Two-hadron correlations in the Color Glass Condensate formalism

Jamal Jalilian-Marian

Baruch College, 17 Lexington Ave., New York NY 10010, USA and The Graduate School and University Center, City University of New York, 365 Fifth Avenue, New York, NY 10016, USA

Two-hadron correlations are a sensitive probe of the dynamics of gluon saturation and the Color Glass Condensate formalism where the degrees of freedom are Wilson lines of the gluon field. It is shown that unlike structure functions in DIS and single hadron production in proton-nucleus collisions, higher point functions of Wilson lines appear in two-hadron production cross section. We investigate the energy (x) evolution of these higher point functions using the JIMWLK evolution equations and show that dipole approximation, employed commonly in the literature to fit the di-hadron data measured by the STAR collaboration in the forward rapidity region, breaks down. This necessitates an investigation of the full hierarchy of the JIMWLK evolution equations for these higher point functions and their solutions. This can then be used to make a quantitative analysis of the di-hadron correlations in the forward rapidity region in deuteron-gold collisions at RHIC and in the long range rapidity correlations observed in proton-proton collisions at the LHC.

§1. Introduction

It is an experimental fact, established by HERA experiments on electron-proton collisions, that the number of gluons in the wave function of a proton grows fast with Bjorken $x_{Bj} \sim \frac{Q^2}{S}$ where Q^2 is the virtuality of the exchanged photon and S is the photon-hadron center of mass energy squared. Bjorken x_{Bj} can also be understood as the ratio of the energy of the proton carried by a gluon (or any parton in general) denoted by x. This came as a surprise and had led to an explosion of ideas on the fate of gluons in the limit when $x \to 0$, also known as the high energy limit of QCD.

The fast growth of the gluon distribution function can be understood in perturbative QCD to be due to sequential radiation of large number of soft (in longitudinal momentum) gluons which populate the wave function. In other words, the gluon splitting function is singular in the limit $x \to 0$ so that due to the large longitudinal phase space available at small x, probability for gluon radiation is large even though the coupling constant may still be small. Technically speaking, one needs to re-sum quantum corrections which are enhanced by $ln 1/x$ which in turn leads to a power growth of the gluon distribution function with $(1/x)^\lambda$ with $\lambda \sim 0.3$.

The power-like rise of the gluon distribution function would eventually lead to a power growth of hadronic cross sections which would violate the Froissart bound and unitarity. This fast growth however is believed to be tamed by non-linear effects, expected to be important when the density of gluons in a proton is so large that the probability for gluon recombination becomes of the same order as the bremsstrahlung radiation, responsible for the fast growth of the gluon distribution function.

The gluon recombination effects were first considered by Gribov-Levin-Ryskin in a pioneering paper[1)] where they argued that recombination diagrams should be as important as the bremsstrahlung ones when the gluon distribution function satisfies the following condition

$$\frac{\alpha_s \, xG(x, Q^2, b_t)}{S_\perp \, Q^2} \sim 1 \tag{1·1}$$

where $S_\perp$ is the unit transverse area. Mueller and Qiu[2)] then calculated the relevant diagrams in the double logarithm region and confirmed the GLR expectation. Solving equation (1·1) self-consistently leads to a scale $Q_s^2(x)$ where this relation is satisfied. This scale is nowadays called the saturation scale and depends on x as well as the impact parameter b_t. In case of a nucleus, it will also depend on A, the nucleon number, as $A^{1/3}$ so that one expects that the saturation scale is large at high energy $(x \to 0)$ and for large nuclei $(A^{1/3} \to \infty)$. This means that this high gluon density system is weakly coupled so that one can still use weak coupling techniques even though the system is non-perturbative. This is a highly non-trivial observation first made by McLerran and Venugopalan.[3)]

In this limit the standard expressions for particle production using collinear factorization in perturbative QCD breakdown due to two reasons, firstly due to the large energy effects which appear as large logs of $1/x$ and are not re-summed in the standard expressions. Secondly, due to the large gluon density the standard twist expansion is not valid anymore in the sense that all twist terms are as large as the leading twist term. This necessitates introduction of another formalism, capable of including both effects. The formalism which generalizes perturbative QCD to include both of these effects has come to be known the Color Glass Condensate formalism and the high gluon density effects are referred to as gluon saturation.

