Nonperturbative gluon and ghost propagators in $d = 3$

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Abstract.

We study the nonperturbative gluon and ghost propagators in $d = 3$ Yang-Mills, using the Schwinger-Dyson equations of the pinch technique. The use of the Schwinger mechanism leads to the dynamical generation of a gluon mass, which, in turn, gives rise to an infrared finite gluon propagator and ghost dressing function. The propagators obtained are in very good agreement with the results of $SU(2)$ lattice simulations.

INTRODUCTION

Even though QCD_3 differs from QCD_4 in several aspects, both theories share a crucial nonperturbative property: they cure their infrared (IR) instabilities through the dynamical generation of a gauge boson (gluon) mass, without affecting the local gauge invariance, which remains intact [1, 2]. The nonperturbative dynamics that gives rise to the generation of such a mass can be ultimately traced back to a subtle realization of the Schwinger mechanism [3, 4]. The gluon mass generation manifests itself at the level of the fundamental Green's functions of the theory in a very distinct way, giving rise to an IR behavior that would be difficult to explain otherwise. Specifically, in the Landau gauge, both in $d = 3, 4$, the gluon propagator and the ghost dressing function reach a finite value in the deep IR [5, 6, 7, 8]. However, the gluon propagator of QCD_3 displays a local maximum at relatively low momenta [9, 10], before reaching a finite value at $q = 0$. This characteristic behavior is qualitatively different to what happens in $d = 4$, where the gluon propagator is a monotonic function of the momentum in the entire range between the IR and UV fixed points [5].

Given that the gluon mass generation is a purely nonperturbative effect, it can be naturally treated within the framework of the Schwinger-Dyson equations (SDE). These complicated dynamical equations are best studied in a gauge-invariant framework based on the pinch technique (PT) [1, 11, 12], and its profound correspondence with the background field method (BFM) [13]. As has been explained in detail in the recent literature [14], this latter formalism allows for a gauge-invariant truncation of the SD series, in the sense that it preserves manifestly, and at every step, the transversality of the gluon self-energy.

In the present talk we report on a recent study of the gluon and ghost propagators of pure Yang-Mills in $d = 3$, using the SDEs of the PT-BFM formalism in the Landau gauge [8] (for different approaches see, e.g.,[15, 16]).

GLUON MASS GENERATION IN YANG-MILLS THEORIES

In order to understand the basic concept underlying the Schwinger mechanism, let us consider the gluon propagator (in the Landau gauge),

$$\Delta_{\mu\nu}(q) = -i\mathrm{P}_{\mu\nu}(q)\Delta(q^2), \qquad (1)$$

where $\mathrm{P}_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2$. The scalar factor $\Delta(q^2)$ is given by $\Delta^{-1}(q^2) = q^2 + i\Pi(q^2)$, where $\Pi_{\mu\nu}(q) = \mathrm{P}_{\mu\nu}(q)\Pi(q^2)$ is the gluon self-energy. One usually defines the dimensionless vacuum polarization, to be denoted by $\boldsymbol{\Pi}(q^2)$, as $\Pi(q^2) = q^2\boldsymbol{\Pi}(q^2)$, and thus

$$\Delta^{-1}(q^2) = q^2[1 + i\boldsymbol{\Pi}(q^2)]. \qquad (2)$$

As Schwinger pointed out long time ago [3], the gauge invariance of a vector field does not necessarily imply zero mass for the associated particle, if the current vector coupling is sufficiently strong. According to Schwinger's fundamental observation, if $\boldsymbol{\Pi}(q^2)$ acquires a pole at zero momentum transfer, then the vector meson becomes massive, even if the gauge symmetry forbids a mass at the level of the fundamental Lagrangian. Indeed, it is clear that if the vacuum polarization $\boldsymbol{\Pi}(q^2)$ has a pole at $q^2 = 0$ with positive residue m^2, i.e.,

$$\boldsymbol{\Pi}(q^2) = m^2/q^2, \qquad (3)$$

then (in Euclidean space)

$$\Delta^{-1}(q^2) = q^2 + m^2. \qquad (4)$$

Thus, the vector meson becomes massive, $\Delta^{-1}(0) = m^2$, even though it is massless in the absence of interactions

($g = 0$). There is *no* physical principle that would preclude $\mathbf{\Pi}(q^2)$ from acquiring such a pole, even in the absence of elementary scalar fields. In a *strongly-coupled* theory, like nonperturbative Yang-Mills in $d = 3, 4$, this may happen for purely dynamical reasons, since strong binding may generate zero-mass bound-state excitations [4]. The latter act *like* dynamical Nambu-Goldstone bosons, in the sense that they are massless, composite, and *longitudinally coupled*; but, at the same time, they differ from Nambu-Goldstone bosons as far as their origin is concerned: they do *not* originate from the spontaneous breaking of any global symmetry [1]. In what follows we will assume that the theory can indeed generate the required bound-state poles; the demonstration of the existence of such bound states is a difficult dynamical problem, that must be addressed by means of Bethe-Salpeter equations.

