Symmetry energy of dense matter in holographic QCD

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We study the nuclear symmetry energy of dense matter using holographic QCD. To this end, we consider two flavor branes with equal quark masses in a D4/D6/D6 model. We find that at high density the symmetry energy monotonically increases without softening. For small density, it shows power law behavior $E_{\rm sym} \sim \rho^{1/2}$.

Nuclear symmetry energy is one of key words in nuclear physics as well as in astrophysics. Its density dependence is a core quantity of asymmetric nuclear matter which has important effects on heavy nuclei and is essential to understand neutron star properties. Although much efforts have been given to match its importance, it is still very poorly understood especially in the supra-saturation density regime, see [1–6] for a review and for a recent discussion.

From experimental side, the available data do not constrain much the value of the symmetry energy at supra-saturation densities. Recently, using the FOPI data on π^-/π^+ ratio in central heavy ion collisions, Xiao et al. [7] obtained a circumstantial evidence for a soft nuclear symmetry energy at $\rho \geq 2\rho_0$, where the nuclear symmetry energy increases with the density up to the saturation density ρ_0 and then starts to decrease afterwards. Theoretically, almost all possible tools in many-body theory, (Dirac)-Brueckner-Hartree-Fock approach, self-consistent Green's function approach, etc, as well as phenomenological approaches, relativistic mean field theory, Thomas-Fermi approximations, etc, were employed to study the density dependence of the symmetry energy, see [3] for a review. While they showed similar behaviors up to the nuclear saturation density, at supra-saturation densities, all possible results one can imagine were predicted and no consensus could be reached: some showed stiff dependence (increasing monotonically with density), while others showed soft one, see Fig. 1 for a typical example. Given this situation, it is very important to determine the behavior of the nuclear symmetry energy at high densities with a model or theory which provides a reliable calculational tool.

The gauge/gravity duality [9–11] provides a new tool to study strongly interacting dense matter, and many models for QCD [12, 13] based on the duality were constructed. Although the true holographic dual of QCD is yet to be constructed, it is worthwhile to find out what the new tool says about QCD, and this is the goal of this paper.

Based on the treatment of dense matter in confined phase suggested in [14], a simple model for nuclear matter to strange matter transition was proposed in [15], where two D6 branes for light and intermediate mass (strange) flavors were introduced. The dense matter was introduced by uniformly distributed compact D4 branes with N_c fundamental strings attached. By considering energy minimization, transition from nuclear to strange matter could be studied. To calculate the symmetry energy in nuclear matter, we study the case where two flavors have the same quark masses, $m_1 = m_2$. We find that the symmetry energy is increasing with the total charge Q, showing that symmetry energy of our system has a stiff dependence on the density.

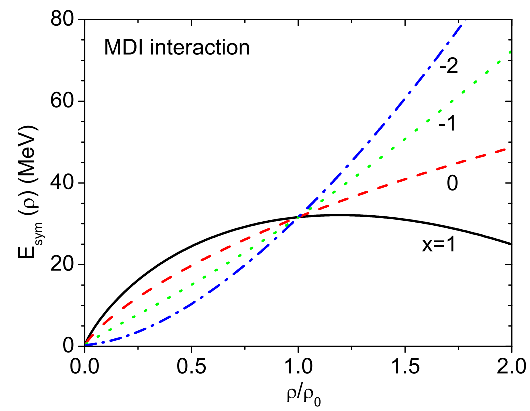

FIG. 1. (Color online) Example of density dependence of the nuclear symmetry energy, taken from [8]. Depending on the value of the parameter x, various high density behaviors are possible.

The nuclear symmetry energy is defined as the energy per nucleon required to change isospin symmetric nuclear matter to pure neutron matter. In the Bethe-Weizsäcker mass formula for the nuclear binding energy, it represents the amount of binding energy that a nucleus has to lose when the numbers of protons and neutrons are not equal. The semi-empirical mass formula based on the liquid drop model has the form:

$$E_{\rm B} = a_{\rm v} A - a_a (N-Z)^2/A - a_c Z^2/A^{1/3} - a_s A^{2/3} \pm a_\delta/A^{3/4}\,. \qquad (1)$$

Here Z (N) is the number of protons (neutrons) in a nucleus. The first term is called the volume energy since the volume of a nucleus is proportional to A, total nucleon number. The origin of this volume term is the strong nuclear force. The second is known as the asymmetry term, which defines the symmetry energy. If there were no Coulomb repulsions between protons, we would expect to have equal number of neutrons and protons in nuclei in general. The term with a_c accounts for the Coulomb interaction of $Z(Z-1)/2$ pairs of protons in the nucleus. The last two terms represent the surface energy and pairing effect, respectively. Using data for nucleus binding energies, one can determine a set of coefficients in Eq. (1).