§2. The Color Glass Condensate

In order to go beyond the leading-twist perturbative QCD and to include both large logs of energy and gluon density, McLerran and Venugopalan introduced an effective action which treats the large x degrees of freedom as color charges ρ to which the gluon fields A^μ couple. Due to the small x approximation, the coupling between the color charges ρ and the gluon field is assumed to be eikonal and given by a Wilson line of the gauge field A_μ. The distribution of color charges ρ is non-perturbative and assumed to be given by a weight functional $W[\rho]$. To calculate an observable, one solves the classical equations of motion for a fixed color charge ρ and then averages over all color charges ρ with the weight functional $W[\rho]$. The weight functions $W[\rho]$ satisfies the so-called JIMWLK evolution equation[4)] which describes the evolution of W with energy or equivalently with rapidity y $(y = log\, 1/x)$. The JIMWLK evolution equation for rapidity evolution of any operator O can be written as

$$\frac{d}{dy}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x \, d^2y \, \frac{\delta}{\delta\alpha^b_x} \, \eta^{bd}_{xy} \, \frac{\delta}{\delta\alpha^d_y} \, O \right\rangle \,, \tag{2·1}$$

where

$$\eta^{bd}_{xy} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2} \frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2} \left[1 + U^\dagger_x U_y - U^\dagger_x U_z - U^\dagger_z U_y\right]^{bd} \tag{2·2}$$

and $U_x \equiv U(x_t)$ is the Wilson line in the adjoint representation. This formalism has been applied to any high energy process which involves at least one proton or nucleus in the initial state. Examples are Deeply Inelastic Scattering (DIS) of electrons on protons and nuclei, and particle production in hadronic/nuclear collisions. The simplest process to consider is DIS structure functions which are related to the total (virtual) photon-hadron cross section. In the rest frame of the target, this process can be factored into two parts; first the virtual photon splits into a quark anti-quark dipole which then scatters on the target. The probability for the virtual photon to split into a quark anti-quark dipole is given by the square of the photon wave function and is calculable in QED. The subsequent scattering of the quark anti-quark dipole on the target is described by the CGC formalism which describes its energy (rapidity or x) dependence. The JIMWLK evolution equation for the rapidity evolution of the quark anti-quark dipole can be written as

$$\frac{d}{dy}\langle \mathrm{tr}\, V_r^\dagger\, V_s\rangle = -\frac{N_c\,\alpha_s}{2\pi^2}\int d^2z\,\frac{(r-s)^2}{(r-z)^2(s-z)^2}\,\left\langle \mathrm{tr}\, V_r^\dagger\, V_s - \frac{1}{N_c}\mathrm{tr}\, V_r^\dagger\, V_z\;\mathrm{tr}\; V_s\, V_z^\dagger\right\rangle \tag{2·3}$$

where r, s are transverse coordinates of the quark and anti-quark. The left hand side of the equation describes the rapidity dependence of the probability for scattering of a fundamental (quark anti-quark) dipole on the target proton or nucleus. Due to the non-linear term on the right hand side, this probability is unitary in the sense that it can never be larger than one, unlike probabilities calculated in leading-twist pQCD. There are, however, no known analytic solutions to this equation and one has to resort to approximate methods in order to gain further understanding of its properties. One very common approximation is the leading N_c and dipole approximation such that the color average (denoted by the brackets in the above equation) of the product of any number of Wilson lines is replaced by products of color averages of two Wilson lines. With this approximation, the JIMWLK equation for the fundamental dipole (2-pt function) reduces to the Balitsky-Kovchegov (BK) equation.[5] Normalizing the 2-pt function as

$$S(r-s) \equiv \frac{1}{N_c} < \mathrm{tr}\; V_r^\dagger\, V_s > \tag{2·4}$$

the BK equation for the evolution of a fundamental dipole is written as

$$\frac{d}{dy}S(r-s) = -\frac{N_c\,\alpha_s}{2\pi^2}\int d^2z\,\frac{(r-s)^2}{(r-z)^2(s-z)^2}\left[S(r-s) - S(r-z)\,S(z-s)\right] \tag{2·5}$$