The Schwinger mechanism is incorporated into the SDE of the gluon propagator through the form of the nonperturbative three-gluon vertex (Fig.1). In fact, in order for the gauge symmetry to be preserved, the three-gluon vertex must satisfy the same Ward identity as in the massless case, but now with massive, as opposed to massless, gluon propagators on its rhs. The way this crucial requirement is enforced is precisely through the incorporation into the three-gluon vertex of the Nambu-Goldstone (composite) massless excitations mentioned above. To see how this works with a simple example, let us consider the standard tree-level vertex

$$\Gamma_{\mu\alpha\beta}(q,p,r) = (q-p)_\beta g_{\mu\alpha} + (p-r)_\mu g_{\alpha\beta} + (r-q)_\alpha g_{\mu\beta},$$

(5)

which satisfies the simple Ward identity

$$q^\mu \Gamma_{\mu\alpha\beta}(q,p,r) = P_{\alpha\beta}(r)\Delta_0^{-1}(r) - P_{\alpha\beta}(p)\Delta_0^{-1}(p) \quad (6)$$

where $\Delta_0^{-1}(q) = q^2$ is the inverse of the tree-level propagator. After the dynamical mass generation, the inverse gluon propagator becomes, roughly speaking,
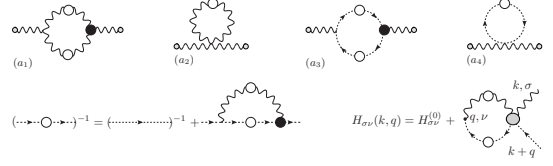
$$\Delta_m^{-1}(q^2) = q^2 - m^2(q^2), \qquad (7)$$

and the new vertex, $\mathbf{\Gamma}_{\mu\alpha\beta}^m(q,p,r)$ that must replace $\Gamma_{\mu\alpha\beta}(q,p,r)$ must still satisfy the Ward identity of (6), but with $\Delta_0^{-1} \to \Delta_m^{-1}$ on the rhs. This is accomplished if

$$\mathbf{\Gamma}_{\mu\alpha\beta}^m(q,p,r) = \Gamma_{\mu\alpha\beta}(q,p,r) + V_{\mu\alpha\beta}(q,p,r), \qquad (8)$$
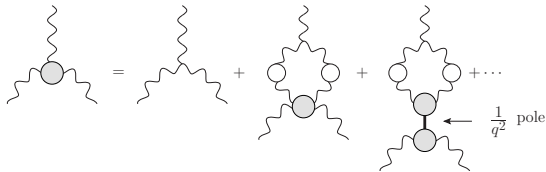
FIGURE 1. Vertex with nonperturbative massless excitations triggering the Schwinger mechanism.

FIGURE 2. The SDEs for the various quantities involved.

where $V_{\mu\alpha\beta}(q,p,r)$ contains the massless poles. A standard Ansatz for $V_{\mu\alpha\beta}(q,p,r)$ is [17]

$$
\begin{aligned}
V_{\mu\alpha\beta}(q,p,r) &= m^2(r)\frac{q_\mu p_\alpha (q-p)_\rho}{2q^2 p^2} P_\beta^\rho(r) \\
&\quad - \left[m^2(p) - m^2(q)\right]\frac{r_\beta}{r^2} P_\rho^\mu(q) P_\alpha^\rho(p) \\
&\quad + \text{c.p.},
\end{aligned}
$$

(9)

It is easy to check that

$$q^\mu V_{\mu\alpha\beta}(q,p,r) = P_{\alpha\beta}(p)m^2(p) - P_{\alpha\beta}(r)m^2(r), \quad (10)$$

and cyclic permutations. Therefore, one has

$$q^\mu \Gamma_{\mu\alpha\beta}^m(q,p,r) = P_{\alpha\beta}(r)\Delta_m^{-1}(r) - P_{\alpha\beta}(p)\Delta_m^{-1}(p),$$

(11)

as announced.

