

Frame dependence of ^3He transverse (e, e') response functions at intermediate momentum transfers

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Abstract

The transverse electron scattering response function of ^3He was recently studied by us in the quasi-elastic peak region for momentum transfers q between 500 and 700 MeV/c. Those results, obtained using the Active Nucleon Breit frame (ANB), are here supplemented by calculations in the laboratory, Breit and ANB frames using the two-fragment model discussed in our earlier work on the frame dependence of the longitudinal response function $R_L(q, \omega)$. We find relatively frame independent results and good agreement with experiment especially for the lower momentum transfers. This agreement occurs when we neglect an ω -dependent piece of the one-body current relativistic correction. An inclusion of this term leads however to a rather pronounced frame dependence at $q = 700$ MeV/c. A discussion of this term is given here. This report also includes a correction to our previous ANB results for $R_T(q, \omega)$.

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In a recent publication Ref. [1] we presented calculations of $R_T(q, \omega)$, the transverse electron scattering response function of ${}^3\text{He}$. That calculation was based on the Lorentz integral transform (LIT) technique [2–4] and included the AV18 [5] NN + UrbanaIX [6] NNN potentials, the lowest order relativistic corrections to the one-body current and consistent two-body currents (MEC) due to meson exchange. Detailed references to these aspects can be found in [1]. The laboratory (lab) frame response function was calculated by first computing in the Active Nucleon Breit (ANB) frame and then transforming to the lab frame. Our earlier work [7] on the frame dependence of the longitudinal response function $R_L(q, \omega)$ showed that the ANB frame appeared to diminish the effects of using non-relativistic nuclear dynamics. In addition, a two-fragment model was introduced which was found to significantly reduce frame dependence. In that model one assumes a quasi-elastic knock-out of a nucleon such that the residual nucleus remains in its lowest energy state. The relative momentum of the two fragments is determined in a relativistically correct way and the energy that is used as input to the non-relativistic calculation is obtained from that momentum by the usual non-relativistic relation. Considering for example the lab system, the quasi-elastically knocked out nucleon has momentum q , thus a difference between non-relativistic and relativistic kinetic energies of $\Delta T_{1b} = q^2/2M - (M^2 + q^2)^{1/2} + M$ (M is nucleon mass). Taking for example $q = 700$ MeV/ c one has $\Delta T_{1b} = 29$ MeV. This relativistic effect is automatically taken into account in the two-fragment model. It is important to state that this model only fixes a kinematical input while the full three-nucleon dynamics is still treated completely via the LIT technique. Thus a comparison of calculations done in the ANB frame with those done in any other frame (including the ANB frame) but with the two-fragment model provides some indication of uncertainties due to non-covariance.

In dealing with the transverse response function of ${}^3\text{He}$ near the quasi-elastic peak where the energy-momentum of the virtual photon is absorbed predominantly by the single ejected nucleon one can again adopt the two-fragment model. Compared to the longitudinal response function there is however a slight difference in applying the two-fragment model to the transverse response. For the former we did not include a two-body charge operator, while for the latter we do consider, as mentioned above, an MEC. It is not evident that such a two-body current should be taken into account in the two-fragment model, since it violates the assumption of a quasi-elastic knock-out. Instead one may assume the following scenario (lab-system): initially two nucleons have opposite and equal momenta \mathbf{p} and $-\mathbf{p}$ while the

third nucleon is at rest. If the photon momentum is transferred to the 1-2 pair then their momenta in the final state become $\mathbf{p} + \mathbf{q}/2$ and $-\mathbf{p} + \mathbf{q}/2$. Setting the total kinetic energy of the 1-2 pair equal to $q^2/2M$, i.e. the excitation energy of the quasi-elastic peak, one can solve for \mathbf{p} and hence the kinetic energy of the 1-2 pair using either relativistic or non-relativistic kinematics. If one takes $q = 700$ MeV/c then one can show this difference varies between 16 MeV (\mathbf{p} and \mathbf{q} orthogonal) and 29 MeV (p and \mathbf{q} parallel). As long as MEC matrix elements are small their main contribution arises from interference with the one-body current operator. Thus most probably their most significant effect is in the forward direction (\mathbf{p} and \mathbf{q} parallel) which is the assumption of the two-fragment model. If, however, MEC and one-body current contributions are similar in size, a part of the MEC contribution is not taken into account with the proper final-state energy in the two-fragment model. Even then one may assume in most cases that the MEC changes only mildly with energy and thus errors remain small. Moreover, as already pointed out in [1] MEC have only a minor effect in the peak region.