The dipole profile function $S(r-s)$ is the building block for many observables such as DIS structure functions at small x and forward particle production in proton-nucleus collisions at high energy. Quite recently the next-to-leading order corrections to the BK equation have been derived and the running coupling solution is obtained numerically.[6] The single inclusive hadron production cross section is given by[7),8]

$$\frac{d\sigma^{pA\to hX}}{dY\,d^2P_t\,d^2b} = \frac{1}{(2\pi)^2}\int_{x_F}^1 dx\,\frac{x}{x_F}\left\{f_{q/p}(x,Q^2)\;S[\frac{x}{x_F}P_t, b]\;D_{h/q}(\frac{x_F}{x},Q^2) + \right.$$

$$f_{g/p}(x,Q^2)\ S_A[\frac{x}{x_F}P_t,b]\ D_{g/h}(\frac{x_F}{x},Q^2)\Big\}\qquad(2{\cdot}6)$$

where S and S_A are the fundamental and adjoint dipoles. Using phenomenological models of the dipole profiles above has led to a quantitative description of the RHIC data on forward rapidity hadron production in dA collisions (the result for $y = 4$ was a prediction[8] which was later confirmed experimentally). Most recently the solution to the running coupling BK equation has been used to describe the data successfully[9] which gives one more confidence about the applicability of the CGC formalism in the forward rapidity region of RHIC. Nevertheless, models based on the standard collinear factorization can also describe the forward rapidity data by including shadowing and cold matter energy loss.[10] Measurement of other processes, for instance photon and dilepton production in the forward rapidity region[11] will help clarify the dynamics of the forward rapidity particle production (see[12] for a review of CGC and its applications to particle production in proton-nucleus collisions).

2.1. *Two-particle correlations*

Two-particle correlations are expected to contain more information about the dynamics of the process than single inclusive production. In addition to transverse momentum dependence of the cross section, one can investigate the angular dependence of the cross section which can shed more light on the dynamics of the process. For instance, in the standard collinear factorization approach to two hadron production, the produced partons are back to back (in Leading Order) so that one expects a sharp peak on the away side (π). On the other hand, in the CGC formalism, one expects to have a disappearance of the peak, due to shadowing generated by small x re-summation and p_t broadening due to multiple scattering. The simplest examples of two-parton productions in the CGC framework include quark anti-quark [13] and 2-gluon[14] production in DIS.

Two-hadron angular correlations in deuteron-gold collisions in the forward rapidity region have been recently measured by the STAR collaboration at RHIC. For central collisions a disappearance of the away side hadron is observed as expected in the CGC formalism. The underlying partonic process is a projectile quark scattering from the target nucleus and radiating a gluon either before or after the scattering. The expressions for this process are given in[14] in momentum space. Later they were also derived in the coordinate space which leads to a more compact form of the equations.[15] We refer the reader to[14],[15] for the details of derivation and the explicit form of the production cross section. Here we just note that production of a quark and gluon in dA collisions involves higher point (more than 2) functions of Wilson lines which were not present in DIS structure functions or single inclusive hadron production in dA collisions. For instance, the following products of Wilson lines, denoted O_4 and O_6 appear in the production cross section

$$O_4(r,\bar{r}:s) \equiv \mathrm{tr}\ V_r^\dagger\, t^a\, V_{\bar{r}}\, t^b\, [U_s]^{ab} = \frac{1}{2}\left[\mathrm{tr}\ V_r^\dagger\, V_s\ \ \mathrm{tr}\ V_{\bar{r}}\, V_s^\dagger - \frac{1}{N_c}\mathrm{tr}\ V_r^\dagger\, V_{\bar{r}}\right]\qquad(2{\cdot}7)$$

and

$$O_6(r,\bar r : s,\bar s) \equiv \mathrm{tr}\, V_r\, V_{\bar r}^\dagger\, t^a\, t^b\, [U_s\, U_{\bar s}^\dagger]^{ba} = \frac{1}{2}\left[\mathrm{tr}\, V_r\, V_{\bar r}^\dagger\, V_{\bar s}\, V_s^\dagger\ \ \mathrm{tr}\, V_s\, V_{\bar s}^\dagger - \frac{1}{N_c}\mathrm{tr}\, V_r\, V_{\bar r}^\dagger\right] \tag{2·8}$$

where the following identity is used

$$U^{ab}\, t^b = V^\dagger\, t^a\, V. \tag{2·9}$$

In principle one will need to know the equation describing the evolution of these operators with rapidity, the same way that the JIMWLK-BK equations describes the rapidity evolution of the weight function and the 2-pt function. The leading N_c part of the equation describing the rapidity evolution of 4 Wilson lines is derived in[14] while the complete evolution equation for the operators O_4 and O_6 (which appear in the di-jet production) is derived in.[16] Here we just write the results and refer the reader to[16] for the details. The evolution equation for the 4-pt function is

