

Proton size anomaly

Vernon Barger¹, Cheng-Wei Chiang^{2,3,1}, Wai-Yee Keung⁴, and Danny Marfatia^{5,1}

¹ *Department of Physics, University of Wisconsin-Madison, Madison, WI 53706, USA*

² *Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chungli, Taiwan 32001, ROC*

³ *Institute of Physics, Academia Sinica, Nankang, Taipei 11925, ROC*

⁴ *Department of Physics, University of Illinois, Chicago, IL 60607, USA*

⁵ *Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045, USA*

A measurement of the Lamb shift in muonic hydrogen yields a charge radius of the proton that is smaller than the CODATA value by about 5 standard deviations. We explore the possibility that new scalar, pseudoscalar, vector, axial-vector, and tensor flavor-conserving nonuniversal interactions may be responsible for the discrepancy. We consider exotic particles that among leptons, couple preferentially to muons, and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1 and spin-2 particles as an explanation.

Lamb shift. The success of quantum electrodynamics (QED) is apparent in the explanation of the Lamb shift [1] which is the observation that the $2S_{1/2}$ state of hydrogen is higher than the $2P_{1/2}$ state by about 1 GHz.¹ Precision measurements in atomic spectra have tested bound-state QED to the extent that the charge distribution of the proton needs to be taken into account [2]. The root-mean-square charge radius of the proton compiled by CODATA from the spectroscopy of atomic hydrogen and electron-proton scattering is [3]

$$\sqrt{\langle r_p^2 \rangle} = 0.8768 \pm 0.0069 \text{ fm}. \quad (1)$$

It has been a long-held goal to measure the corresponding Lamb shift in muonic hydrogen which is even more sensitive to the structure of the proton due to its smaller Bohr radius $(\alpha m_\mu)^{-1}$, (where $\alpha \sim 1/137$ is the electromagnetic fine structure constant and $m_\mu \simeq 105$ MeV). Recently, the $2P_{3/2} \rightarrow 2S_{1/2}$ Lamb shift in muonic hydrogen was measured to be [4],

$$\Delta \tilde{E}_{LS} \equiv E(2P_{3/2}) - E(2S_{1/2}) = 206.2949 \pm 0.0032 \text{ meV}, \quad (2)$$

while the predicted value is [2, 5]

$$\Delta \tilde{E}_{LS} = 209.9779(49) - 5.2262 \langle r_p^2 \rangle + 0.0347 \langle r_p^2 \rangle^{3/2}, \quad (3)$$

where radii are in fm and energy in meV, and the number in parenthesis indicates the 1σ uncertainty of the last two decimal places of the given number. (Note that $\Delta \tilde{E}_{LS}$ is defined to be positive.) Equations (2) and (3) yield the order of magnitude more precise result [4],

$$\sqrt{\langle r_p^2 \rangle} = 0.84184 \pm 0.00067 \text{ fm}, \quad (4)$$

which is smaller than the CODATA value by about 5σ . A partial resolution of the discrepancy may be found in a correlation between $\langle r_p^2 \rangle$ and the r_p^3 -dependent third Zemach moment (since they contribute to the Lamb shift with opposite signs) and perhaps unreliable extractions of these from electron-proton scattering data [6]. Nevertheless, a 4σ difference remains. The possibility that the 4% difference is a hint of a new gauge interaction with a natural scale αm_μ has been entertained in Ref. [7].

In this Letter, we postulate the existence of a new interaction between muons and nucleons, and study its nature, bearing in mind the many experimental constraints. The interaction must be attractive since $\Delta \tilde{E}_{LS}$ measured in muonic hydrogen is larger than expected, signaling that the $2S_{1/2}$ state is subject to a stronger attraction than electromagnetic.

Scalar and spin-2 boson exchanges produce an attractive potential, giving positive contributions to $\Delta \tilde{E}_{LS}$. Pseudoscalar boson exchange is a derivative interaction involving the spins and velocities of the lepton and the nucleus, which becomes insignificant in the nonrelativistic limit, and irrelevant to the Lamb shift. A boson with both scalar and pseudoscalar couplings violates CP conservation. Such a scenario faces strong constraints from electric dipole moment measurements of leptons and nucleons, and is disfavored [8]. Vector boson exchange (like photon exchange) can produce an attractive potential if the quantum numbers associated with the lepton and the nucleus are opposite in sign, such as in the gauged $B-3L$ interaction. Then Lamb shift phenomenology is like that of scalar exchange. Axial-vector exchange couples the