In principle one should also consider implicit MEC contributions, which arise for example by application of the Siegert theorem. Though here we do not make use of the classical Siegert operators we need to better investigate the actual form of our one-body current. It is described in Eqs. (11) to (16) of Ref. [1]. One of the pieces of this current, referred to here as the ω -dependent piece, appears as $(\omega/M)\mathbf{j}_\omega$ where ω is the energy transferred to the nuclear system by the virtual photon and where

$$\mathbf{j}_\omega = e^{i\mathbf{q}\mathbf{r}'} \frac{G_E - 2G_M}{8M} (\mathbf{q} + i\kappa[\vec{\sigma} \times \mathbf{q}] + 2i[\vec{\sigma} \times \mathbf{p}']). \quad (1)$$

Here κ is given by

$$\kappa = 1 + 2P_i/Aq, \quad (2)$$

\mathbf{r}' and \mathbf{p}' are relative coordinates and momenta of a single particle, P_i is the magnitude of the target momentum, and A is the nucleon number of the target. This current contains an implicit MEC contribution. For example for deuteron photodisintegration in [8] it was shown that the contribution due to the spin-orbit term is almost exclusively an MEC contribution. For quasi-elastic kinematics, however, one has a completely different situation. In this respect it is helpful to realize that the operator form of the spin-orbit current, i.e. $\vec{\sigma} \times \mathbf{q}$, is identical to that of the non-relativistic spin current and thus, in the quasi-elastic region, should lead like the latter mainly to one-body knock-out in forward direction.

For the explicit MEC we have studied the interference with the one-body current. As to be expected we find a rather weak interference effect only. Nonetheless it is the interference, not the MEC by itself, which leads to the bulk of the MEC contribution. Thus we may conclude that for quasi-elastic kinematics MEC contributions are quite small and due to interference with the one-body current and hence can be safely included in the two-fragment model.

In Fig. 1 we show various results for R_T in the ANB frame. These results are obtained by calculating in the ANB frame and then transforming to the lab frame. The effect of \mathbf{j}_ω is negligibly small in the ANB frame since $\kappa=0$ in this case and $\omega=0$ at the quasi-elastic peak, thus its contribution is not shown separately. The ANB frame results in the left-hand panels of Fig. 1 replace those of [1] where an error resulting from an incorrect form factor argument gave results higher than experiment. In the present corrected version the ANB results are now lower than experiment. In order to show that the difference does not result from an insufficient consideration of higher multipole contributions we show in Fig. 2 convergence tests for electric and magnetic LIT strength for both isospin channels as a function of final state multipolarity at $q=700$ MeV/c. It is clear that the results are essentially fully converged by $J_f=35/2$. As an additional accuracy test we calculated the non-energy weighted sum rules for $R_T(q, \omega)$ with the non-relativistic one-body current operator. Comparing with the well known non-energy weighted spin sum rule [12] we obtain 98.4%, 98.5%, and 98.0% at $q = 500, 600, \text{ and } 700$ MeV/c, respectively. It shows that the sum rule is quite well satisfied.

It is of interest to compare this ANB calculation with one where the ANB calculation uses the two-fragment approximation for setting the kinematics. The results of this are depicted in the right-hand panels of Fig. 1 where one notes some difference between the two concerning the peak height only. Indeed whereas in other frames the two-fragment model is effective in obtaining the quasi-elastic peak in the correct position this occurs automatically in the ANB frame without the need for the two-fragment model. On the other hand the proper treatment of relativistic kinematics leads to a slight increase of the peak heights, namely by 3.3%, 4.6%, and 6.1% at $q = 500, 600, \text{ and } 700$ MeV/c, respectively. Such an increase improves the agreement with experimental data, however, the theoretical peak heights are still somewhat too low, but one should also take into account that some additional strength could come from currents involving the Δ -resonance.