$$\begin{aligned}
\frac{d}{dy}\langle O_4(r,\bar r : s)\rangle = &-\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z \Bigg\langle 2\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right] O_4(r,\bar r : s) \\
&- \frac{1}{N_c}\Bigg[\frac{(r-s)^2}{(r-z)^2(s-z)^2}\,\mathrm{tr}\, V_r^\dagger\, V_z\ \ \mathrm{tr}\, V_s^\dagger\, V_{\bar r}\ \ \mathrm{tr}\, V_z^\dagger\, V_s + \\
&\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\mathrm{tr}\, V_r^\dagger\, V_s\ \ \mathrm{tr}\, V_z^\dagger\, V_{\bar r}\ \ \mathrm{tr}\ \ V_s^\dagger\, V_z\ - \\
&\frac{1}{2}\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2} - \frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}\right] \\
&\left[\mathrm{tr}\, V_r^\dagger\, V_z\, V_s^\dagger\, V_{\bar r}\, V_z^\dagger\, V_s + \mathrm{tr}\, V_r^\dagger\, V_s\, V_z^\dagger\, V_{\bar r}\, V_s^\dagger\, V_z\right]\Bigg] \\
&+ \frac{1}{N_c^2}\,\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}\,\mathrm{tr}\ \ V_r^\dagger\, V_z\ \ \mathrm{tr}\ \ V_z^\dagger\, V_{\bar r}\ \Bigg\rangle
\end{aligned} \tag{2·10}$$

while the evolution equation for the 6-pt function can be written as

$$\begin{aligned}
\frac{d}{dy}\langle O_6(r,\bar r : s,\bar s)\rangle = &-\frac{N_c\,\alpha_s}{2(2\pi)^2}\int d^2z \Bigg\langle 2\,\Big[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2} + \\
&\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2} + 3\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}\Big]\, O_6(r,\bar r : s,\bar s) - \frac{1}{N_c}\Bigg[\\
&\Big[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2} + \frac{(r-s)^2}{(r-z)^2(s-z)^2} - \frac{(s-\bar r)^2}{(s-z)^2(\bar r-z)^2}\Big]\,\mathrm{tr}\, V_z\, V_{\bar r}^\dagger\, V_{\bar s}\, V_s^\dagger\ \ \mathrm{tr}\, V_r\, V_z^\dagger\ \ \mathrm{tr}\, V_s\, V_{\bar s}^\dagger \\
&+\Big[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2} + \frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2} - \frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\Big]\,\mathrm{tr}\, V_r\, V_z^\dagger\, V_{\bar s}\, V_s^\dagger\ \ \mathrm{tr}\, V_z\, V_{\bar r}^\dagger\ \ \mathrm{tr}\, V_s\, V_{\bar s}^\dagger \\
&+\Big[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2} - \frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\Big]\,\mathrm{tr}\, V_r\, V_{\bar r}^\dagger\, V_{\bar s}\, V_z^\dagger\ \ \mathrm{tr}\, V_z\, V_s^\dagger\ \ \mathrm{tr}\, V_s\, V_{\bar s}^\dagger
\end{aligned}$$

$$+[\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}]\,\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\,V_z\,V_s^\dagger\;\;\mathrm{tr}\,V_{\bar{s}}\,V_z^\dagger\;\;\mathrm{tr}\,V_s\,V_{\bar{s}}^\dagger$$
$$+2\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\,\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\,V_{\bar{s}}\,V_s^\dagger\;\;\mathrm{tr}\,V_s\,V_z^\dagger\,\mathrm{tr}\,V_z\,V_{\bar{s}}^\dagger$$
$$+[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}]$$
$$\mathrm{tr}\,V_r\,V_s^\dagger\;\;\mathrm{tr}\;V_{\bar{r}}^\dagger\,V_{\bar{s}}\;\;\mathrm{tr}\,V_s\,V_{\bar{s}}^\dagger$$
$$-[-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}+\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}]$$
$$\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\,\mathrm{tr}\,V_{\bar{s}}\,V_s^\dagger\;\;\mathrm{tr}\,V_s\,V_{\bar{s}}^\dagger$$
$$-[\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}]\,\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\,V_{\bar{s}}\,V_s^\dagger\,V_z\,V_{\bar{s}}^\dagger\,V_s\,V_z^\dagger$$
$$+[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}]\,\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\,V_z\,V_{\bar{s}}^\dagger\,V_s\,V_z^\dagger\,V_{\bar{s}}\,V_s^\dagger$$
$$+[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}]$$
$$\mathrm{tr}\,V_r\,V_z^\dagger\,V_s\,V_{\bar{s}}^\dagger\,V_z\,V_{\bar{r}}^\dagger\,V_{\bar{s}}\,V_s^\dagger-2\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\,\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\,V_{\bar{s}}\,V_z^\dagger\,V_s\,V_{\bar{s}}^\dagger\,V_z\,V_s^\dagger$$
$$-[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}]$$
$$\mathrm{tr}\,V_r\,V_{\bar{s}}^\dagger\,V_s\,V_{\bar{r}}^\dagger\,V_{\bar{s}}\,V_s^\dagger$$
$$-[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}-\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}]\,\mathrm{tr}\,V_r\,V_{\bar{r}}^\dagger\Bigg]$$
$$+\frac{1}{N_c^2}\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}\,\mathrm{tr}\,V_r^\dagger\,V_z\;\;\mathrm{tr}\,V_z^\dagger\,V_{\bar{r}}\Bigg\rangle-\frac{1}{4\,N_c}\,\frac{d}{dy}\;<\mathrm{tr}\,V_r^\dagger\,V_{\bar{r}}>. \qquad (2\cdot11)$$

