M}S} = 1.31$, the results unexpectedly coincide.)

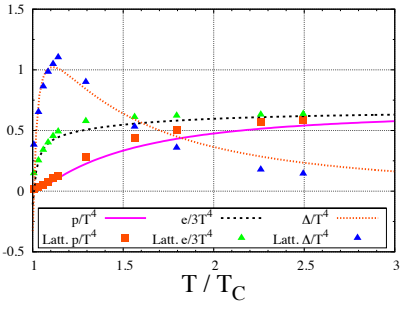

FIG. 2: Comparison of lattice results for $SU(2)$ pure gauge theory to the model, for the pressure, energy density, and interaction measure.

Including $\xi(g^2)$, the semi-classical computation of the 't Hooft loop in the complete QGP agrees with lattice simulations above $\sim 4.0\,T_c$; below that temperature, they agree with the result in the semi-QGP [23]. To obtain agreement, however, it is necessary to include the correction $\xi(g^2)$; this is computed in the complete QGP, which is incorrect. Two things are required to compute $\xi(g^2)$ in the semi-QGP. First, the potential for constant q_a needs to be computed to two loop order, expanding about the full potential, $\mathcal{V}_{pt}(q_a) + \mathcal{V}_{non}(q_a)$. Second, corrections to one loop order need to be computed for the kinetic term. In the complete QGP this brings in new functions, the $\psi(q_a)$ [18]. Other functions could arise in the semi-QGP. For now, we defer these involved computations; since the corrections $\sim g^2$ are large, $\sim 50\%$, our results should be considered as tentative.

Besides the 't Hooft loop, which is an interface tension for an order-order interface at $T \geq T_c$, the interface tension for the order-disorder interface, at T_c, is also computable in our model. This only exists for a first order transition; for three colors,

$$\sigma_{dis} = 0.0258012\,\frac{T_c^2}{\sqrt{g^2}}\;. \qquad (11)$$

It is necessary to compute the corrections $\sim g^2$ before comparing to lattice data, though.

The parameters for three colors, Eq. (8), and two colors, Eq. (9), are similar; the difference is commensurate with a dependence on $\sim 1/N^2$, with the coefficient of order one. We have then assumed that the parameters for three colors are close to those for higher N. We find reasonable agreement for the interaction measure to lattice results [16]. When $N \geq 4$, there is more than one 't Hooft loop. Lattice simulations find that they obey Casimir scaling to good approximation [23]. We have not checked this explicitly, but suspect that in our model, 't Hooft loops respect Casimir scaling.

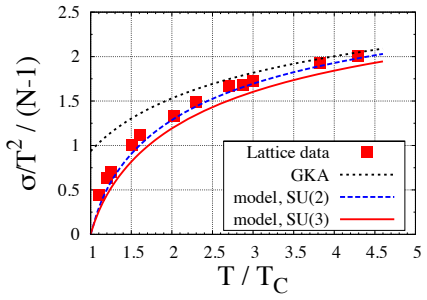

FIG. 3: The 't Hooft loop for $SU(3)$ pure gauge theory: lattice data from Ref. [23], and GKA, Giovannangeli and Korthals Altes, Ref. [21], the semi-classical computation in the complete QGP, including corrections of $\sim g^2$. In our model we show results for two and three colors, assuming that the corrections of $\sim g^2$ are identical in the complete and semi-QGP; see text.

The most novel prediction of our model is that there is a Higgs effect in the semi-QGP. This was noted first in Ref. [32], and in theories at a femtoscale [12]. To understand it, consider the quantum fluctuations about the background field of Eq. (3):

$$\left\langle \left(A_0^{qu}\right)_{ab}(\vec{x})\ \left(A_0^{qu}\right)_{ba}(0)\right\rangle \sim \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{x}} \sum_{n=-\infty}^{+\infty} \Delta_{00} \tag{12}$$

where Δ_{00} is the quantum propagator

$$\Delta_{00} = \frac{e^{-ip_0\tau}}{(\vec{p})^2 + p_0^2 + m_D^2(q)}\,;\ p_0 = 2\pi T\,(n + q_a - q_b)\ . \tag{13}$$

The shift in the energy, $p_0 = 2\pi Tn \to 2\pi T(n+q_a-q_b)$, is because we are expanding about a background field. The background field A_0^{cl} acts upon quantum fluctuations like an adjoint Higgs field. Because the field is diagonal in color, Eq. (3), diagonal fields do not feel the background field. Thus for diagonal fields, the only mass they develop is the Debye mass, m_D. This is of order $\sim gT$ times a function of the q_a's [33]. In contrast, off diagonal fields have non-trivial commutators with a diagonal field, and so they develop "masses" which are large, $\sim 2\pi T(n + q_a - q_b)$.