Our previous work [7] on frame dependence in $R_L(q, \omega)$ compared ANB frame results with

those obtained in the lab and Breit frames all calculated using the two-fragment model. In Fig. 3 we show the corresponding results for $R_T(q, \omega)$ (MEC and \mathbf{j}_ω left out). As expected the experimental quasi-elastic peak position is reproduced in all three cases. For the quasi-elastic peak height one finds differences between 7% ($q=500$ MeV/c) and 10% ($q=700$ MeV/c) showing an existing but not excessive frame dependence, which is about twice as large as the experimental error. The range of theoretical results covers the experimental data in the peak region.

Our conclusion concerning the relatively mild frame dependence relies on the assumption that contributions due to MEC and \mathbf{j}_ω remain small in other frames. While this assumption seems to be rather safe for the MEC it could be not correct for \mathbf{j}_ω , since it depends on the frame dependent variables κ and total energy transfer ω . To better investigate this question we show in Fig. 4 the contributions of the various currents for the lab frame calculation using the two-fragment model. One can see that \mathbf{j}_ω has a significant effect on the lab system calculations and that the total reduction due to relativistic effects becomes very large (37% at $q=700$ MeV/c). Comparing the result with the \mathbf{j}_ω contribution to the corresponding ANB frame results in the right-hand panels of Fig. 1, one finds reductions of the peak heights of about 5% ($q=500$ MeV/c), 9% ($q=600$ MeV/c), and 16% ($q=700$ MeV/c). Thus one has a rather pronounced frame dependence at $q=700$ MeV/c. At this point we should remind the reader that the two-fragment model takes into account relativistic effects for the kinematics only, and that dynamical relativistic effects are not considered at all.

In such a situation with rather frame dependent results one should choose the frame which leads to the smallest relativistic corrections. In [7] we already pointed that the ANB frame is preferable in case of quasi-elastic kinematics, since in this frame the nucleon momenta in initial and final states are only of the order of $q/2$, while in other frames one finds larger momenta, e.g., in the lab frame the knocked out nucleon has momentum q . In fact the reduction of R_T due to relativistic effects is relatively small in the ANB frame (13% instead of the 37% in the lab frame at $q=700$ MeV/c). Therefore we consider the ANB results to be more realistic than, e.g., the lab results. On the other hand it would be desirable to further reduce the frame dependence. This is possibly achieved by taking into account boost corrections. This could be done in a way similar to that used in deuteron electrodisintegration [13, 14].

