

Potential for measurement of the tensor electric and magnetic polarizabilities of the deuteron in storage-ring experiments with polarized beams

Vladimir G. Baryshevsky and Alexander J. Silenko

*Research Institute for Nuclear Problems,
Belarusian State University, Minsk 220080, Belarus**

Abstract

Measurement of the tensor electric and magnetic polarizabilities of the deuteron is of great interest, especially in connection with the possibilities of COSY and GSI. These polarizabilities can be measured in storage rings by the frozen spin method providing a disappearance of $g - 2$ precession. This method will be used in the planned deuteron electric-dipole-moment experiment in storage rings. The tensor electric polarizability of the deuteron significantly influences the buildup of the vertical polarization in the above experiment. The spin interactions depending on the electric dipole moment, the tensor electric polarizability, and main systematical errors caused by field misalignments have very different symmetries. For the considered experimental conditions, the sensitivity to the deuteron EDM of 1×10^{-29} e-cm corresponds to measuring the both of tensor polarizabilities with an accuracy of $\delta\alpha_T \approx \delta\beta_T \approx 5 \times 10^{-42}$ cm³. This conservative estimate can be improved by excluding the systematical error caused by the field instability which is negligible for the measurement of the tensor polarizabilities. To find the tensor magnetic polarizability, the horizontal components of the polarization vector should be measured.

*Electronic address: bar@inp.minsk.by; silenko@inp.minsk.by

I. INTRODUCTION

Tensor electric and magnetic polarizabilities are important properties of the deuteron and other nuclei defined by spin interactions of nucleons. Their measurement provides a good possibility to examine the theory of spin-dependent nuclear forces. Methods for determining these important electromagnetic properties of the deuteron based on the appearance of interactions quadratic in the spin have been proposed by V. Baryshevsky and co-workers [1–6]. It has been shown that the tensor polarizabilities cause oscillations of spin characteristics and transitions between vector and tensor polarizations of the deuteron. Additional investigations have been performed in Refs. [7–9]. Interactions quadratic in the spin and proportional to the tensor electric and magnetic polarizabilities affect spin dynamics. When an electric field in the particle rest frame oscillates at the resonant frequency, an effect similar to the nuclear magnetic resonance takes place. This effect stimulates the buildup of the vertical polarization (BVP) of the deuteron beam [1–6]. More general formulas describing the BVP caused by the tensor electric polarizability of the deuteron in storage rings have been derived in Ref. [7]. The problem of influence of the tensor electric polarizability on spin dynamics in such a deuteron electric-dipole-moment experiment in storage rings has been investigated [1–7]. It has been proved that doubling the resonant frequency used in this experiment dramatically amplifies the effect and provides the opportunity to make high-precision measurements of the deuterons tensor electric polarizability [7].

The tensor magnetic polarizability, β_T , produces the spin rotation with two frequencies instead of one, beating with a frequency proportional to β_T , and causes transitions between vector and tensor polarizations [2, 4–6]. In Ref. [8], the existence of these effects has been confirmed and a detailed calculation of deuteron spin dynamics in storage rings has been carried out. The use of the matrix Hamiltonian derived in Ref. [7] is very helpful for calculating general formulas describing the evolution of the spin. It is important that the results obtained by different methods agree.

Significant improvement in the precision of possible experiments can be achieved if initial deuteron beams are tensor-polarized [7, 8].

The frozen spin method [10, 11] provides another possibility to measure the tensor polarizabilities of the deuteron and other nuclei. This method ensures that the spin orientation relative to the momentum direction remains almost unchanged. We also analyze additional

advantages ensured by the use of tensor-polarized beams and compute the related spin evolution. Since the spin interactions depending on the EDM, the tensor electric polarizability, and the main systematical errors have very different symmetries, all these interactions can be properly distinguished.

The system of units $\hbar = c = 1$ is used.

