

Scalar-isoscalar states, gravitational form factors, and dimension-2 condensates in a large- N_c Regge approach

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Abstract. Scalar-isoscalar states ($J^{PC} = 0^{++}$) are analyzed within the large- N_c Regge approach. We find that the lightest $f_0(600)$ scalar-isoscalar state fits very well into the pattern of the radial Regge trajectory. We confirm the obtained mass values from an analysis of the pion and nucleon spin-0 gravitational form factors, recently measured on the lattice. We find that a simple two-state model suggests a meson nature of $f_0(600)$, and a glueball nature of $f_0(980)$, which naturally explains the ratios of various coupling constants. Finally, matching to the OPE requires a fine-tuned mass condition of the vanishing dimension-2 condensate in the Regge approach with infinitely many scalar-isoscalar states.

Keywords: σ meson, scalar-isoscalar states, large- N_c Regge models, pion and nucleon gravitational form factors, dimension-2 condensate
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INTRODUCTION

Hadron resonances appearing in the PDG tables increase their mass while their width remains constant [1]. On the other hand, in the large- N_c limit, with $g^2 N_c$ fixed, mesons and glueballs are stable, their masses become independent on N_c , $m = \mathcal{O}(N_c^0)$, while their widths Γ are suppressed as $1/N_c$ and $1/N_c^2$, respectively, which means that Γ/m is suppressed (see e.g. [2] for a review). This suggests that excited states in the mesonic spectrum may follow a reliable large- N_c pattern. The main feature of a resonance is that it corresponds to a mass distribution, with values approximately spanning the $m \pm \Gamma/2$ mass interval. The lowest resonance in QCD is the 0^{++} state $f_0(600)$ or the σ -meson, which appears as a complex pole in the second Riemann sheet of the $\pi\pi$ scattering amplitude at $s_\sigma = m_\sigma^2 - im_\sigma\Gamma_\sigma$ with $m_\sigma = 347(17)\text{MeV}$ and $\Gamma_\sigma/2 = 345(24)\text{MeV}$ [3] (see also Ref. [4]). Higher 0^{++} states are listed in Table 1.

For scalar states a measure of the spectrum is given in terms of the trace of the energy momentum tensor [5]

$$\Theta_\mu^\mu \equiv \Theta = \frac{\beta(\alpha)}{2\alpha} G^{\mu\nu a} G_{\mu\nu}^a + \sum_q m_q [1 + \gamma_m(\alpha)] \bar{q}q. \quad (1)$$

Here $\beta(\alpha) = \mu^2 d\alpha/d\mu^2$ denotes the beta function, $\alpha = g^2/(4\pi)$ is the running coupling constant, $\gamma_m(\alpha) = d \log m/d \log \mu^2$ is the anomalous dimension of the current quark mass m_q , and $G_{\mu\nu}^a$ is the field strength tensor of the gluon field. This operator connects scalar states to

the vacuum through the matrix element

$$\langle 0 | \Theta | n \rangle = m_n^2 f_n. \quad (2)$$

The two-point correlator reads

$$\begin{aligned} \Pi_{\Theta\Theta}(q) &= i \int d^4x e^{iq \cdot x} \langle 0 | T \{ \Theta(x) \Theta(0) \} | 0 \rangle \\ &= \sum_n \frac{f_n^2 q^4}{m_n^2 - q^2} = q^4 \left[C_0 \log q^2 + \sum_n \frac{C_{2n}}{q^{2n}} \right], \end{aligned} \quad (3)$$

where in the second line we saturate with scalar states and the large $q^2 \gg \Lambda_{\text{QCD}}$ limit is taken. Comparison with the Operator Product Expansion (OPE) [6] leads to

$$C_0 = - \lim_{n \rightarrow \infty} \frac{f_n^2}{dm_n^2/dn} = - \frac{N_c^2 - 1}{2\pi^2} \left(\frac{\beta(\alpha)}{\alpha} \right)^2, \quad (4)$$

$$C_2 = \sum_n f_n^2 = 0, \quad (5)$$

$$C_4 = \sum_n f_n^2 m_n^2 = \left(\frac{\beta(\alpha)}{\alpha} \right)^2 \langle G^2 \rangle. \quad (6)$$

Equation (4) requires infinitely many states, while Eq. (5) suggests a positive and non-vanishing gauge-invariant dimension-2 object, $C_2 = i \int d^4x x^2 \langle \Theta(x) \Theta \rangle$, which is generally non-local, as it should not appear in the OPE.