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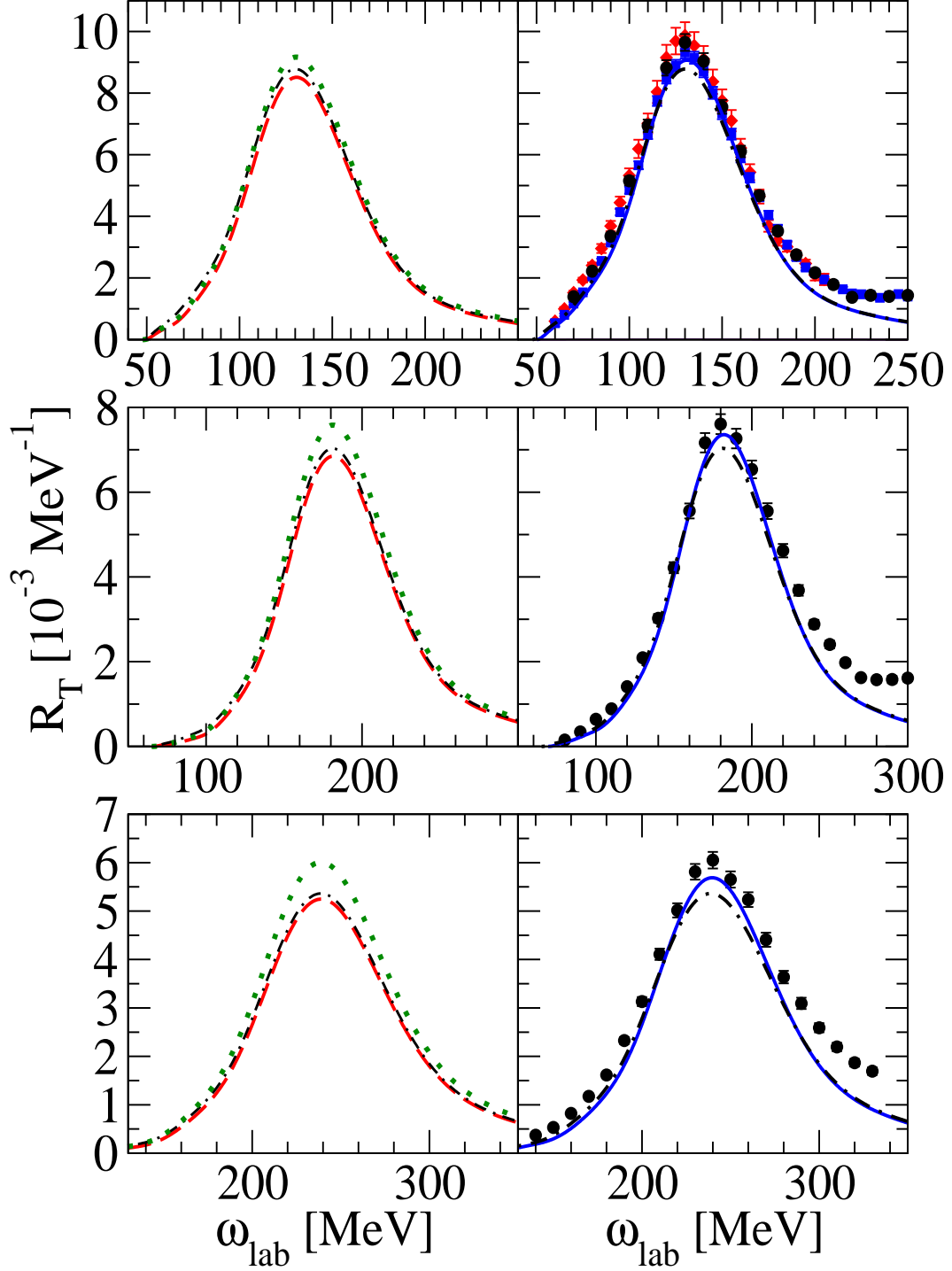


FIG. 1: $R_T(q_{\text{lab}}, \omega_{\text{lab}})$ at $q_{\text{lab}} = 50, 600, \text{ and } 700 \text{ MeV}/c$ from ANB frame calculation. In left-hand panels results without use of two-fragment model with non-relativistic one-body current (dotted), relativistic one-body current (dashed), and relativistic one-body current + MEC (dash-dotted). In right-hand panels results with relativistic one-body current + MEC without (dash-dotted) and with (solid) use of two-fragment model. Experimental data from [9] (squares), [10] (diamonds), [11] (circles).