We note that these equations are free of power divergences in the limit the internal transverse coordinate approaches any of the external coordinates. Unlike the evolution equation for the dipole profile (2-pt function) which is rather simple, these equations are quite involved and it is not easy to investigate their properties analytically (see[17]) for particular kinematics in which these relations get a bit simplified is investigated). In phenomenological applications of CGC to two-hadron production it is assumed[18]) that in the leading N_c approximation one can write the product of higher point functions as the products of two point functions only (the dipole approximation):

$$\langle O_6(r,\bar{r}:s,\bar{s})\rangle\;\simeq\;\langle O_2(r-s)\rangle\;\langle O_2(\bar{r}-\bar{s})\rangle\;\langle O_2(s-\bar{s})\rangle+\langle O_2(r-\bar{r})\rangle\;\langle O_2(\bar{s}-s)\rangle\;\langle O_2(s-\bar{s})\rangle. \qquad (2\cdot12)$$

This assumption has been made by Albacete and Marquet[18]) in order to fit the data on forward rapidity di-jet correlations in dA collisions at RHIC. Unfortunately, *this (dipole) assumption is wrong* and misses many leading N_c contributions to the evolution equation as proven in.[16] Therefore, a quantitative understanding of the forward rapidity two-hadron correlations from CGC is still lacking.

There are several issues that need to be understood. First, we note that the number (n) of terms in the evolution equations for the higher point functions which are formally N_c suppressed becomes very large so that when $n >> N_c$, large N_c approximation may cease to be a good approximation. To see this, it is convenient to define normalized operators S_6 and S_4 such that

$$S_4(r, \bar{r} : s) \equiv \frac{1}{C_A\, C_F}\, \langle O_4 \rangle$$
$$S_6(r, \bar{r} : s, \bar{s}) \equiv \frac{1}{C_A\, C_F}\, \langle O_6 \rangle\ . \qquad (2\cdot 13)$$

Here we focus on S_6 since it is the more interesting one. Rewriting the evolution for O_6 in terms of S_6, we note that the terms in the first few lines in eq. (2·11) which are products of three traces will end up being leading order in N_c while the next few lines which involve only one trace will be suppressed by N_c^2. We note the number of N_c^2 suppressed terms is 8 for O_4 and ~ 20 for O_6. Making a Gaussian approximation (see[19]) to these higher point functions would result in further proliferation of these N_c suppressed terms.

The second point that needs to be understood better is the energy dependence of expectation value of trace of a large number of Wilson lines that appear on the right hand side of eq. (2·11). One may expect that these higher point functions will grow faster with energy than the two-point function. There is an example where this happens, energy dependence of a state of four reggeized gluons has been investigated in[20] where it is found that it has a faster rate of growth with energy than the state of two reggeized gluons. Whether a similar thing happens in our case is not known for sure but is likely to be true. This would mean that the terms with larger number of Wilson lines on the right hand side of evolution equation will grow faster with energy than the other terms which have fewer Wilson lines. This stronger energy dependence may eventually compensate for the N_c^2 suppression.

Furthermore we note the presence of terms on the right hand side of eq. (2·11) which involve lower point functions, namely the 2-pt and 4-pt functions, reminiscent of pomeron loop contribution to the BK equation for the dipole profile.[21] We note that these terms are in addition to the original 2-pt function which was present in the definition of O_6. The origin of these terms seems to be due to kinematics but needs to be better understood. Therefore, di-jet production in the forward rapidity region of deuteron-gold collisions offer a rich context and a unique opportunity to investigate CGC correlations. For this, one needs to solve the full JIMWLK equation for the weight function $W[\rho]$ which can then be used to compute any n-pt function in the CGC formalism.