¹ The dominant contributions to the Lamb shift arise from vacuum polarization and the vertex charge form factor of the lepton. Vacuum polarization contributes negatively to $\Delta E_{LS} \equiv E(2S_{1/2}) - E(2P_{1/2})$ since more of the lepton's bare charge is revealed for the S state (than the P state) due to its greater overlap with the nucleus. On the other hand, the vertex charge form factor is related to the zitterbewegung of the lepton which causes the effective Coulomb potential to be smeared out and less attractive. The effect is greater for the S state, so that the contribution to ΔE_{LS} is positive. The latter contribution is dominant for ordinary hydrogen but plays a minor role in muonic hydrogen because of the smaller Compton wavelength of the muon. Consequently, ΔE_{LS} is positive in ordinary hydrogen and negative in muonic hydrogen.

spins of the lepton and the nucleus in the nonrelativistic limit. It affects the hyperfine structure, but not the Lamb shift.

Suppose the interaction between fermions f and χ is given by $C_f^{S,V,T} \bar{f} f \chi$, where S, V, T denote scalar, vector and tensor χ , respectively, and f can be a muon μ or a nucleon n ; we assume isospin is conserved. Throughout, we take the couplings C to be real and positive. In the nonrelativistic limit, the muon-nucleon interaction is given by the Yukawa-type potential,

$$\Delta V(r) = -\alpha_\chi \frac{e^{-m_\chi r}}{r}, \quad (5)$$

where $\alpha_\chi = C_\mu^{S,V,T} C_n^{S,V,T} / (4\pi)$ and m_χ is the mass of the particle χ . Physical systems in which tensor interactions (by which we mean spin-2 exchange) are governed by a Yukawa potential allow the identification, $C_f^T \equiv C_f^S$. This will be valid for all our constraints except those that involve the anomalous magnetic moment of the muon $a_\mu \equiv (g_\mu - 2)/2$. The correction to the muonic Lamb shift is [9]

$$\delta(\Delta \tilde{E}_{LS}) = \alpha_\chi m_\chi \frac{\frac{m_\chi}{\alpha m_r}}{2 \left(1 + \frac{m_\chi}{\alpha m_r}\right)^4}, \quad (6)$$

where m_r is the reduced mass of the muon-proton system, and its use is numerically important as m_r is smaller than m_μ by more than 10%. The green shaded region in Fig. 1 shows the 95% C. L. region that accommodates the difference between Eqs. (1) and (4). We do not consider $m_\chi > 10$ GeV since the required α_χ becomes larger than 20α , in the nonperturbative regime.

Upsilon decay. For scalar χ in the mass range $2m_\mu - 9.3$ GeV, the nonobservance of radiative decays of the $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances: $\Upsilon \rightarrow \gamma\chi$, $\chi \rightarrow \mu^+\mu^-$, strongly constrains the $\Upsilon - \chi$ coupling [10], which we expect to be no smaller than C_n^S ; for a Higgs-like χ , the coupling is naturally $\mathcal{O}(m_b/m_n) \times C_n^S$, while for a universal interaction, it should be $\mathcal{O}(C_n^S)$. We conservatively take the $\Upsilon - \chi$ coupling to be C_n^S . In obvious notation [11],

$$\frac{BF(\Upsilon \rightarrow \gamma\chi)}{BF(\Upsilon \rightarrow \mu^+\mu^-)} = \frac{(C_n^S)^2}{4\pi\alpha} \left(1 - \frac{m_\chi^2}{m_\Upsilon^2}\right). \quad (7)$$

Under our assumption that the branching fraction of $\chi \rightarrow \mu^+\mu^-$ is unity, the 90% C. L. upper limit on C_n^S ranges from $(0.94 - 9.4) \cdot 10^{-3}$ [10], where the lower end of the range corresponds to smaller m_χ . The values of α_χ/α needed to explain the muonic Lamb shift constrain C_μ^S to lie above $\mathcal{O}(1)$, $\mathcal{O}(10)$ and $\mathcal{O}(100)$ for $m_\chi \sim 2m_\mu$, 1 GeV and 9 GeV, respectively, couplings which are too large.