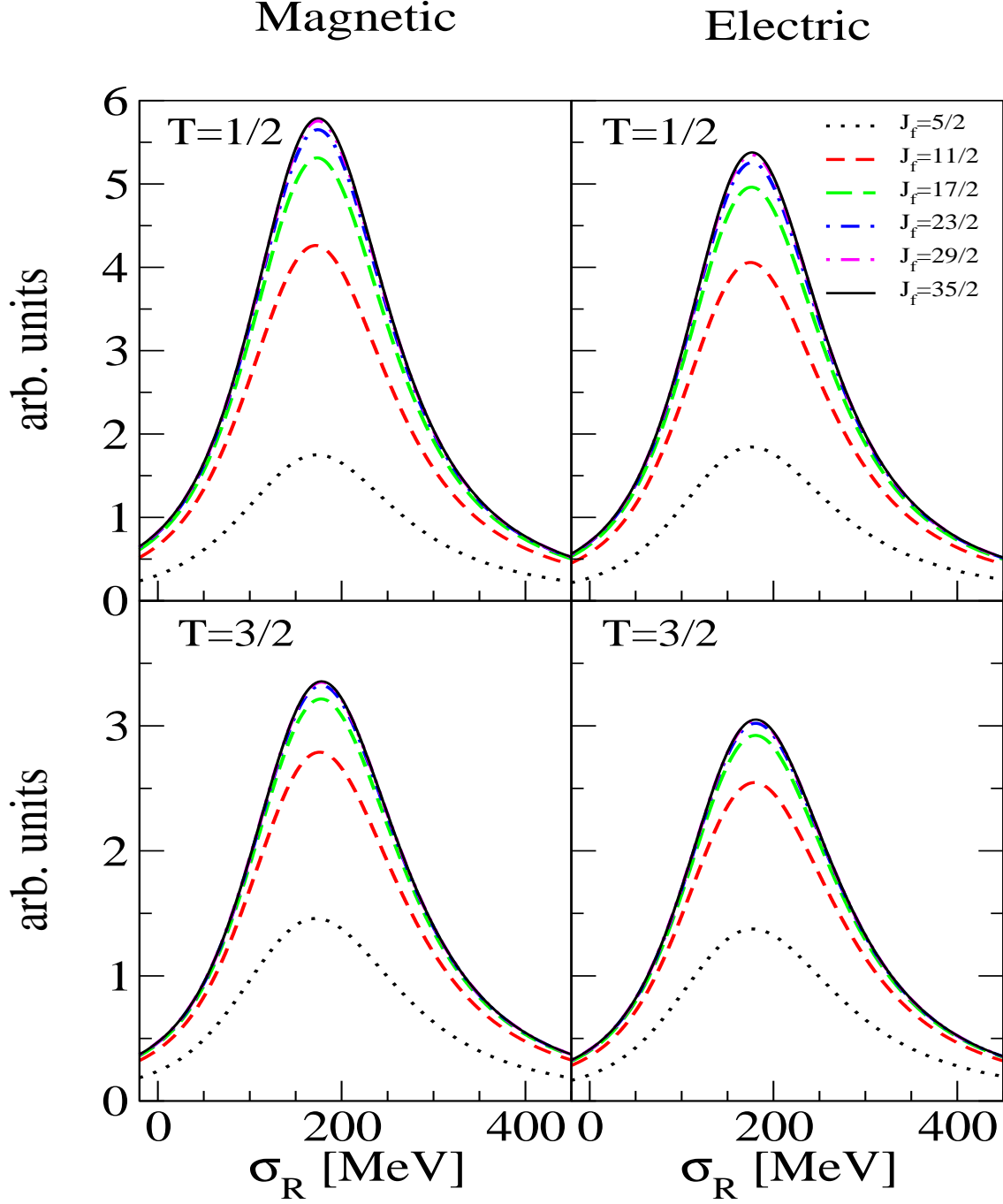


FIG. 2: Convergence of Lorentz integral transforms with maximal final state angular quantum number J_f for magnetic (left panels) and electric (right panels) transition strength with final state isospin quantum number $T_f=1/2$ (upper panels) and $T_f=3/2$ (lower panels); the LIT resolution parameter σ_I [4] is equal to 50 MeV.