Due to the invariance of nuclear forces under neutron-proton interchange, iso-scalar quantities in a nuclear system are function of only even powers of the asymmetry factor. Here we define the asymmetry factor $\tilde{\alpha}$ as $\tilde{\alpha} \equiv (N-Z)/A$. Then we can express the energy density per nucleon $E(\rho, \tilde{\alpha})$ as

$$E(\rho, \tilde{\alpha}) \simeq E(\rho, 0) + S_2(\rho)\tilde{\alpha}^2 \,, \tag{2}$$

where ρ is the nucleon number density and $S_2(\rho) = \frac{1}{2}\frac{\partial^2 E}{\partial \tilde{\alpha}^2}|_{\tilde{\alpha}=0}$ is the symmetry energy.

Now we study the symmetry energy in the D4/D6/D6 model [15] with baryon vertices which consist of D4 branes and fundamental strings. In our approach gluon dynamics is replaced by the gravity sourced by the N_c color D4 branes, and two probe D6 branes are used to describe the up and down quarks. The bare quark masses are the distances between the D4 and two D6's in the absence of the string coupling.

We can write the metric of the confining D4 background as

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(-dt^2 + d\vec{x}^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{U}{\xi}\right)^2\left(d\xi^2 + \xi^2 d\Omega_4^2\right), \tag{3}$$

where $f(U) = 1 - (U_{KK}/U)^3$ and $(U/U_{KK})^{3/2} = (\xi^{3/2} + \xi^{-3/2})/2 \equiv \xi^{3/2}\omega_+/2$.

We wrap the D4 brane on S^4 which is transverse to the original D4 brane. Due to the Chern-Simons interaction with RR-field, $U(1)$ gauge field is induced on the D4 brane world volume. The source of gauge field is interpreted as the end point of fundamental strings. Substituting the equation of motion for gauge field to the Dirac-Born-Infeld action of D4 brane with the Chern-Simons, we get 'Hamiltonian' for D4 brane as

$$\mathcal{H}_{D4} = \tau_4 \int d\theta \sqrt{\omega_+^{4/3}(\xi^2 + \xi'^2)}\sqrt{D(\theta)^2 + \sin^6\theta}\,, \tag{4}$$

where $\tau_4 = \frac{1}{2^{2/3}}\mu_4\Omega_3 g_s^{-1}R^3U_{KK}$, $D(\theta) = -2 + 3\cos\theta - \cos^3\theta$ and the prime denotes the derivative with respect to θ. We assume that the radial coordinate ξ depends only on the polar angle θ of S^4.

Now, we consider the other end points of fundamental strings that are attached to two D6 branes with given number ratio. The end points of fundamental strings provide the source of $U(1)$ gauge field on the D6 brane. By taking the Legendre transformation for the gauge field, we get 'Hamiltonian' for D6 brane as

$$\mathcal{H}_{D6} = \tau_6 \int d\rho \sqrt{1 + \dot{y}^2}\sqrt{\omega_+^{4/3}\left(\tilde{Q}^2 + \rho^4\omega_+^{8/3}\right)}, \tag{5}$$

where $\tau_6 = \frac{1}{4}\mu_6 V_3 \Omega_2 g_s^{-1} U_{KK}^3$. $\tilde{Q}$ is dimensionless and related to the number of fundamental strings Q, $\tilde{Q} = \frac{U_{KK}Q}{2\cdot 2^{2/3}\pi\alpha'\tau_6}$. Two D6 branes are connected to a D4 brane by fundamental strings. Therefore, D6 branes are pulled down and compact D4 brane is pulled up. As discussed in [14], the length of the fundamental strings becomes zero since the tension of the fundamental strings is always larger than that of D-branes. Finally, the position of the cusp of D6 branes should be located at the same position of the cusp of D4 brane, ξ_c. We consider Q_1 fundamental strings attached on one D6 brane and Q_2 strings attached on another D6 brane. We denote the slope at the cusp of each brane as $\dot{y}_c^{(1)}$ and $\dot{y}_c^{(2)}$. The force at the cusp of D6 branes can be obtained as

$$F_{D6} = \left.\frac{\partial \mathcal{H}(Q_1)_{D6}}{\partial U_c}\right|_{\partial} + \left.\frac{\partial \mathcal{H}(Q_2)_{D6}}{\partial U_c}\right|_{\partial} \equiv F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2). \tag{6}$$

To make the system stable, the force balancing condition should be satisfied;

$$\frac{Q}{N_c}F_{D4} = F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2), \tag{7}$$

where $Q_1 = (1-\alpha)Q$ and $Q_2 = \alpha Q$ with $0 \le \alpha \le 1$, and F_{D4} is the force at the cusp due to the D4 brane.