A vector χ can mediate leptonic decays of spin-1 quarkonia. Since the only lepton that χ couples to is the muon, one expects nonuniversality in leptonic decays. For $\Upsilon(1S)$ decays, $R_{\tau\mu} \equiv \Gamma_{\tau\tau}/\Gamma_{\mu\mu} = 1.005 \pm 0.013 \pm$

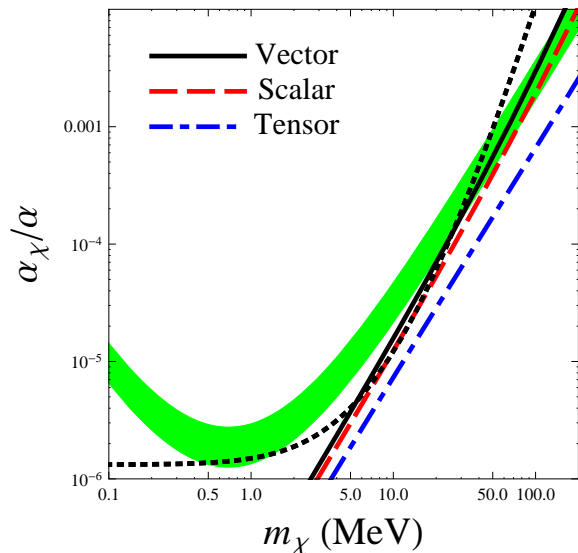


FIG. 1. The 95% C. L. range of α_χ/α required to reproduce the muonic Lamb shift is indicated by the green shaded region. The black solid, red dashed and blue dot-dashed lines are the *upper limits* for vector, scalar and spin-2 particles, respectively, from a combination of n-²⁰⁸Pb scattering data and the anomalous magnetic moment of the muon. The black dotted curve is the upper bound obtained from atomic X-ray transitions. All bounds are at the 95% C. L.

0.022 [12], whereas the SM expectation is 0.992 [13]. The inclusion of χ modifies the SM value of $R_{\tau\mu}$ by a factor,

$$\left[\left(1 \pm \frac{\alpha_\chi}{\alpha Q_b}\right) - (m_\chi/m_\Upsilon)^2 \right]^2 \left[1 - (m_\chi/m_\Upsilon)^2 \right]^{-2}, \quad (8)$$

where the + (−) sign corresponds to destructive (constructive) interference and Q_b is the electric charge of the b quark. For $m_\chi \lesssim 1$ GeV, a conservative 95% C. L. (one-sided) upper bound on α_χ/α (assuming the SM and χ contributions destructively interfere) is 8.8×10^{-3} . In the range $1 \text{ GeV} \lesssim m_\chi < m_\Upsilon$, the upper bound becomes even more stringent, falling monotonically with m_χ . The mass of a vector χ is restricted to be less than about 230 MeV in order to explain the muonic Lamb shift. Henceforth, we only consider $m_\chi < 2m_\mu$.

Neutron scattering. Very precise neutron scattering experiments on heavy nuclei in the keV regime have been performed to study the electric polarizability of the neutron. The goal is to measure interference effects between the nuclear potential and the r^{-4} potential produced by electric polarizability. One can then see that a Yukawa potential $\mp A (C_n^{S,V})^2 e^{-m_\chi r} / (4\pi r)$ may also be probed by such experiments; the minus and plus signs apply to scalar/tensor and vector interactions, respectively. Stringent bounds are obtainable because the p-wave amplitude due to the short range strong interaction depends linearly on energy and differs markedly from that due to the new longer range interaction. A n-²⁰⁸Pb scatter-

ing experiment [14] in the neutron energy range 1 to 26 keV measured the differential cross section (under the assumption that the scattering amplitude can be expanded in s and p waves) to be

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{4\pi} (1 + \omega E \cos \theta), \quad (9)$$

with $\sqrt{\sigma_0/4\pi} \simeq 10$ fm and $\omega = (1.91 \pm 0.42) \cdot 10^{-3}$ keV $^{-1}$. The measured values are in line with expectations so that the Yukawa potential contribution ought to be subdominant. Denoting the strong interaction contribution to ω by ω_s , and the contribution of the new interaction by $\Delta\omega$, clearly, $\omega = \omega_s + \Delta\omega$ with [15]

$$\Delta\omega = \mp \frac{16}{m_\chi^4} \frac{(C_n^{S,V,T})^2}{4\pi} \frac{A m_n^2}{\sqrt{\sigma_0/4\pi}}, \quad (10)$$

in the Born approximation (not valid for $m_\chi \lesssim 0.1$ MeV), and m_n is the neutron mass and A is the atomic mass number. For scalar/tensor exchange, it is possible that a cancellation between ω_s and $\Delta\omega$ produces the experimental result. However, this cannot be the case for a vector χ . A conservative 95% C. L. (one-sided) upper limit can be obtained by requiring that $\Delta\omega \leq 2.6 \times 10^{-3}$ keV, *i.e.*,

$$C_n^V \leq (m_\chi/206)^2, \quad (11)$$

with m_χ in MeV. It is the shaded region of Fig. 2.