We illustrate this in Fig. (4) for three colors. The masses of the two diagonal gluons are equal, and decrease as $T \to T_c^+$. There are two types of off-diagonal gluons: four with $|a-b| = 1$, and two with $|a-b| = 2$. The splitting of the masses is evident only close to T_c, for $T < 1.1\,T_c$.

We do not plot lattice data, because it is somewhat contradictory. Lattice measurements of a gauge invariant quantity, the two point function between Polyakov loops, shows that the associated mass goes down [15]. In contrast, the two point function of gluons indicate that the (gauge dependent) mass increases as $T \to T_c^+$ [26].

The static, spatial gluon fields, the A_i, also undergo a Higgs effect. This also happens in a monopole gas [7].

We have not discussed the most obvious application of our model: the computation of the Polyakov loop. We plot this quantity, and the lattice results, for three colors in Fig. (5). A direct comparison of the two is somewhat misleading. We have not computed perturbative corrections to the Polyakov loop, which enter at $\sim g^3$ [38]. This contribution is positive, and will increase the result. Nevertheless, while the two coincide at T_c — which is presumably coincidence — they immediately diverge from one another. From Fig. (5), in our model the loop quickly goes up to a constant value by $\sim 1.2\,T_c$; this is very different from lattice measurements, for which it is not constant until a much higher temperature, $\sim 4.0\,T_c$ [27–29].

Our computations show that determining the width of the semi-QGP depends sensitively upon the quantity considered. Even for purely thermodynamic quantities, Figs. (1) and (2), p/T^4 and Δ/T^4 are not constant until temperatures $\sim 3.0\,T_c$; in contrast, e/T^4 is nearly constant above $\sim 1.5\,T_c$. The 't Hooft loop in Fig. (3) is like the pressure, and shows deviations from the complete QGP up to $\sim 4.0\,T_c$. The Higgs effect, Fig. (4), most closely follows the results for the Polyakov loop in our model, and shows no significant splitting above $\sim 1.2\,T_c$.

With this qualification, we define the width of the semi-QGP from the behavior of the Polyakov loop. We comment that a similar rapid growth in the Polyakov loop is found in solutions of the Schwinger-Dyson equations [39]. Our results do not coincide numerically, though.

Why does the value of the Polyakov loop, computed from our model, differ so significantly from lattice measurements of the (renormalized) Polyakov loop? There is an ambiguity associated with the renormalized Polyakov loop, from the zero point energy. In Ref. [33] we argued that perturbatively, the zero point energy vanishes for a straight Polyakov loop. This argument fails for a "smeared" loop (see, *e.g.*, the appendix of Ref. [28]). If so, then the effects of smearing must be very dramatic.

It is worth emphasizing that the rapid growth of the Polyakov loop in our model is *not* an accident of the particular values of Eqs. (8) and (9), but dictated by the peak in the interaction measure above T_c. To understand this, consider a model which involves only the Polyakov

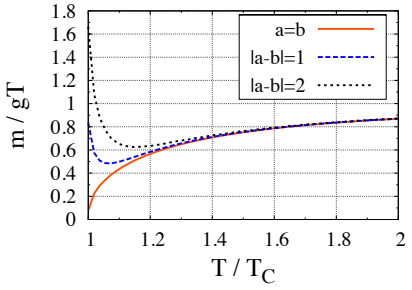

FIG. 4: Gluon masses, $m_{ab}/(gT)$, for $SU(3)$ pure gauge theory: diagonal gluons, with $a = b$, are light; while there are two off diagonal gluons with heavy masses, for $|a-b| = 1$ and $|a-b| = 2$.