To find the ground state of our system, we need to consider the energy minimization together with the force balancing condition. The total energy of our system can be written as

$$E_{tot} = \frac{Q}{N_c}\mathcal{H}_{D4} + \mathcal{H}_{D6}(Q_1) + \mathcal{H}_{D6}(Q_2)\,. \tag{8}$$

With given Q, we can determine the value of α, which satisfies the force balancing condition and minimizes the total energy. As in [15], for $m_2/m_1 = 50$ there exists a transition from a matter with $\alpha = 0$ to $\alpha \neq 0$ at some value of Q. This was identified as a transition from nuclear to strange matter. For very large Q, α saturates to 0.5, as expected. If we take $m_1 = m_2$ and if we do not consider isospin violating interactions or electromagnetic interactions, then the ground state of the matter would be always with $\alpha = 0.5$.

Here we consider only two cases: $m_2/m_1 = 1$ and $m_2/m_1 = 2$. The former is for the nuclear matter with isospin invariance and the latter is for the non-invariant case. The energy density per nucleon $E(\rho, \alpha)$ is given by E_{tot} in Eq. (8) divided by Q/N_c. Then we define the symmetry energy as in Eq. (2) with $\tilde{\alpha} = 1 - 2\alpha$.

We start with $m_2/m_1 = 1$ case. The explicit form of symmetry energy per nucleon can be written as

$$S_2 = \frac{2\tau_6}{N_B}\int d\rho \frac{\sqrt{1+\dot{y}^2}\tilde{Q}^2\omega_+^{10/3}\rho^4}{(\tilde{Q}^2 + 4\omega_+^{8/3}\rho^4)^{3/2}}, \tag{9}$$

where y is the embedding solution of D6 brane with $\tilde{\alpha} = 0$. Notice that $N_B = Q/N_c$ and so the symmetry energy (9) contains N_c factor. We need to factor this N_c out for the reason we discuss later. Our results are given in Fig. 2. Note that so far we use ρ for both the coordinate and the density. Hereafter ρ is only for the density. To fix the energy scale, we used the value of the 't Hooft coupling λ and the compactification scale M_{KK} determined in [19], see Eq. (12) for details. Now,

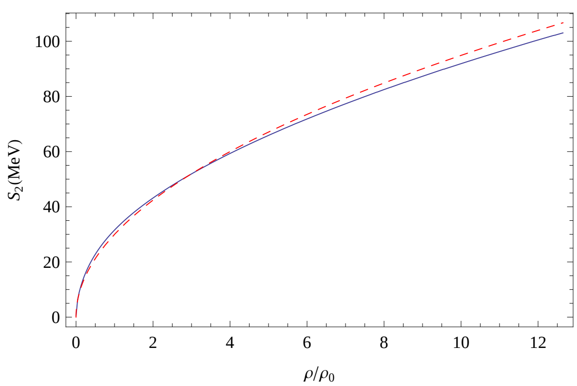

FIG. 2. (Color online) Solid line is the symmetry energy as a function of the density. The dotted line is the best fit of S_2 with $\rho^{1/2}$.

we lay emphasis on two aspects of our results, which are rather insensitive to the choice of λ and M_{KK}. One is the stiffness of the symmetry energy S_2 in supra-saturation density regime, and another is its low density power law behavior $S_2 \sim \rho^{1/2}$.

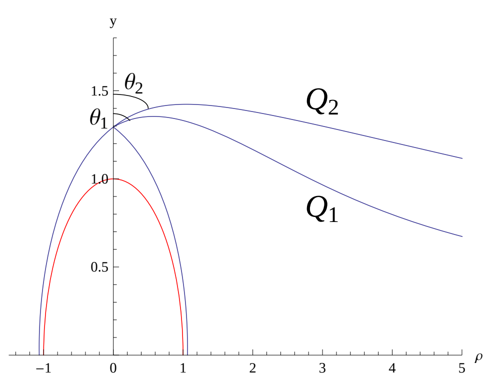

FIG. 3. (Color online) Embedding of D-branes with $\alpha \neq 0.5$. The asymptotic heights of two branes are the same ($m_1 = m_2 = 0.1$). Red curve denotes to the position of U_{KK}.

One can understand the stiffness based on the property of the branes: suppose two D6 branes meet with compact D4 at a point with different polar angles θ_1 and θ_2, see Fig. 3. Here Q_1 and Q_2 with $Q_1 + Q_2 = Q$ fixed are the number of the strings attached to each D6 brane. To balance the pulling force of the compact D4 with a minimum energy, each D6 brane balances roughly half of the pulling force. Since the upward force is proportional to $\cos\theta$, we have $Q_1 \cos\theta_1 \sim Q_2 \cos\theta_2$. Since $\theta_1 > \theta_2$, more strings should be attached to the lower brane. Therefore, the asymmetry in the number of the attached string is due to the angle difference. Notice that each end point provides the 'electric' flux which contributes to the energy of the brane. Since each flux of charges Q_1, Q_2 are confined in each brane, Coulomb energy increases like $\sim (Q_1^2 + Q_2^2)$. Minimum energy requests $Q_1 = Q_2 = Q/2$. As the number of attached strings increases, the brane gets stiffer and the maintaining the angle difference costs more and more energy.