While reliable bounds for a scalar/tensor χ are not extractable from the differential cross section, the total cross section measured between 10 eV and 10 keV [16, 17] may be employed with confidence. The energy dependence of the n - ^{208}Pb cross section for neutron energies below 10 keV can be parameterized by

$$\sigma(k) = \sigma(0) + \sigma_2 k^2 + \mathcal{O}(k^4), \quad (12)$$

where $k = 2.1968 \times 10^{-4} \sqrt{EA}/(A+1)$ is the wave vector of the incoming neutron (with k in fm $^{-1}$ and E in eV). The cross section in the limit of vanishing momentum transfer $\sigma(0)$ is directly related to the scattering length, and σ_2 gives the effective range of the potential. The $\mathcal{O}(k)$ contribution to $\sigma(k)$ arises from the electric field of the nuclear charge distribution and is negligible. The measured values $\sigma(0) = 12.40 \pm 0.02$ b and $\sigma_2 = -448 \pm 3$ b fm 2 give a 95% C. L. bound on $C_n^{S,T}$ [17] that is almost identical to Eq. (11) in the mass range of interest (and is not shown separately in Fig. 2), but without the ambiguity from the cancellation mentioned above.

Muon anomalous magnetic moment. We now consider the independent constraint on C_μ from a_μ . In fact, since the experimental value of a_μ is above the SM expectation by more than three standard deviations: $\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (29 \pm 9) \times 10^{-10}$ [18], the new interaction may explain this difference. From Ref. [19],

$$\Delta a_\mu = \frac{(C_\mu^{S,V})^2}{8\pi^2} \int_0^1 \frac{2x^2 - \beta x^3}{x^2 + (m_\chi^2/m_\mu^2)(1-x)} dx, \quad (13)$$

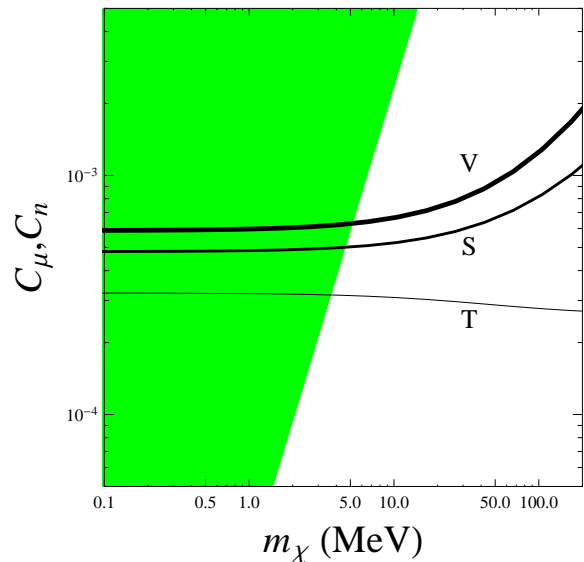


FIG. 2. The curves are 95% C. L. upper bounds on the muonic couplings C_μ^S , C_μ^V , C_μ^T from Δa_μ . The green shading marks the values of the nucleon coupling $C_n^{S,V,T}$ excluded by n - ^{208}Pb scattering at the 95% C. L.

where $\beta = 1$ for a scalar and $\beta = 2$ for a vector. For a tensor interaction, we trivially modify the result of Ref. [20]. In the limit $m_\chi \ll m_\mu$,

$$C_\mu^S = 4\pi \sqrt{\Delta a_\mu/3} \lesssim 4.8 \times 10^{-4}, \quad (14)$$

$$C_\mu^V = 4\pi \sqrt{\Delta a_\mu/2} \lesssim 5.9 \times 10^{-4}, \quad (15)$$

$$C_\mu^T = 4\pi \sqrt{3\Delta a_\mu/20} \lesssim 3.2 \times 10^{-4}, \quad (16)$$

where the one-sided upper bounds are at the 95% C. L. From Fig. 2 it is evident that Eqs. (14)-(16) apply for $m_\chi \lesssim 10$ MeV.