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II. GENERAL EQUATIONS

We use the matrix Hamiltonian equation and the matrix Hamiltonian H for determining the evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = H\Psi, \quad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}. \quad (1)$$

The three-component wave function Ψ , which is similar to a spinor, consists of the amplitudes $C_i(t)$ characterizing states with definite spin projections onto the preferential direction (z axis). Correction to the Hamilton operator caused by the tensor polarizabilities has the form [7]

$$V = -\frac{\alpha_T}{\gamma}(\mathbf{S} \cdot \mathbf{E}')^2 - \frac{\beta_T}{\gamma}(\mathbf{S} \cdot \mathbf{B}')^2, \quad (2)$$

where α_T is the tensor electric polarizability, γ is the Lorentz factor, and \mathbf{E}' and \mathbf{B}' are the electric and magnetic fields in the rest frame of the deuteron. Let us note that the components of \mathbf{E}' and \mathbf{B}' orthogonal to the deuteron momentum contain the Lorentz factor (see below). As a result, the effect does not decrease at high energies [1–6].

The spin motion in storage rings is measured relative to the axes of the cylindrical coordinate system. Therefore, cylindrical coordinates are used in the present work. The horizontal axes of the cylindrical coordinate system are connected with the position of the particle and rotate at the instantaneous frequency of its revolution. The description of spin effects in the cylindrical coordinate system strongly correlates with that in the frame rotating at the instantaneous frequency of orbital revolution of the deuteron. The frequency of spin precession in the rotating frame coincides with that in the cylindrical coordinate system (see Refs. [7–9]).

In the rotating frame, the motion of deuterons is relatively slow because it can be caused only by beam oscillations and other deflections from the ideal trajectory.

The Hamiltonian operator is defined by [7]

$$\mathcal{H} = \mathcal{H}_0 + \mathbf{S} \cdot \boldsymbol{\omega}_a + V, \quad (3)$$

where $\boldsymbol{\omega}_a$ is the angular velocity of the spin precession relatively to the momentum direction ($g - 2$ precession).

In the considered case, the expressions for \mathbf{E}' and \mathbf{B}' in terms of the unprimed laboratory fields have the form

$$\mathbf{E}' = \gamma(E_\rho + \beta_\phi B_z)\mathbf{e}_\rho, \quad \mathbf{B}' = \gamma(\beta_\phi E_\rho + B_z)\mathbf{e}_z, \quad (4)$$

where $\beta_\phi = \boldsymbol{\beta} \cdot \mathbf{e}_\phi \equiv \mathbf{v} \cdot \mathbf{e}_\phi / c$.

When the frozen spin method is used, the quantity $\boldsymbol{\omega}_a$ is very small and the fields satisfy the following relation [11]:

$$E_\rho = \frac{a\beta_\phi\gamma^2}{1 - a\beta^2\gamma^2} B_z. \quad (5)$$

Since the main electric field is radial and almost orthogonal to the particle (nucleus) trajectory, its effect on a change of the γ factor can be neglected. This factor can be supposed to be constant.

Therefore,

$$V = -\frac{\gamma B_z^2}{(1 - a\beta^2\gamma^2)^2} [\alpha_T(1 + a)^2\beta^2 S_\rho^2 + \beta_T S_z^2]. \quad (6)$$

The matrix Hamiltonian has the form [7, 9]

$$H = \begin{pmatrix} E_0 + \omega_0 + \mathcal{A} + \mathcal{B} & 0 & \mathcal{A} \\ 0 & E_0 + 2\mathcal{A} & 0 \\ \mathcal{A} & 0 & E_0 - \omega_0 + \mathcal{A} + \mathcal{B} \end{pmatrix}, \quad (7)$$

where E_0 is the zero energy level, $\omega_0 = (\omega_a)_z$,

$$\mathcal{A} = -\alpha_T \frac{(1 + a)^2 \beta^2 \gamma B_z^2}{2(1 - a\beta^2\gamma^2)^2}, \quad \mathcal{B} = -\beta_T \frac{\gamma B_z^2}{(1 - a\beta^2\gamma^2)^2}. \quad (8)$$

Equations (7) and (8) are basic equations defining the dynamics of the deuteron spin in storage rings when the frozen spin method is used [9].

We are interested in the case when the particle or nucleus has a fixed spin projection ($S_l = +1, 0$, or -1) onto the certain direction \mathbf{l} defined by the spherical angles θ and ψ .

The azimuth ψ is determined in relation to the cylindrical axes \mathbf{e}_ρ and \mathbf{e}_ϕ . The $\psi = 0$ case characterizes the spin directed radially outward.

The polarization of particles (nuclei) is described by the three-component polarization vector \mathbf{P} and the polarization tensor P_{ij} , which has five independent components.