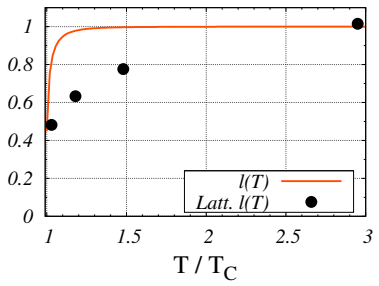

FIG. 5: The Polyakov loop for a $SU(3)$ pure gauge theory from lattice simulations [29] and in our model.

loop of Eq. (4),

$$\mathcal{V}_{eff}(\ell) = \left(-\frac{b_2}{2} |\ell|^2 + \frac{1}{4} (|\ell|^2)^2 \right) b_4 \, T^4 \; ; \qquad (14)$$

see, *e.g.*, Eq. (2) of Ref. [31], and Polyakov Nambu-Jona-Lasino (PNJL) models [34]. For three colors, the $Z(3)$ symmetry also allows a cubic term, $\sim \ell^3 + (\ell^*)^3$, but its addition would only complicate the algebra, and not our qualitative conclusion. There is no cubic term for two colors.

The minimum of the potential is $\ell_0 = \sqrt{b_2}$, which we choose to be real. As it is related to the pressure of an ideal gas of gluons, we assume that the coefficient b_4 is independent of the temperature, and that only b_2, or equivalently ℓ_0, depends upon T. The pressure and the interaction measure are then

$$\frac{p}{p_{ideal}} = \ell_0^4 \; ; \; \frac{e - 3\,p}{4\,p_{ideal}} = \ell_0^3 \, T \frac{\partial \, \ell_0}{\partial \, T} \; . \qquad (15)$$

Consider first high temperature, where the expectation value of the loop is near one. Assuming that $\ell_0 - 1 \sim -(T_c/T)^2 + \ldots$, then from Eq. (15), $p/p_{ideal} - 1 \sim 4(\ell_0 - 1) + \ldots$, while $e/e_{ideal} - 1 \sim (4/3)(\ell_0 - 1) + \ldots$. That is, differences from ideal gas behavior are greatest for the pressure and the interaction measure, and smallest for the energy density. This agrees with results from the lattice: above T_c, e/T^4 quickly shoots up to a nearly constant value, while p/T^4 is not nearly constant until a much higher temperature [14–17].

Now consider the region near T_c. The lattice data, for two [14], three [15], and four or more [16] colors, show a sharp peak in the interaction measure just above T_c. From Eq. (15), this must correspond to rapid change in ℓ_0. This is similar to what we find in our matrix model, Fig. (5). Moreover, in the loop model $\sqrt{b_2}$ in Eq. (14) is proportional to the mass for the ℓ field. Since $\sqrt{b_2} = \ell_0$, then, a rapid increase in ℓ implies the same for the mass of ℓ. This is similar to the mass of the diagonal modes in the matrix model, Fig. (4).

There are significant differences between Polyakov loop and matrix models, though. In Ref. [31] and Polyakov loop models [34], in order to fit the pressure the temperature dependence of b_2 must have a complicated form. For our matrix model, though, the coefficients are just $\sim T^4$, Eq. (5), and $\sim T^2$, Eq. (7). In a mean field theory such as this, simplicity is a virtue. Second, the splitting of gluon masses near T_c is special to a matrix model, as a Higgs effect for the adjoint scalar A_0 field. (See, *e.g.*, the loop model of Ref. [10], where the splitting of masses does not occur.) Notably, moving up from T_c, the masses of the off-diagonal modes decrease, while those of the diagonal modes increase, Fig. (4).

Our analysis is a preliminary first step. In deriving our results, we balance the perturbative potential, $\mathcal{V}_{pt}(q_a)$, against the non-perturbative potential, $\mathcal{V}_{non}(q_a)$. In powers of g^2, the perturbative potential is of order one, so implicitly we have assumed that the non-perturbative is as well. Since the non-perturbative potential represents a resummation of effects to all orders, this is a strong assumption. Nevertheless, it allows us to envisage computing to higher order in g^2. Corrections at least to order $\sim g^2$ and $\sim g^3$ are needed in order to make a serious comparison to lattice data. This also requires precise lattice data, close to the continuum limit, not just for the pressure, but also for the 't Hooft loop and gluon masses.

There are several formal questions raised by our analysis. The parameters of effective theories can be computed from lattice simulations [35]; doing so for elements of the Lie algebra, instead of for elements of the Lie group, may be useful. It is also necessary to extend the analysis of Hard Thermal Loops in the complete QGP to the semi-QGP. This is equivalent to understanding the analytic continuation of the thermal Wilson line from imaginary to real time.