The bound in Eq. (11) can be combined with those in Eqs. (14)-(16) to give the following 95% C. L. constraints for $m_\chi \lesssim 10$ MeV:

$$\alpha_\chi/\alpha \lesssim (m_\chi/2847)^2 \quad \text{scalar}, \quad (17)$$

$$\alpha_\chi/\alpha \lesssim (m_\chi/2573)^2 \quad \text{vector}, \quad (18)$$

$$\alpha_\chi/\alpha \lesssim (m_\chi/3477)^2 \quad \text{tensor}, \quad (19)$$

with m_χ in MeV. A similar (numerical) procedure can be applied for the entire range of m_χ to obtain the upper bounds shown in Fig. 1. We see that a vector χ with mass between 25 MeV (with $\alpha_\chi \simeq 10^{-4}\alpha$) and 210 MeV (with $\alpha_\chi \sim 10^{-2}\alpha$) is a viable candidate. While a scalar χ with mass between 70 MeV and 210 MeV (with $\alpha_\chi \sim (10^{-3} - 10^{-2})\alpha$) is marginally allowed, a spin-2 χ is excluded.

Muonic atom transitions. Measurements of the muonic $3D_{5/2} - 2P_{3/2}$ X-ray transition in ^{24}Mg and ^{28}Si atoms directly constrain α_χ for scalar, vector and tensor particles [21]. For the Yukawa form of Eq. (5) with the

coupling enhanced by a factor of A , the shift in the difference in energy levels from the QED expectation is [21],

$$\frac{\Delta E}{E} = \frac{2\alpha_\chi A}{5\alpha Z} [9f(2) - 4f(3)], \quad (20)$$

where $f(j) = [1 + jm_\chi/(2\alpha Z m_\mu)]^{-2j}$, Z is the atomic number, and j is the principle quantum number of the muonic state. The measured value obtained by averaging the results for ^{24}Mg and ^{28}Si , $\Delta E/E = (0.2 \pm 3.1) \cdot 10^{-6}$ [21], gives the 95% C. L. bound (dotted curve) in Fig. 1. No additional area of the relevant parameter space is excluded by this constraint.

J/ψ decay. For $m_\chi < 2m_\mu$, the decay of scalar $\chi \rightarrow \mu^+\mu^-$ is kinematically forbidden so that a constraint from the nonobservance of the decay $J/\psi \rightarrow \gamma\chi$, with χ invisible [22], may be employed to exclude the marginally allowed region with $70 \text{ MeV} < m_\chi < 210 \text{ MeV}$; preliminary data also exist for the decay $\Upsilon(3S) \rightarrow \gamma\chi$ [23]. A trivial modification of Eq. (7) applies to J/ψ decay. The 90% C. L. upper limit on $BF(J/\psi \rightarrow \gamma\chi)$ is $\sim 4.5 \times 10^{-6}$ [22], which when combined with $BF(J/\psi \rightarrow \mu^+\mu^-) = (5.93 \pm 0.06) \cdot 10^{-2}$ [24], gives $C_n^S < 0.029$. Then, the muonic Lamb shift dictates that C_μ^S be larger than 3.4×10^{-3} which is excluded at the 95% C. L. by a_μ ; see Fig. 2. Thus, scalars are also disfavored.

Pion decay. The 90% C. L. experimental upper limit on the decay $\pi^0 \rightarrow \gamma\chi$, where χ is a vector particle, is $(3.3 - 1.9) \cdot 10^{-5}$ for m_χ ranging from 0 to 120 MeV [25]. Equivalently, $C_n^V < 4.5 \times 10^{-4} (1 - m_\chi^2/m_\pi^2)^{-3/2}$ [26], and the corresponding values of C_μ^V required to explain the muonic Lamb shift are excluded by a_μ . This leaves m_χ between 120 MeV and 210 MeV.

Eta decay. For vector χ , the 90% C. L. experimental upper limit on invisible decays, $BF(\eta \rightarrow \chi\chi)/BF(\eta \rightarrow \gamma\gamma) < 1.65 \times 10^{-3}$ [27], translates into $C_n^V \lesssim 0.05$ [28]. The corresponding C_μ^V for $120 < m_\chi < 210 \text{ MeV}$ is excluded by a_μ , so that vector χ is also ruled out.

Conclusions. We have found that new spin-0, spin-1 or spin-2 particles that mediate flavor-conserving nonuniversal interactions are excluded by several low energy constraints as an explanation of the proton radius anomaly. We assumed that among leptons, the new particles couple only to the muon so as to avoid the large number of constraints involving the interaction of the electron with exotica. We also supposed that the coupling of the new particle to nucleons represents the minimal hadronic coupling, and employed it to mesons.