SDE ANALYSIS AND COMPARISON WITH THE LATTICE

In the "one-loop dressed" approximation, the PT-BFM gluon self-energy is given by the subset of diagrams shown in Fig.2. As explained in detail in various works (see, e.g., [14, 12], the resulting gluon self-energy is manifestly transverse, due to the simple Ward identities satisfied by the PT-BFM fully dressed vertices. In addition to the SDE of the gluon, we consider the corresponding SDEs for (i) the ghost propagator, denoted by $D(p)$, (or its dressing function, $F(p)$, given by $D(p) = iF(p)/p^2$), and (ii) the auxiliary function $G(q)$, defined as the $g_{\mu\nu}$ component of the function $H_{\mu\nu}$, shown in Fig.2. $G(q)$ enters into the important relation

$$\Delta(q) = [1 + G(q)]^2 \widehat{\Delta}(q), \qquad (12)$$

relating the PT-BFM gluon propagator, $\widehat{\Delta}(q)$, and the conventional one, $\Delta(q)$ (simulated on the lattice). The closed expressions for all these SDEs have been given in [8].

The way we proceed is the following. Instead of actually solving the system of coupled integral equation, we follow an approximate procedure, which is operationally less complicated, and appears to capture rather well the underlying dynamics.
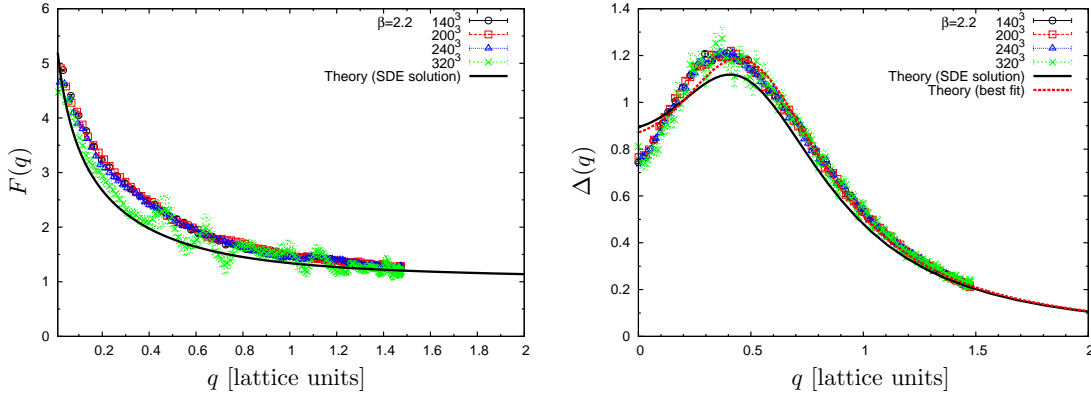
FIGURE 3. Comparison with the lattice results of [9].

Specifically, we will assume that the PT-BFM gluon propagator has the form

$$\widehat{\Delta}^{-1}(q) = q^2 + m^2 + \widehat{\Pi}_m(q). \qquad (13)$$

where

$$(d-1)\widehat{\Pi}_m(q) = [(a_1) + (a_2) + (a_3) + (a_4)]_\mu^\mu. \qquad (14)$$

The function $\widehat{\Pi}_m(q)$ will be determined by calculating the Feynman graphs given in Fig.2, using inside the corresponding integrals $\Delta \to (q^2 + m^2)^{-1}$, and $D \to 1/q^2$ In order to maintain gauge invariance intact, ensuring that $q_\mu \widehat{\Pi}_m^{\mu\nu}(q) = 0$, we will use the $\boldsymbol{\Gamma}_{\mu\alpha\beta}^m(q,p,r)$ given in (8) as the fully-dressed three-gluon vertex; in $V_{\mu\alpha\beta}^m(q,p,r)$ we will use a constant (instead of a running) mass, m. As for the ghost dressing function, we set up an approximate version of the ghost SDE, and *solve* it self-consistently for the unknown function $F(p)$. As explained in [8], this procedure allows for a very good *simultaneous* fit of the available lattice data. The best possible fit we have found is shown in Fig.3, furnishing the ratio $m/2g^2 = 0.15$.

CONCLUSIONS

We have presented a nonperturbative study of the (Landau gauge) gluon and ghost propagator for $d = 3$ Yang-Mills, using the "one-loop dressed" SDEs of the PT-BFM formalism. One of the most powerful features of this framework is that the transversality of the truncated gluon self-energy is guaranteed, by virtue of the QED-like Ward identities satisfied by the fully-dressed vertices entering into the dynamical equations. The central dynamical ingredient of our analysis is the assumption that the Schwinger mechanism is indeed realized in $d = 3$ Yang-Mills. The propagators obtained from these non-perturbative equations agree rather well with the results of $SU(2)$ lattice simulations.

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