SCALAR REGGE SPECTRUM

Radial and rotational Regge trajectories were analyzed in Ref. [7]. In Ref. [8] the scalar sector was studied in more detail. Two parallel radial trajectories could then

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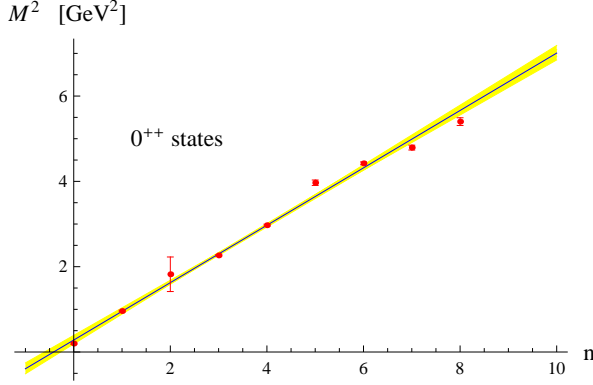


FIGURE 1. Radial Regge trajectory corresponding to the squared mass of all $J^{PC} = 0^{++}$ scalar-isoscalar states listed in the PDG tables [1]. The four heaviest 0^{++} states are not yet well established and are omitted from the PDG summary table. The error bars correspond to the errors in the determination of the square of mass, listed in [1]. The straight line is the result of our fit.

be identified, including three states per trajectory. In a recent work [9] we have analyzed all the 0^{++} states which appear in the PDG tables (see Fig. 1) and found that *all* fit into a single radial Regge trajectory of the form

$$M_S(n)^2 = \frac{a}{2}n + m_\sigma^2. \quad (7)$$

The mass of the σ state can be deduced from this trajectory as the mass of the lowest state. The resonance nature of these states suggests using the corresponding half-width as the mass uncertainty by minimizing

$$\chi^2 = \sum_n \left(\frac{M_{f,n} - M_S(n)}{\Gamma_{f,n}/2} \right)^2, \quad (8)$$

which yields $\chi^2/\text{DOF} = 0.12$ (see also Table 1) with

$$a = 1.31(12) \text{ GeV}^2, \quad m_\sigma = 556(127) \text{ MeV}. \quad (9)$$

Formula (7) is equivalent to two parallel radial Regge trajectories with the *standard* slope

$$M_{S,-}(n)^2 = an + m_\sigma^2, \quad (10)$$

$$M_{S,+}(n)^2 = an + m_\sigma^2 + \frac{a}{2}, \quad (11)$$

where $a = 2\pi\sigma$, and σ is the string tension associated to the potential $V(r) = \sigma r$ between heavy colored sources. The value $\sqrt{\sigma} = 456(21) \text{ MeV}$ agrees well with lattice determinations of $\sqrt{\sigma} = 420 \text{ MeV}$ [10].

This situation suggests the existence of a hidden symmetry in the 0^{++} sector. In the holographic approach based on the AdS/CFT correspondence the symmetry corresponds to parity in the fifth-dimensional variable.

TABLE 1. PDG values of resonance parameters [1], compared to the fit to the radial Regge spectrum of the 0^{++} scalar-isoscalar states, $M_n^2 = \frac{1}{2}an + m_\sigma^2$, given by Eq. (8). The lowest $n = 0$ state, corresponding to the $f_0(600)$ resonance, is excluded from the fit.

Resonance	M [MeV]	Γ [MeV]	n	M (Fit)
$f_0(600)$	400 – 1200	500 – 1000	0	556
$f_0(980)$	980(10)	70(30)	1	983
$f_0(1370)$	1350(150)	400(100)	2	1274
$f_0(1500)$	1505(6)	109(7)	3	1510
$f_0(1710)$	1724(7)	137(8)	4	1714
$f_0(2020)$	1992(16)	442(60)	5	1896
$f_0(2100)$	2103(8)	209(19)	6	2062
$f_0(2200)$	2189(13)	238(50)	7	2215
$f_0(2330)$	2321(30)	223(30)	8	2359

This is similar to the one-dimensional harmonic oscillator; all states with the energy $E_n = \hbar\omega(n + 1/2)$ can be separated in parity *even* and *odd* states, with energies $E_n^{(+)} = 2\hbar\omega(n + 1/4)$ and $E_n^{(-)} = 2\hbar\omega(n + 3/4)$, respectively, having twice the slope of E_n .

Besides, there seems to be no obvious difference between mesons and glueballs, as far as the spectrum is concerned. Note that the Casimir scaling suggests that the string tension is $\sigma_{\text{glueball}} = \frac{9}{4}\sigma_{\text{meson}}$, but this holds in the case of fixed and heavy sources. The fact that we have light quarks might explain why we cannot allocate easily the Casimir scaling pattern in the light-quark scalar-isoscalar spectrum.