The Coulomb repulsion discussed above is, of course, not the electromagnetic one. However, the repulsion in the dual bulk theory means repulsion in 4 dimension as well. From the boundary theory point of view, such repulsion is simply due to the presence of the baryon charge at the boundary. Therefore, it may be interpreted as the realization of the Pauli principle, which was suggested in [17] for different reason. Indeed, in the boundary theory it is the Pauli principle that requires $N = Z$ when we do not introduce any isospin violations. While, in the dual bulk theory it is the Coulomb interaction that requests $N = Z$, and therefore our interpretation of the Coulomb repulsion as the holographic Pauli principle is very natural [20].

Now we discuss the power law behavior of S_2 in low density. We can understand the numerical result by calculating analytically in special limit, $m_q \to \infty$ and $\rho \to 0$. In this case, the solution of D6 brane embedding becomes trivial, $\dot{y} = 0$ and we can integrate (9) analytically to have

$$S_2 = \left(\Gamma(\frac{5}{4})\right)^2 \sqrt{\frac{\lambda \rho_0}{2M_{KK}}} \sqrt{\frac{\rho}{\rho_0}}. \tag{10}$$

The current experimental result of the symmetry energy can be summarized by a fitting formula

$$S_2(\rho) = c(\rho/\rho_0)^\gamma \tag{11}$$

with $c \simeq 31.6$ MeV and $\gamma = 0.5 - 0.7$ in the low density regime, $0.3\rho_0 \le \rho \le \rho_0$, see [16] for example.

To fix the energy scale, we rewrite the energy density per nucleon as

$$E(\rho, \alpha) = \frac{\lambda N_c M_{KK}}{2^{2/3}(9\pi)} \frac{\tilde{E}}{\tilde{Q}}, \quad \rho = \frac{2 \cdot 2^{2/3}}{81(2\pi)^3} \lambda M_{KK}^3 \tilde{Q}\,, \tag{12}$$

where $\tilde{E} = E_{tot}/\tau_6$. For our purpose, we have to fix the value of λ and M_{KK}. One may determine those values by using the quark mass or meson mass as inputs. From the the non-anomalous η' mass, ~ 390 MeV [18], and the quark mass, $M_q = 7$ MeV, we can determine parameter of the D4/D6 model: $\lambda = 1.905$, and $M_{KK} = 1.039$ GeV [19]. With this choice as an example, we obtain $\gamma \simeq 0.48$ and $c \simeq 31$ MeV from Eq. (9). In this case, $\tilde{Q} = 4$ corresponds to $\rho \simeq 1.03\rho_0$. Notice that the value of γ is rather insensitive to the value of λ and M_{KK}.

Now, we study the effect of small isospin violation by considering $m_1 \neq m_2$. In this case, α for physical configuration becomes 0.5 only after certain density. Once α reaches 0.5, the symmetry energy seems to be the same for any values of m_2/m_1.

In summary, we calculated the symmetry energy of dense matter in the D4/D6/D6 model. To obtain the symmetry energy in nuclear matter with charge symmetry, we considered the case with $m_1 = m_2$ and found that the symmetry energy is increasing with the total charge Q, showing a stiff nuclear symmetry energy. It is universal in the sense that the result is independent of the value of λ and M_{KK}. We also studied the low density behavior with power γ to be $\sim 1/2$, which is again independent of the value of λ and M_{KK} and is close to the value suggested by experiments, $\gamma = 0.5 - 0.7$.

One subtle point we mentioned in the main text was about the factor N_c. The reason we divided this factor out is as follows. In our model, the same flavor quarks form a nucleon; for instance, proton in our model consists of $N_c m_1$ quarks and neutron has $N_c m_2$ quarks in it. Hence, the total number difference of quarks is N_c times the number difference of neutrons and protons, resulting in the overall N_c factor in the symmetry energy. However, in reality, where $N_c = 3$, proton consists of two up quarks and one down quark, and neutron contains one up quark and two down quarks. The total number difference of quarks is the same as the number difference of neutrons and protons. Therefore, in order to compare our result with the realistic case, we have to divide the symmetry energy (9) by N_c.

Finally, we studied the effect of isospin violation by considering $m_1 \neq m_2$. However, the symmetry energy in this case turned out to be almost the same with the case with isospin invariance for the mass ratios of order one.

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[20] Footnote: In the supersymmetric situation, however, there is a subtlety in this interpretation, because bosonic counter part of the quark also carries the baryon charge.