III. EVOLUTION OF VECTOR POLARIZATION OF THE DEUTERON BEAM

In Ref. [8], off-diagonal components of the Hamiltonian (8) were not taken into account, because their effect on the rotating spin did not satisfy the resonance condition. These components cannot, however, be neglected in the considered case because the resonant frequency ω_0 can be very small.

The best conditions for a measurement of the tensor polarizabilities of the deuteron and other nuclei can be achieved with the use of tensor-polarized initial beams. In this case, we may confine ourselves to the consideration of a zero projection of the deuteron spin onto the preferential direction. When this direction is defined by the spherical angles θ and ψ , the general equation describing the evolution of the polarization vector has the form [9]

$$\begin{aligned} P_\rho(t) &= \sin(2\theta) \left\{ \left[\cos(\omega't) \sin\psi + \frac{\omega_0}{\omega'} \sin(\omega't) \cos\psi \right] \sin(bt) + \frac{\mathcal{A}}{\omega'} \sin(\omega't) \cos(bt) \sin\psi \right\}, \\ P_\phi(t) &= \sin(2\theta) \left\{ \left[-\cos(\omega't) \cos\psi + \frac{\omega_0}{\omega'} \sin(\omega't) \sin\psi \right] \sin(bt) + \frac{\mathcal{A}}{\omega'} \sin(\omega't) \cos(bt) \cos\psi \right\}, \\ P_z(t) &= -\frac{2\mathcal{A}}{\omega'} \sin^2\theta \sin(\omega't) \left[\cos(\omega't) \sin(2\psi) + \frac{\omega_0}{\omega'} \sin(\omega't) \cos(2\psi) \right], \end{aligned} \quad (9)$$

where

$$\omega' = \sqrt{\omega_0^2 + \mathcal{A}^2}, \quad b = \mathcal{B} - \mathcal{A}. \quad (10)$$

When the frozen spin method is used,

$$b = -\frac{\gamma B_z^2}{(1 - a\beta^2\gamma^2)^2} \left[\beta_T - \frac{1}{2}\alpha_T(1 + a)^2\beta^2 \right]. \quad (11)$$

As a rule, we can neglect \mathcal{A}^2 as compared with ω_0^2 and use the approximation $|b|t \ll 1$. In this case [9]

$$\begin{aligned} P_\rho(t) &= \sin(2\theta) \left[bt \sin(\omega_0 t + \psi) + \frac{\mathcal{A}}{\omega_0} \sin(\omega_0 t) \sin\psi \right], \\ P_\phi(t) &= \sin(2\theta) \left[-bt \cos(\omega_0 t + \psi) + \frac{\mathcal{A}}{\omega_0} \sin(\omega_0 t) \cos\psi \right], \\ P_z(t) &= -\frac{2\mathcal{A}}{\omega_0} \sin^2\theta \sin(\omega_0 t) \sin(\omega_0 t + 2\psi). \end{aligned} \quad (12)$$

When the initial deuteron beam is vector-polarized, the direction of its polarization can be also defined by the spherical angles θ and ψ . Such a polarization (with $\theta = \pi/2$) will be used in the planned deuteron electric-dipole-moment (EDM) experiment [12]. The EDM manifests in an appearance of a vertical component of the polarization vector.

The evolution of this component defined by the tensor polarizabilities of the deuteron is given by

$$P_z(t) = \left[1 - \frac{2\mathcal{A}^2}{\omega'^2} \sin^2(\omega't) \right] \cos\theta + \frac{\mathcal{A}}{\omega'} \sin^2\theta \sin(\omega't) \left[\cos(\omega't) \sin(2\psi) + \frac{\omega_0}{\omega'} \sin(\omega't) \cos(2\psi) \right]. \quad (13)$$

The tensor magnetic polarizability does not influence on P_z .