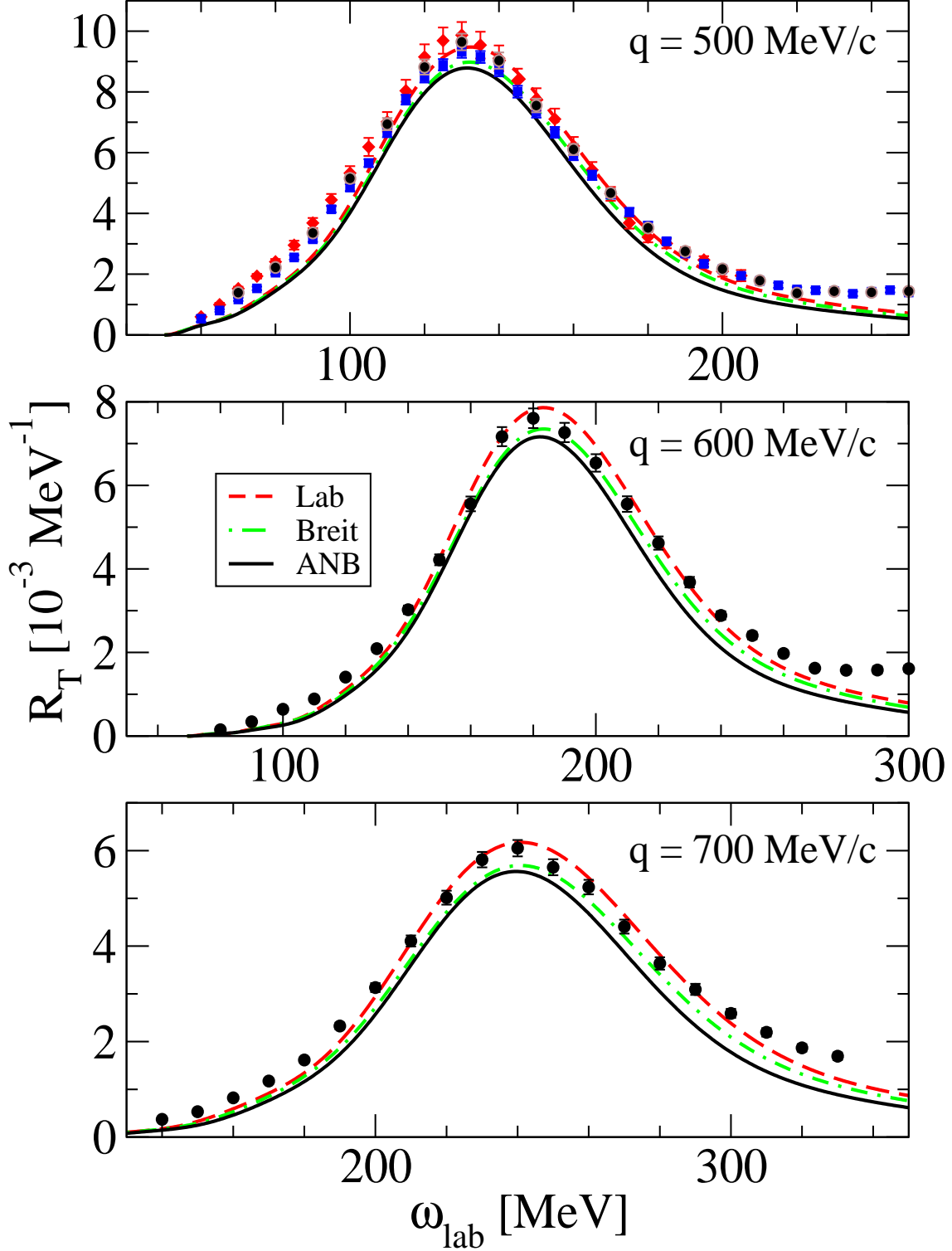


FIG. 3: $R_T(q_{\text{lab}}, \omega_{\text{lab}})$ at $q_{\text{lab}} = 500, 600,$ and $700 \text{ MeV}/c$ with relativistic one-body current less j_ω with use of two-fragment model from calculations in various frames, namely ANB (solid), lab (dashed), and Breit (dot-dashed) frames. Data as in Fig. 1.