We note that photon-hadron correlations[22] in the forward rapidity region in deuteron-gold collisions is another process which is more sensitive to saturation dynamics than single inclusive production. It also has the advantage, compared to two-hadron production, that it is sensitive only to the 2-pt function so that one could use the latest results for running coupling BK equation in order to make quantitative predictions.

Another example of the importance of higher point functions in the CGC framework is the two-hadron production process in proton-proton or nucleus-nucleus collisions. In this case one can not solve the problem analytically in the full kinematics of CGC and has to resort to numerical methods. Nevertheless, it is common to use the so-called k_t factorization in the dilute region, i.e., where both the target and projectile are dilute but the energy of the collision is high enough so that a re-summation in x is required (the BFKL region). In this case the two-gluon production cross section is given by[23)]

$$\left\langle \frac{dN_2}{d^2p dy_p\, d^2q dy_q} \right\rangle = \frac{g^{12}}{64(2\pi)^6} \left(f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'}\right) \int \prod_{i=1}^{4} \frac{d^2k_i}{(2\pi)^2 k_i^2}$$
$$\frac{L_\mu(p,k_1)L^\mu(p,k_2)}{(p-k_1)^2(p-k_2)^2} \frac{L_\nu(q,k_3)L^\nu(q,k_4)}{(q-k_3)^2(q-k_4)^2} \left\langle {\rho^*}_A^a(k_2) {\rho^*}_A^b(k_4) {\rho_A}^c(k_1) {\rho_A}^d(k_3) \right\rangle$$
$$\times \left\langle {\rho^*}_B^{a'}(p-k_2) {\rho^*}_B^{b'}(q-k_4) {\rho_B}^{c'}(p-k_1) {\rho_B}^{d'}(q-k_3) \right\rangle \qquad (2\cdot14)$$

where $p, y_p,\ q, y_q$ are the transverse momenta and rapidities of the two produced gluons, A and B label the color charge ρ of the projectile and target proton or nucleus and the Lipatov vertex is denoted by L^μ. The color charge ρ is the source for the classical gluon field which satisfies the relation (in the covariant gauge)

$$A^\mu(x^+, r) \equiv \delta^{\mu -} \alpha(x^+, r) = -g\, \delta^{\mu -} \delta(x^+) \frac{1}{\nabla_\perp^2} \rho(x^+, r)\ , \qquad (2\cdot15)$$

The standard un-integrated gluon distribution function $\Phi(x, p_t^2)$ is defined in terms of the color average of two ρ's as

$$\left\langle \rho^{*a}(k) \rho^b(k') \right\rangle (x) = \frac{1}{\alpha_s} \frac{\delta^{ab}}{N_c^2 - 1} (2\pi)^3 \delta(k-k')\, \Phi(x, k^2) \qquad (2\cdot16)$$

Due to the complexity of the expression, it is common[23)] to make the dipole approximation and write (symbolically)

$$\left\langle \rho^a \rho^b \rho^c \rho^d \right\rangle = \delta^{ab}\delta^{cd}(\rho^2)^2 + \delta^{ac}\delta^{bd}(\rho^2)^2 + \delta^{ad}\delta^{bc}(\rho^2)^2 + \cdots \qquad (2\cdot17)$$

where ρ^2 denotes $\langle \rho\rho \rangle$. Again, the dipole approximation breaks down under rapidity evolution. The correct evolution equation for the product of four ρ's was derived in[24)] and reads

$$\frac{d}{dY} \langle \alpha_r^a \alpha_{\bar r}^b \alpha_s^c \alpha_{\bar s}^d \rangle = \frac{g^2 N_c}{(2\pi)^3} \int d^2 z$$
$$\left\langle \frac{\alpha_z^a \alpha_{\bar r}^b \alpha_s^c \alpha_{\bar s}^d}{(r-z)^2} + \frac{\alpha_r^a \alpha_z^b \alpha_s^c \alpha_{\bar s}^d}{(\bar r - z)^2} + \frac{\alpha_r^a \alpha_{\bar r}^b \alpha_z^c \alpha_{\bar s}^d}{(s-z)^2} + \frac{\alpha_r^a \alpha_{\bar r}^b \alpha_s^c \alpha_z^d}{(\bar s - z)^2} - 4 \frac{\alpha_r^a \alpha_{\bar r}^b \alpha_s^c \alpha_{\bar s}^d}{z^2} \right\rangle$$
$$+ \frac{g^2}{\pi} \int \frac{d^2 z}{(2\pi)^2}$$