There are ways to relax some of the bounds at the expense of introducing complication. For example, since the contributions of scalars and pseudoscalars to a_μ are opposite in sign, allowing both a scalar boson and a pseudoscalar boson with appropriately tuned couplings can lead to a cancellation that permits a rather large muonic coupling. Then, although the hadronic couplings are highly restricted, the muonic Lamb shift can be ac-

commodated. Another possibility is that the new interaction violates isospin or CP , so that additional freedom is garnered.

In response to I. Rabi's gibe about the existence of the muon, "Who ordered that?" we declare, "The proton!"

Acknowledgments. This work was supported by DoE Grant Nos. DE-FG02-84ER40173, DE-FG02-95ER40896 and DE-FG02-04ER41308, by NSF Grant No. PHY-0544278, by NSC Grant No. 97-2112-M-008-002-MY3, by NCTS, and by the WARF.

-
- [1] W. Lamb and R. Retherford, Phys. Rev. **79**, 549 (1950).
 - [2] For a review, see S. G. Karshenboim, Phys. Rept. **422**, 1 (2005) [arXiv:hep-ph/0509010].
 - [3] P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008) [arXiv:0801.0028 [physics.atom-ph]].
 - [4] R. Pohl *et al.*, Nature **466**, 213 (2010).
 - [5] A. P. Martynenko, Phys. Atom. Nucl. **71**, 125 (2008) [arXiv:hep-ph/0610226].
 - [6] A. De Rujula, arXiv:1010.3421 [hep-ph].
 - [7] W. Maricano, talk at INT, Seattle, 2010.
 - [8] For a review, see W. Bernreuther, M. Suzuki, Rev. Mod. Phys. **63**, 313 (1991) [Erratum-ibid. **64**, 633 (1992)].
 - [9] J. Jaeckel and S. Roy, arXiv:1008.3536 [hep-ph].
 - [10] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **103**, 081803 (2009). [arXiv:0905.4539 [hep-ex]].
 - [11] F. Wilczek, Phys. Rev. Lett. **39**, 1304 (1977).
 - [12] P. del Amo Sanchez *et al.* [BaBar Collaboration], Phys. Rev. Lett. **104**, 191801 (2010) [arXiv:1002.4358 [hep-ex]].
 - [13] R. Van Royen and V. F. Weisskopf, Nuovo Cim. A **50**, 617 (1967) [Erratum-ibid. A **51**, 583 (1967)].
 - [14] Y. A. Aleksandrov, G. S. Samosvat, J. Sereeter and Tsoy Gen Sor, JETP Letters **4**, 134 (1966).
 - [15] R. Barbieri and T. Ericson, Phys. Lett. B **57**, 270 (1975).
 - [16] J. Schmiedmayer, P. Riehs, J. A. Harvey and N. W. Hill, Phys. Rev. Lett. **66**, 1015 (1991).
 - [17] H. Leeb and J. Schmiedmayer, Phys. Rev. Lett. **68**, 1472 (1992).
 - [18] For a review, see F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, 1 (2009) [arXiv:0902.3360 [hep-ph]].
 - [19] J. P. Leveille, Nucl. Phys. B **137**, 63 (1978).
 - [20] M. L. Graesser, Phys. Rev. D **61**, 074019 (2000) [arXiv:hep-ph/9902310].
 - [21] I. Beltrami *et al.*, Nucl. Phys. A **451**, 679 (1986).
 - [22] J. Insler *et al.* [CLEO Collaboration], Phys. Rev. D **81**, 091101 (2010) [arXiv:1003.0417 [hep-ex]].
 - [23] B. Aubert *et al.* [BaBar Collaboration], arXiv:0808.0017 [hep-ex].
 - [24] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).
 - [25] J. Altegoer *et al.* [NOMAD Collaboration], Phys. Lett. B **428**, 197 (1998) [arXiv:hep-ex/9804003].
 - [26] S. N. Gninenko and N. V. Krasnikov, Phys. Lett. B **427**, 307 (1998) [arXiv:hep-ph/9802375].
 - [27] M. Ablikim *et al.* [BES Collaboration], Phys. Rev. Lett. **97**, 202002 (2006) [arXiv:hep-ex/0607006].
 - [28] P. Fayet, Phys. Rev. D **74**, 054034 (2006) [arXiv:hep-ph/0607318].