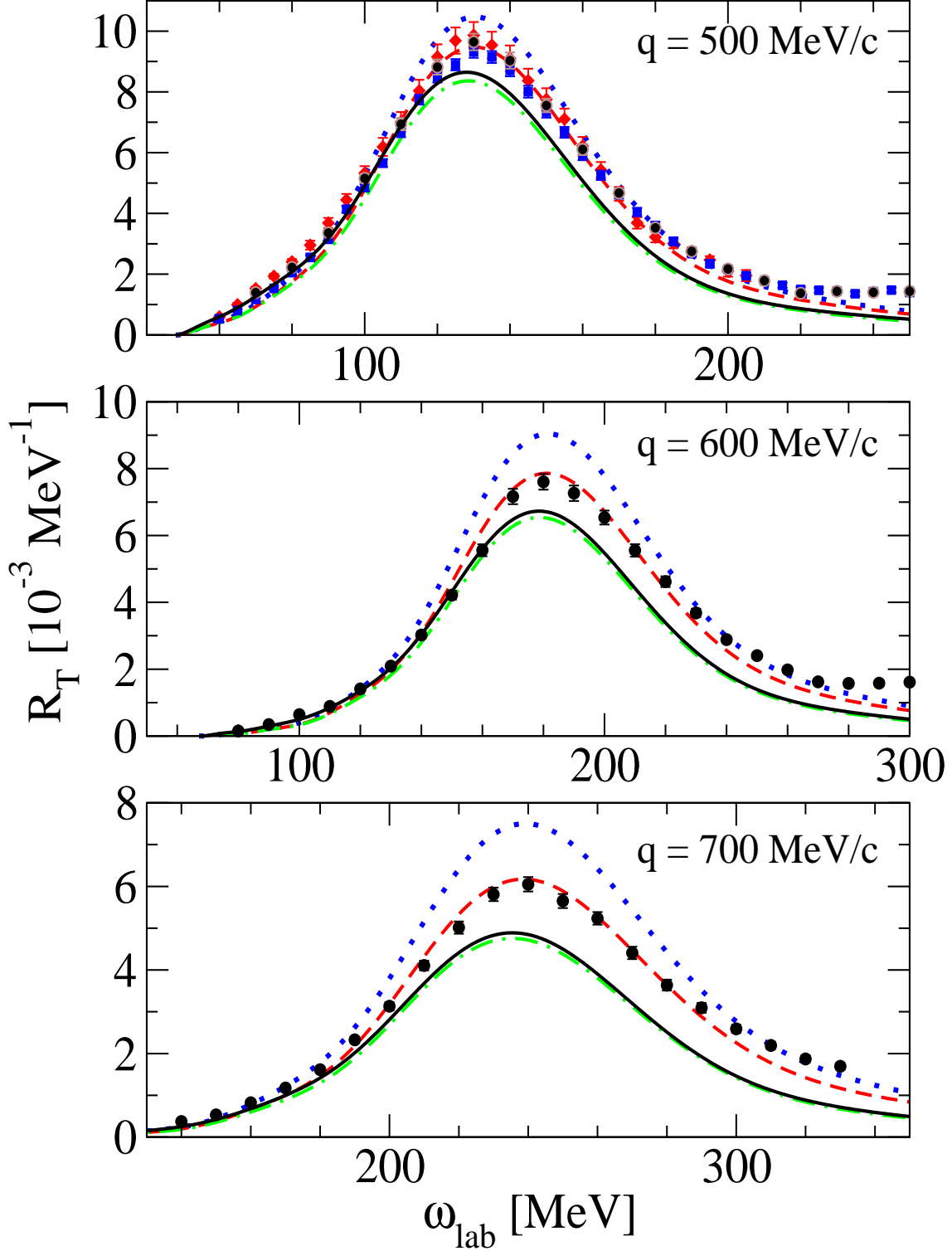


FIG. 4: $R_T(q_{\text{lab}}, \omega_{\text{lab}})$ at $q_{\text{lab}} = 500, 600,$ and $700 \text{ MeV}/c$ from lab frame calculation with use of two-fragment model: non-relativistic one-body current (dotted), relativistic one-body current less j_ω (dashed), relativistic one-body current (dash-dotted), and relativistic one-body current + MEC (solid). Data as in Fig. 1.