$$
\begin{aligned}
&\Big\langle f^{e\kappa a} f^{f\kappa b} \frac{(r-z)\cdot(\bar{r}-z)}{(r-z)^2(\bar{r}-z)^2} \left[\alpha_r^e \alpha_{\bar{r}}^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_{\bar{r}}^f + \alpha_z^e \alpha_z^f\right] \alpha_s^c \alpha_{\bar{s}}^d \\
&+ f^{e\kappa a} f^{f\kappa c} \frac{(r-z)\cdot(s-z)}{(r-z)^2(s-z)^2} \left[\alpha_r^e \alpha_s^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_s^f + \alpha_z^e \alpha_z^f\right] \alpha_{\bar{r}}^b \alpha_{\bar{s}}^d \\
&+ f^{e\kappa a} f^{f\kappa d} \frac{(r-z)\cdot(\bar{s}-z)}{(r-z)^2(\bar{s}-z)^2} \left[\alpha_r^e \alpha_{\bar{s}}^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f\right] \alpha_{\bar{r}}^b \alpha_s^c \\
&+ f^{e\kappa b} f^{f\kappa c} \frac{(\bar{r}-z)\cdot(s-z)}{(\bar{r}-z)^2(s-z)^2} \left[\alpha_{\bar{r}}^e \alpha_s^f - \alpha_{\bar{r}}^e \alpha_z^f - \alpha_z^e \alpha_s^f + \alpha_z^e \alpha_z^f\right] \alpha_r^a \alpha_{\bar{s}}^d \\
&+ f^{e\kappa b} f^{f\kappa d} \frac{(\bar{r}-z)\cdot(\bar{s}-z)}{(\bar{r}-z)^2(\bar{s}-z)^2} \left[\alpha_{\bar{r}}^e \alpha_{\bar{s}}^f - \alpha_{\bar{r}}^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f\right] \alpha_r^a \alpha_s^c \\
&+ f^{e\kappa c} f^{f\kappa d} \frac{(s-z)\cdot(\bar{s}-z)}{(s-z)^2(\bar{s}-z)^2} \left[\alpha_s^e \alpha_{\bar{s}}^f - \alpha_s^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f\right] \alpha_r^a \alpha_{\bar{r}}^b \Big\rangle \ . \quad (2\cdot 18)
\end{aligned}
$$

This equation is valid in the dilute region of the proton or nucleus so that all higher order terms in the field α have been neglected on the right hand side of the equation. Solving this equation (which can be done using numerical methods), one can use the solution in the expression for two-gluon production (2·14) and investigate the dependence of the production cross section on the transverse momenta and rapidities of the two gluons. This will be extremely interesting due to the recent results from heavy ion collisions at RHIC and proton-proton collisions at LHC where long range rapidity correlations are observed.[25)] These long range rapidity correlations must be generated very early after the collision and subsequent re-scattering can not change these due to causality.[23)] Furthermore, in proton-proton collisions one does not expect to create a medium with a size and life time much larger than a Fermi, which is how the heavy ion community defines a Quark-Gluon Plasma. Thus, viscous corrections to ideal hydrodynamics must necessarily be *very large* for transverse momenta larger than a few hundred MeV. Therefore, one does not expect to see flow effects above one $1\,GeV$.

In the CGC formalism long range rapidity correlations are due to the production of boost-invariant longitudinal color fields at the very early stages of the collision. These classical longitudinal fields are boost invariant which leads to production of gluons independent of rapidity. Furthermore, the transverse size (correlation length) of these fields is inversely proportional to the saturation momentum Q_s and decreases with increasing collision energy. This means that most produced particles will have transverse momenta of the order of Q_s and will be produced approximately independently of rapidity. These features of the data from RHIC and LHC are in qualitative agreement with the expectations from the CGC formalism.[26)] Nevertheless, a truely quantitative comparison with the data requires knowledge of the higher point functions in CGC formalism.

Acknowledgements

We would like to thank F. Dominguez and A.H. Mueller for interesting discussions. The results presented here are based on work in collaboration with A. Dumitru and have been presented by the author in the program "High Energy Strong Interactions 2010" at the Yukawa Institute for Theoretical Physics, July-August 2010, Kyoto, Japan and in the "5th International Workshop on high p_t Physics at LHC 2010", September - October, 2010, Mexico City, Mexico. This work is supported by the DOE Office of Nuclear Physics through Grant No. DE-FG02-09ER41620 and by The City University of New York through the PSC-CUNY Research Award Program, grant number 62625-40.