In the same approximation as before,

$$P_z(t) = \cos\theta + \frac{\mathcal{A}}{\omega_0} \sin^2\theta \sin(\omega_0 t) \sin(\omega_0 t + 2\psi). \quad (14)$$

IV. DISTINGUISHING FEATURES OF THE TENSOR POLARIZABILITIES AND ELECTRIC-DIPOLE-MOMENT EFFECTS

The tensor electric polarizability can in principle imitate the presence of the EDM because both of them stimulate the buildup of the vertical polarization of the deuteron beam. The exact equation of spin motion with allowance for the EDM has been obtained in Ref. [10] specifically for the EDM experiment. In the considered case, the angular velocity of spin rotation is equal to

$$\boldsymbol{\omega}_a = \omega_0 \mathbf{e}_z + \mathcal{C} \mathbf{e}_\rho, \quad \mathcal{C} = -\frac{e\eta}{2m} \cdot \frac{1+a}{1-a\beta^2\gamma^2} \beta_\phi B_z, \quad (15)$$

where $\eta = 2dm/(eS)$ is the factor similar to the g factor for the magnetic moment with d being the EDM.

When the tensor polarizabilities are not taken into account, the spin rotates about the direction

$$\mathbf{e}'_z = \frac{\mathcal{C}}{\omega'} \mathbf{e}_\rho + \frac{\omega_0}{\omega'} \mathbf{e}_z$$

with the angular frequency $\omega' = \sqrt{\omega_0^2 + \mathcal{C}^2}$.

When the initial beam is vector-polarized, the final polarization vector is defined by [9]

$$\begin{aligned}
P_\rho(t) &= \frac{\omega_0 \mathcal{C}}{\omega'^2} [1 - \cos(\omega't)] \cos \theta + \left[1 - \frac{2\omega_0^2}{\omega'^2} \sin^2 \frac{\omega't}{2} \right] \sin \theta \cos \psi - \frac{\omega_0}{\omega'} \sin(\omega't) \sin \theta \sin \psi, \\
P_\phi(t) &= \sin(\omega't) \left(\frac{\omega_0}{\omega'} \sin \theta \cos \psi - \frac{\mathcal{C}}{\omega'} \cos \theta \right) + \cos(\omega't) \sin \theta \sin \psi, \\
P_z(t) &= \left[1 - \frac{2\mathcal{C}^2}{\omega'^2} \sin^2 \frac{\omega't}{2} \right] \cos \theta + \frac{\omega_0 \mathcal{C}}{\omega'^2} [1 - \cos(\omega't)] \sin \theta \cos \psi + \frac{\mathcal{C}}{\omega'} \sin(\omega't) \sin \theta \sin \psi.
\end{aligned} \tag{16}$$

If we neglect terms of the order of \mathcal{C}^2 , the vertical component of the polarization vector takes the form

$$P_z(t) = \cos \theta + \frac{2\mathcal{C}}{\omega_0} \sin \theta \sin \frac{\omega_0 t}{2} \sin \frac{\omega_0 t + 2\psi}{2}. \tag{17}$$

Although Eqs. (14) and (17) are similar, the effects of the tensor electric polarizability and the EDM have different angular dependencies and can be properly separated.

Carrying out the dEDM experiment in storage rings [12] ensures the wonderful possibility of measurement of the tensor electric and magnetic polarizabilities. The EDM collaboration plans to use the frozen spin method and to have multiple deuteron beam bunches circulating in the storage ring at the same time with opposite states of polarization. Main systematical errors caused by field misalignments will be canceled by clockwise (CW) and counterclockwise (CCW) consecutive beam injections [12]. Initial beam polarization will be horizontal and the final vertical polarization will be measured. Spin interactions depending on the EDM, the tensor electric polarizability, and the main systematical errors have very different symmetries. In the Table I, we show the behavior of the final vertical polarization caused by these spin interactions when the direction of either the initial polarization or the beam is reversed. The plus and minus signs denote reversing and conserving the final vertical polarization, respectively. In the dEDM experiment, one will obtain four blocks of data for two opposite states of initial polarization and two beam directions. Evidently, the tensor electric polarizability can be determined with summing up the data for two opposite states of initial polarization. To find the tensor magnetic polarizability of the deuteron, the horizontal components of the polarization vector should be measured. Determination of the deuteron EDM can be made with calculating the difference of the results for two opposite beam directions and taking into account the correction for the tensor electric polarizability.

TABLE I: Reversing (+) or conserving (−) the final vertical polarization when either the initial polarization or the direction of the beam is reversed.