To compare with QCD it is necessary to include the effects of dynamical quarks. It will be especially interesting to see if, upon adding the effects of quarks to the perturbative potential, $\mathcal{V}_{pt}(q_a)$, whether the thermodynamics [17], and the Debye mass, are reproduced using the same parameters for the non-perturbative potential, $\mathcal{V}_{non}(q_a)$, in the pure glue theory. (With dynamical quarks, the 't Hooft loop does not exist as an order parameter.) If need be, one could adjust the parameters of the non-perturbative potential to depend upon the presence of dynamical quarks, but this would be inelegant.

Without detailed computation, we assume that a narrow width for the semi-QGP in the pure glue theory implies the same for QCD. We thus conclude with some speculations for the phenomenology of heavy ion collisions.

If RHIC probes to some temperature in the QGP, then LHC may probe to a temperature approximately $\sim 50\%$ higher. If the AdS/CFT correspondence holds for QCD, then results at the LHC must mimic those at RHIC. With the present analysis, the picture is rather more complicated.

We assume, for the sake of argument, that RHIC probes only to a temperature in the semi-QGP, very near T_c. Then the LHC begins at a temperature well in the complete QGP. Any conclusions are tempered by the fact that even if the LHC starts at a higher temperature, as it cools it must traverse through the semi-QGP.

In the semi-QGP, the ratio of η/s decreases as the square of the Polyakov loop as $T \to T_c^+$; this is true both in the pure glue theory, and with dynamical quarks [33]. Conversely, then, η/s increases as the temperature goes up from T_c. This is in sharp contrast to models based upon the AdS/CFT correspondence, where η/s is constant [3–5]. Unfortunately, a computation beyond leading logarithmic order is required to compute the precise dependence of η/s with temperature.

If the shear viscosity increases strongly from T_c, *and* the system is in thermal equilibrium, then an increased shear viscosity should lead to an increase in particle mul-

tiplicity, and a decrease in the elliptical flow, over the results expected from a (nearly) ideal gas. If the shear viscosity increases significantly, though, a hydrodynamic description could easily break down.

It is also possible that the temperature dependence of η/s is weak; if so, then the particle multiplicity and elliptical flow at LHC should be similar to that expected by an extrapolation from the results at RHIC. There are then other ways to probe the effects of the semi-QGP.

Consider, for example, energy loss, which is controlled by a parameter $\hat{q}$. In the complete QGP, $\hat{q} \sim T^3$, or equivalently, the entropy density, s. In kinetic theory, $\hat{q}/s$ and s/η are each proportional to a cross section, so one expects that a minimum in η/s corresponds to a maximum in the energy loss, $\hat{q}/s$ [7, 40]. Following the methods of Ref. [33], the energy loss of a quark can be computed in the semi-QGP; as $T \to T_c^+$, it vanishes linearly in the Polyakov loop. Thus in the semi-QGP, both η/s and $\hat{q}$ decrease as $T \to T_c^+$; the difference from Refs. [7, 40] is because the kinetic theory for the semi-QGP is in the presence of a background A_0 field. As the temperature increases from T_c, then, excluding the obvious dependence upon the entropy, the energy loss is larger in the complete QGP than in the semi-QGP. As with the shear viscosity, determining the precise dependence upon temperature requires computation beyond leading logarithmic order.

There is also a qualitatively new phenomenon in the semi-QGP: besides energy loss, the propagation of a colored field is suppressed by the background A_0 field [33]. This suppression is universal, independent of the mass or momentum of the colored field. A complete analysis need incorporate this universal suppression as well as energy loss.

Lastly, we note that given the modified propagator of the semi-QGP, Eq. (13), there are also significant modifications to the heavy quark potential [41]. This can also be compared to lattice data, which we defer for now.

In the end, our speculations will soon be rendered moot by the wealth of results which will flow from heavy ion collisions at the LHC. The present approach is based upon constructing an effective theory from the results of lattice simulations; not just of the pressure, but quantities such as the 't Hooft loop and screening masses. This can then be tested against predictions from the AdS/CFT correspondence [2–6] and other models [7].

Acknowledgments

We thank A. Bazavov, O. Kaczmarek, F. Karsch, J. Liao, P. Petreczky, and E. Shuryak for discussions. The research of A.D. was supported by the U.S. Department of Energy under contract #DE-FG02-09ER41620; of Y.G., in part by the Natural Sciences and Engineering Research Council of Canada; of Y.H., by the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence" from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan; of R.D.P., by the U.S. Department of Energy under contract #DE-AC02-98CH10886; R.D.P. also thanks the Alexander von Humboldt Foundation for their support.

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