GRAVITATIONAL FORM FACTORS

Hadronic matrix elements of the energy-momentum tensor, the so-called gravitational form factors (GFF) of the pion and nucleon, correspond to a dominance of scalar states in the large- N_c picture, as ($u(p)$ is a Dirac spinor)

$$\langle \pi(p') | \Theta | \pi(p) \rangle = \sum_n \frac{g_{n\pi\pi} f_n q^2 m_n^2}{m_n^2 - q^2}, \quad (12)$$

$$\langle N(p') | \Theta | N(p) \rangle = \bar{u}(p') u(p) \sum_n \frac{g_{nNN} f_n m_n^2}{m_n^2 - q^2}, \quad (13)$$

where the sum rules $\sum_n g_{n\pi\pi} f_n = 1$ [11] $M_N = \sum_n g_{nNN} f_n$ [12] hold. Unfortunately, the lattice QCD data for the pion [13] and nucleon (LHPC [14] and QCDSF [15] collaborations), picking the valence quark contribution, are too noisy as to pin down the coupling of the excited scalar-isoscalar states to the energy-momentum tensor. Nevertheless, useful information confirming the (Regge) mass estimates for the σ -meson can be extracted using multiplicative QCD evolution of the GFF through the valence quark momentum fraction, $\langle x \rangle_{u+d}$, as seen in deep inelastic scattering or on the

TABLE 2. N_c -scaling.

quantity	glueball	$q\bar{q}$ meson
m_n	1	1
f_n	N_c	$\sqrt{N_c}$
$\Gamma_{n\pi\pi}$	$1/N_c^2$	$1/N_c$
$g_{n\pi\pi}$	$1/N_c$	$1/\sqrt{N_c}$
g_{nNN}	1	$\sqrt{N_c}$

lattice at the scale $\mu = 2\text{GeV}$. For the pion GFF we obtain the fit

$$\langle x \rangle_{u+d}^\pi = 0.52(2), \quad m_\sigma = 445(32) \text{ MeV}, \quad (14)$$

whereas for the nucleon GFF we get

$$\langle x \rangle_{u+d}^N = 0.447(14), \quad m_\sigma = 550_{-200}^{+180} \text{ MeV}. \quad (15)$$

Assuming a simple dependence of m_σ on m_π ,

$$m_\sigma^2(m_\pi) = m_\sigma^2 + c(m_\pi^2 - m_{\pi,\text{phys}}^2), \quad (16)$$

yields $m_\sigma = 550_{-200}^{+180} \text{ MeV}$ and $c = 0.95_{-0.75}^{+0.80}$, or $m_\sigma = 600_{-100}^{+80} \text{ MeV}$ and $c = 0.8(2)$, depending on the lattice data [14] or [15], respectively. *Higher* quark masses might possibly clarify whether or not the state evolves into a glueball or a meson, since in that case one has, respectively, either $m_\sigma/(2m_q) \rightarrow 0$, or $m_\sigma/(2m_q) \rightarrow 1$.

DIMENSION-2 CONDENSATES

It is interesting to discern the nature of the σ state from an analysis of a truncated spectrum. The minimum number of states, allowed by certain sum rules and low energy theorems, is just two. In Ref. [9] we undertake such an analysis, which suggests that $f_0(600)$ (denoted as σ) is a $\bar{q}q$ meson, while $f_0(980)$ (denoted as f_0) is a glueball according to the N_c scaling of various quantities (see Table 2). The argument is based on the fact that one obtains $f_\sigma/f_{f_0} \sim g_{f_0 NN}/g_{\sigma NN} \sim g_{f_0 \pi\pi}/g_{\sigma \pi\pi} \sim 1/\sqrt{3}$, however $C_2 = f_\sigma^2 + f_{f_0}^2 \neq 0$ because the number of states is finite.

The infinite Regge spectrum of Eq. (7) with Eq. (4) may be modeled with a constant $f_{f_0} = f_{n,+} = \mathcal{O}(N_c)$ discarding $f_\sigma = f_{n,-} = \mathcal{O}(\sqrt{N_c})$. Naively, we get $C_2 = \infty$. However, C_2 may vanish, as required by standard OPE, when infinitely many states are considered *after regularization*. Using ζ -function regularization [16], yields [9]

$$C_2 \equiv \lim_{s \rightarrow 0} \sum_{n=0} f_{n,+}^2 M_{s,+}(n)^{2s} = f_{f_0}^2 (1/2 - m_{f_0}^2/a), \quad (17)$$

which at leading N_c implies $C_2 = 0$ for $m_{f_0} = \sqrt{a/2} = 810(40) \text{ MeV}$, a reasonable value to $\mathcal{O}(1/N_c)$.

What should these m_σ values be compared to? Besides the pole definition one also has the Breit-Wigner (BW)

definition, which in the $\pi\pi$ data is disputed for the σ but not for the ρ . Our analysis is driven by large- N_c considerations (see also Refs. [17, 18]). In Ref. [19] it was shown that the difference between the BW and the pole definitions is $\mathcal{O}(1/N_c^2)$ and, further, that for $N_c = 3$ the BW definition works *as good* for the σ as for the ρ . In Ref. [20] it was argued that $m_\sigma - m_\rho = \mathcal{O}(1/N_c)$. Thus, we expect the present estimates to be in between, incorporating a systematic $\mathcal{O}(1/N_c)$ mass shift.

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