Spin interactions	Reversing the initial polarization	Reversing the beam direction
EDM	+	−
Tensor electric polarizability	−	−
Systematical errors caused by field misalignments	+	+

V. DISCUSSION AND SUMMARY

Experimental conditions needed for the measurement of the tensor polarizabilities and the EDMs of nuclei in storage rings [11, 12] are similar. Equation (5) shows that the radial electric field should be sufficiently strong to eliminate the effect of the vertical magnetic field on the spin. As a result, the frozen spin method provides a weaker magnetic field than other methods. This factor is negative because the evolution of the spin caused by both the tensor polarizabilities and the EDMs strongly depends on B_z . Nevertheless, the Storage Ring EDM Collaboration considers the frozen spin method to be capable of detecting the deuteron EDM of the order of $10^{-29} e \cdot \text{cm}$. Another method for searching for the deuteron EDM in storage rings is the resonance method developed in Ref. [13]. This method is based on a strong vertical magnetic field and an oscillatory resonant longitudinal electric field. Both methods can be successfully used for the measurement of the tensor electric and magnetic polarizabilities of the deuteron and other nuclei [1–7].

We can evaluate the precision of measurement of the tensor polarizabilities of the deuteron via its comparison with the expected sensitivity of the deuteron EDM experiment. For the considered experimental conditions [12], the sensitivity to the EDM of $1 \times 10^{-29} e \cdot \text{cm}$ corresponds to measuring the tensor electric polarizability with an accuracy of $\delta\alpha_T \approx 5 \times 10^{-42} \text{ cm}^3$.

There are three independent theoretical predictions for the value of the tensor electric polarizability of the deuteron, namely $\alpha_T = -6.2 \times 10^{-41} \text{ cm}^3$ [14], $-6.8 \times 10^{-41} \text{ cm}^3$ [15], and $3.2 \times 10^{-41} \text{ cm}^3$ [16]. The first two values are very close to each other but they do not agree with the last result. The theoretical estimate for the tensor magnetic polarizability of

deuteron is $\beta_T = 1.95 \times 10^{-40} \text{ cm}^3$ [14, 15].

We can therefore conclude that the expected sensitivity of the deuteron EDM experiment allows us to measure the tensor electric polarizability with an absolute precision of $\delta\alpha_T \approx 5 \times 10^{-42} \text{ cm}^3$ which corresponds to the relative precision of the order of 10^{-1} . This estimate made for the vector-polarized initial beam is rather conservative and can be improved by excluding the systematic error caused by the field instability which is negligible for the measurement of the tensor polarizabilities. The best sensitivity in the measurement of α_T can be achieved with the use of a tensor-polarized initial beam. When the vector polarization of such a beam is zero, spin rotation does not occur. In this case, there are no related systematic errors caused by the radial magnetic field or other factors. In the general case, such systematic errors are proportional to a residual vector polarization of the beam. This advantage leads to a sufficient increase in experimental accuracy [7, 8]. In this case, our preliminary estimate of experimental accuracy is $\delta\alpha_T \sim 10^{-43} \text{ cm}^3$.

The frozen spin method can also be successively used for the measurement of the tensor magnetic polarizability. Equations (9)–(12) show that the preferential direction of initial tensor polarization is defined by $\theta = \pi/2$ and $\theta = \pi/4$ for measuring the tensor electric and magnetic polarizabilities, respectively. In the latter case, the horizontal components of the polarization vector should be measured. Owing to a restriction of spin rotation in the horizontal plane, the achievable absolute precision of measurement of the tensor magnetic polarizability of the deuteron is of the same order ($\delta\beta_T \sim 10^{-43} \text{ cm}^3$). A comparison with the theoretical estimate [14, 15] shows that the relative precision of measurement of this quantity can be rather high ($\delta\beta_T/\beta_T \sim 10^{-3}$).

All the formulas derived here are applicable to any spin-1 nucleus. Moreover, the evolution of the polarization vector defined by spin tensor effects has to be similar for nuclei with any spin $S \geq 1$ despite difference of spin matrices and has to agree with classical spin physics. The description of dynamics of the vector and tensor polarizations for arbitrary S and the investigation of special features of higher spins have been carried out in Ref. [17].

Thus, the frozen spin method can be effectively used for the high-precision determination of the tensor electric and magnetic polarizabilities of the deuteron and other nuclei. The experiments could be carried out at COSY and GSI.

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