References

1) L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100**, 1 (1983).
2) A. H. Mueller and J. w. Qiu, Nucl. Phys. B **268**, 427 (1986).
3) L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994), Phys. Rev. D **49**, 3352 (1994).
4) J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **55**, 5414 (1997); J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Nucl. Phys. B **504**, 415 (1997), Phys. Rev. D **59**, 014014 (1999), Phys. Rev. D **59**, 014015 (1999), Phys. Rev. D **59**, 034007 (1999) [Erratum-ibid. D **59**, 099903 (1999)]; A. Kovner, J. G. Milhano and H. Weigert, Phys. Rev. D **62**, 114005 (2000); A. Kovner and J. G. Milhano, Phys. Rev. D **61**, 014012 (2000); E. Iancu, A. Leonidov and L. D. McLerran, Nucl. Phys. A **692**, 583 (2001), Phys. Lett. B **510**, 133 (2001); E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A **703**, 489 (2002).
5) I. Balitsky, Nucl. Phys. B **463**, 99 (1996); Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999), Phys. Rev. D **61**, 074018 (2000).
6) Y. V. Kovchegov and H. Weigert, Nucl. Phys. A **784**, 188 (2007), Nucl. Phys. A **789**, 260 (2007); I. Balitsky and G. A. Chirilli, Phys. Rev. D **77**, 014019 (2008); I. Balitsky, Phys. Rev. D **75**, 014001 (2007).
7) A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. **89**, 022301 (2002).
8) A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, Nucl. Phys. A **765**, 464 (2006), Nucl. Phys. A **770**, 57 (2006).
9) J. L. Albacete and Y. V. Kovchegov, Phys. Rev. D **75**, 125021 (2007), J. L. Albacete and C. Marquet, Phys. Lett. B **687**, 174 (2010).
10) R. B. Neufeld, I. Vitev and B. W. Zhang, arXiv:1010.3708 [hep-ph].
11) F. Gelis and J. Jalilian-Marian, Phys. Rev. D **76**, 074015 (2007), Phys. Rev. D **66**, 094014 (2002), Phys. Rev. D **66**, 014021 (2002); J. Jalilian-Marian, Nucl. Phys. A **753**, 307 (2005), Nucl. Phys. A **739**, 319 (2004); R. Baier, A. H. Mueller and D. Schiff, Nucl. Phys. A **741**, 358 (2004); A. H. Rezaeian and A. Schafer, Phys. Rev. D **81**, 114032 (2010).
12) F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, arXiv:1002.0333 [hep-ph]; J. Jalilian-Marian and Y. V. Kovchegov, Prog. Part. Nucl. Phys. **56**, 104 (2006).
13) F. Gelis and J. Jalilian-Marian, Phys. Rev. D **67**, 074019 (2003).
14) J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D **70**, 114017 (2004) [Erratum-ibid. D **71**, 079901 (2005)].
15) C. Marquet, Nucl. Phys. A **796**, 41 (2007).
16) A. Dumitru and J. Jalilian-Marian, Phys. Rev. D **82**, 074023 (2010).
17) F. Dominguez, B. W. Xiao and F. Yuan, arXiv:1009.2141 [hep-ph].
18) J. L. Albacete and C. Marquet, Phys. Rev. Lett. **105**, 162301 (2010).
19) J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A **743**, 57 (2004).
20) Z. Chen and A. H. Mueller, Nucl. Phys. B **451**, 579 (1995).
21) E. Iancu and A. H. Mueller, Nucl. Phys. A **730**, 494 (2004); E. Iancu and D. N. Triantafyllopoulos, Nucl. Phys. A **756**, 419 (2005).
22) J. Jalilian-Marian, Eur. Phys. J. C **61**, 789 (2009).
23) A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, Nucl. Phys. A **810**, 91 (2008); K. Dusling, F. Gelis, T. Lappi and R. Venugopalan, Nucl. Phys. A **836**, 159 (2010).
24) A. Dumitru and J. Jalilian-Marian, Phys. Rev. D **81**, 094015 (2010).
25) [CMS Collaboration], JHEP **1009**, 091 (2010),
26) A. Dumitru, K. Dusling, F. Gelis, J. Jalilian-Marian, T. Lappi and R. Venugopalan, arXiv:1009.5295 [hep-ph].